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Generic Modeling of N-Level Pulse Width Modulation Voltage Source Inverters and their Control.

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Abstract: A generic and mathematic tool is adapted and used for pulse width modulation inverter modelling and control. The model is developed on the elementary switching cell and finally extended to some N-level topologies and three-phase inverters. Based on the generalized inverses, the solution set of pulse width modulation carrier based scheme is highlighted. The degree of freedom highlighted for the modulation control are linked to the architecture described. A specific example is illustrated and based on a H-bridge 3 level topology.

1. INTRODUCTION

This study deals with model and Pulse Width Modulation - PWM strategy of Voltage Source Inverters - VSI. For many applications, such as utility interface of renewable energy, voltage regulation or variable drive in transportation systems, the use of multilevel VSI is under interest, (Gemmell, 2008). Many studies that demonstrates their ability to match the requirements can be found, for instance in (Rodriguez, 2007). Indeed, it exists a lot of studies where the relationship between topology and application is detailed. Moreover, one can find what is the best PWM to apply for a given topology as illustrated in (Choi, 2014). Nevertheless, it appears to our knowledge that it does not exist any generic way to model every VSI architecture to establish the PWM scheme that will be later applied. The purpose of this study is to establish a generic model of N-level VSI that can be used to generate PWM strategy, whatever N value is. The defined model should match the following criteria:

- to be applied to carrier based PWM;
- to be extended to SVM principles;
- to highlight how to implement a generic modulation scheme;
- to define modulation admissible solutions.

The first part of the paper is devoted to describe an elementary structure and proposes a mathematical model. The second part details how to extend the elementary model to three-phase VSI. The third is about to generalize the given model to N-level VSI as far as the three main N-level VSI topologies are concerned. Then, the determination of control strategies by using a generic mathematical method is demonstrated. Finally, among the whole solution set and for a given 3-Level H-bridge, a specific modulation strategy is highlighted and discussed.

2. FROM AN ELEMENTARY SWITCHING CELL

Let us consider an elementary switching cell. It is composed of 2 semiconductor switches which are stated to be controlled through a gate signal.

2.1 Description

As illustrated in figure. 1, the switching cell is connected to a constant Direct Current - DC voltage source such as $V_{DC} = E$. Each ideal switch, denoted K_j or K'_j, is
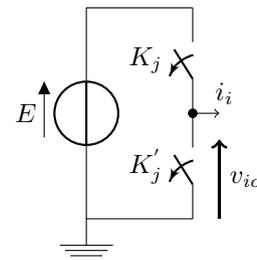
Fig. 1. Elementary switching cell structure.

considered to be assigned an ON/OFF control signal c_j c'_j respectively. To fulfill the electrical sources use conditions, c_j and c'_j are complementary. The figure 2 depicts the leg voltage obtained for a given control signal sequence. It is obvious that K_j is ON when $c_j = 1$. A carrier based PWM defines the control signal, $\in \{0, 1\}$ according to the comparison of a linear carrier signal denoted $h(t)$ and a desired duty cycle named α_i. In symmetric PWM, $h(t)$ is usually taken as a triangle signal, α_i is sampled and hold over a switching period, and finally c_j is computed such as :

$$\begin{cases} c_j = 1 & c'_j = 0 \text{ if } \alpha_i \geq h(t), \\ c_j = 0 & c'_j = 1 \text{ if } \alpha_i \leq h(t) \end{cases} \tag{1}$$

Fig. 2. Example of desired switching sequence.

2.2 Switching cell modelling

Thus, α_i is the desired duty cycle of the output voltage $v_{io}(t)$. The mean value over a switching period $< v_{io} >_{Ts}$ is denoted V_{io}, and expressed as
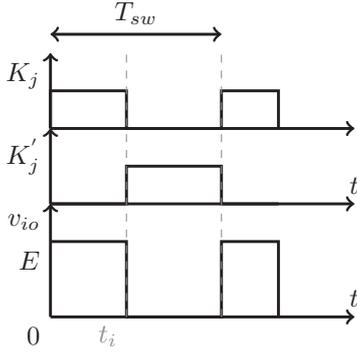
$$< v_{io}(t) >_{T_s} = V_{io} = \frac{1}{T_s} \int_t^{t+T_s} v_{io}(t) dt \qquad (2)$$
$$= E\alpha_i,$$

where

$$\alpha_i = \frac{t_i}{T_s}. \qquad (3)$$

Indeed in such elementary structure the PWM variable, α_i is directly linked to the desired mean voltage value, $V_{i_{ref}}$ so that

$$\alpha_i = \frac{V_{i_{ref}}}{E}. \qquad (4)$$

Finally, in most applications and following the load time constant, a controller can produce $V_{i_{ref}}$ and finally monitor the duty cycle obtained at the leg output. For instance in variable drive applications, for alternating current output, the reference signal is

$$V_{i_{ref}} = V_{i_{max}} \cos(2\pi f_0 t) + \frac{E}{2}, \qquad (5)$$

where f_0 is the fundamental frequency. The duty cycles are expressed as,

$$\alpha_i = \frac{V_{i_{max}}}{E} \cos(2\pi f_0 t) + \frac{1}{2}, \qquad (6)$$

where $\frac{V_{i_{max}}}{E}$ is $\in \left[0, \frac{1}{2}\right]$.

3. TO A THREE-PHASE INVERTER

For variable speed drive applications, the mostly used inverter structure is a three-phase one, as illustrated in figure 3. A previous study, (Vidal, 2013), has demonstrated the potential use of the generalized inverses to generate the PWM solution set. Nevertheless, in the following section the model will be adapted in order to match the generic modeling purpose of this study.

3.1 Three-phase inverter model

The same relationship as (2), is also established for each leg, where $i \in \{a, b, c\}$. They are combined in a column vector $V_{lo} = [V_{ao} \quad V_{bo} \quad V_{co}]^T$ such as

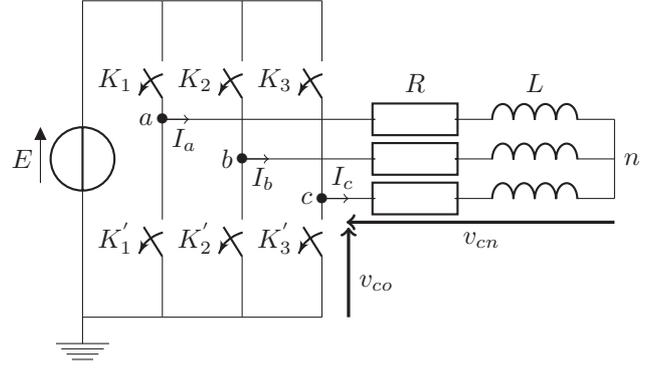$$V_{lo} = E \begin{bmatrix} \alpha_a \\ \alpha_b \\ \alpha_c \end{bmatrix}. \qquad (7)$$

Fig. 3. Three-phase inverter structure.

For generic purpose, the previous expression is wrote in a more complex way. A matrix named S is defined. It is deduced from the series - parallel connection analysis of elementary switching cells within a given leg. As the inverter depicted is the most simplest one - a single elementary cell per leg, $S = 1$. Combined with each part of the duty cycle vector, it produces the line voltages as,

$$V_{io} = E S \alpha_i. \qquad (8)$$

Then the Kroneker product $\otimes$ extends the model to a three-phase inverter.

$$V_{lo} = E (I_3 \otimes S) \begin{bmatrix} \alpha_a \\ \alpha_b \\ \alpha_c \end{bmatrix}. \qquad (9)$$

The Kronecker product performs a weighted duplication of the matrix S on each component of I_3. Obviously in our case this goes to

$$I_3 \otimes S = \begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad (10)$$

As depicted in figure 3, assuming a resistance R, and inductance L as a three-phase and balanced load, the load neutral voltage is given by

$$V_{no} = \frac{1}{3} (V_{ao} + V_{bo} + V_{co}) \qquad (11)$$
$$= \frac{1}{3} [1 \quad 1 \quad 1] V_{lo}.$$

Finally, the three load voltages are expressed in a single vector V_{ln} such as

$$V_{ln} = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} V_{ao} \\ V_{bo} \\ V_{co} \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} V_{no} = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} V_{lo}. \qquad (12)$$

The PWM variable is then expressed when (12) is associated with (7) as

$$V_{ln} = \frac{E}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \alpha. \qquad (13)$$

The matrix M is then defined such as

$$M = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}. \qquad (14)$$

Consequently, a reduced expression of (13) is obtained :

$$V_{ln} = M V_{lo} = E M \alpha. \qquad (15)$$

3.2 Preliminary remark

M stands for three-phase inverters. It means that for other structures, e.g. penta-phase structure etc., another matrix should be defined and combined to (2). Moreover in the previous generalized description, a leg structure representative matrix is defined: S. This will be reused later for multilevel converters.

Unfortunately, while the solution is directly expressed in (4) for a switching cell, the three-phase structure does not allow to obtain α as a function of V_{ln} with a simple matrix inversion. Effectively, M is a singular matrix which does not admit an inverse. The link between experimental and implemented PWM and a generic mathematical solution to express the duty cycle solution set will be demonstrated further.

4. UP TO MULTI-LEVEL INVERTERS

The model defined in the case of an elementary switching cell will be extended to the main three N-level inverter topologies:

- A H-Bridge inverter denoted H-bridge;
- A Neutral Point Clamped inverter denoted NPC;
- A Flying Capacitor inverter, denoted FC.

A generic approach will be considered as the number of level, named N, will not be fixed. As stated in figure 2, an elementary switching cell stands for an output leg voltage with two possible levels: $v_{io} \in \{0, E\}$. Consequently for N-level architecture,

$$N > 2. \tag{16}$$

For the following N-level inverter the average model established previously will be extended. The model for a single leg is developed by defining the corresponding S matrix. The model of the three-phase inverter will be obtained using the same technique given in (10) and (11).

4.1 H-bridge

A H-bridge is a basic structure made of two elementary switching cells. The potential of the middle points of the two cells may take two values each, leading to three levels for the voltage between these points. This voltage is tuned by modulating the duty cycle of each cell. To produce N-level inverters, several H-bridge are linked one to the other. All the H-bridges are connected in series through the middle points of their switching cells. The load is connected between the remaining middle point of the top and bottom of the H-bridge chain, as depicted in 4. Inverters involving such a topology has a odd number of level N

The voltage between the two legs of a basis structure of the leg i of H-bridge inverters is

$$V_{kl} = \frac{2E}{N-1}(\alpha_{i2k+1} - \alpha_{i2k}) = \frac{2E}{N-1}\begin{bmatrix} 1 & -1 \end{bmatrix}\begin{bmatrix} \alpha_{i2k} \\ \alpha_{i2k+1} \end{bmatrix}. \tag{17}$$

where $2k$ is the number of the first switching cell of the H-bridge basis structure considered.
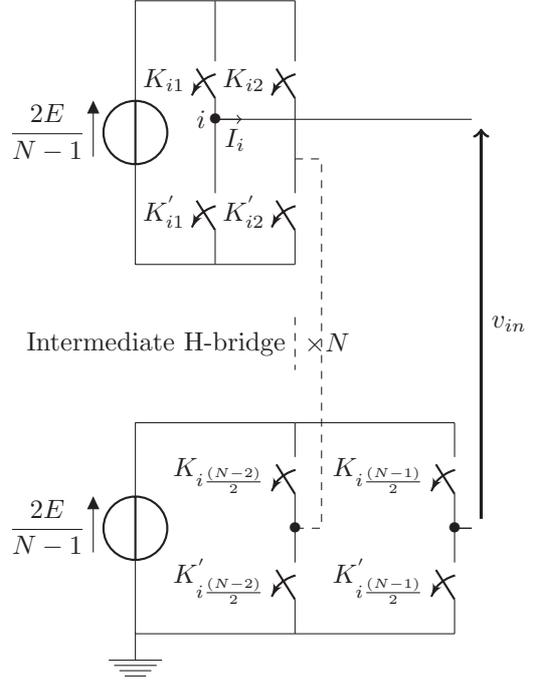

Fig. 4. Isolated leg of a N-level H-bridge inverter.

As soon as all basis structures are serialized, the leg voltage sums up the voltage between the two legs of all the structure of a leg

$$V_{io} = \sum_{j=1}^{\frac{N-1}{2}} \frac{2E}{N-1}(\alpha_{i2k+1} - \alpha_{i2k})$$
$$= \sum_{j=1}^{\frac{N-1}{2}} \frac{2E}{N-1}\begin{bmatrix} 1 & -1 \end{bmatrix}\begin{bmatrix} \alpha_{i2k} \\ \alpha_{i2k+1} \end{bmatrix}. \tag{18}$$

Notice here that the V_{io} voltage make sense only for a leg study because for a multi-phased inverter, the reference potential point has to be determined commonly for all the legs.

Aggregating all the duty cycles α_{ij} of the leg i within a single vector column $\alpha_i = \begin{bmatrix} \alpha_{i1} & \dots & \alpha_{ij} & \dots & \alpha_{iN-1} \end{bmatrix}^T$ leads to

$$V_{io} = \frac{2E}{N-1}\underbrace{\begin{bmatrix} 1 & -1 & \dots & 1 & -1 \end{bmatrix}}_{N-1}\alpha_i. \tag{19}$$

Defining the vector S_H corresponding to the H-bridge topology simplifies expression (19)

$$V_{io} = \frac{2E}{N-1}S_H\alpha_i. \tag{20}$$

Considering distinct reference potential o_i, such a leg model can be duplicated thank to the Kronecker product to get the three phase line voltages

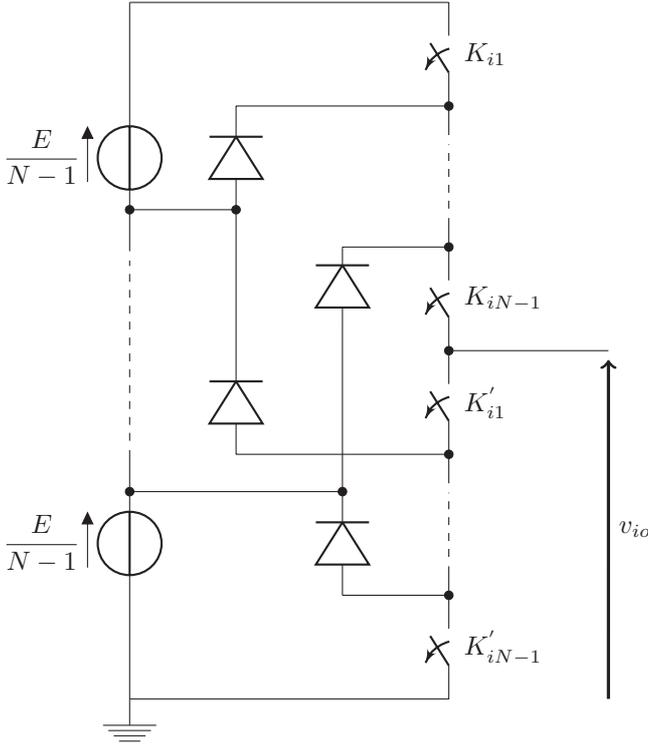$$V_{lo_i} = \frac{2E}{N-1}(I_3 \otimes S_H)\alpha \tag{21}$$

Fig. 5. Isolated leg of a N-level NPC inverter.

where α aggregates the α_i vectors of the 3 legs within a single column vector.

Finally, defining the neutral point N connecting all the o_i together, the model of the line voltages is obtained involving the M matrix introduced in section 3

$$V_{ln} = MV_{lo} = \frac{2E}{N-1}M(I_3 \otimes S_H)\alpha. \qquad (22)$$

The maximal amplitude V_{max} such an inverter modulates is E.

4.2 NPC

The N-levels NPC leg of an inverter is composed of $N-1$ nested switching cells as depicted in figure 5 . Each cell switches a $\frac{E}{N-1}$ voltage source. The network of clamping diodes relays the potential to allow their multiplexing by the switching cells.

The particularity of such a topology is that $c_{ij} = 0$ and $c_{ij+1} = 1$ is not desirable because in such a case, the line voltage of the leg i depends on the sign of its current. As soon as we do not consider the current for the control of the inverter, these configurations are called *undetermined configurations* and avoided. Let us denote α_{ij} the duty cycle of the jth switching cell of the leg i. Defining that all the command order c_{ij} of a given leg i are generated by comparing a single carrier waveform to all the duty cycles α_{ij} of the switching cell j of the leg i, the *undetermined configurations* are avoided by ensuring the following condition

$$\alpha_{ij} \leq \alpha_{ij+1}. \qquad (23)$$

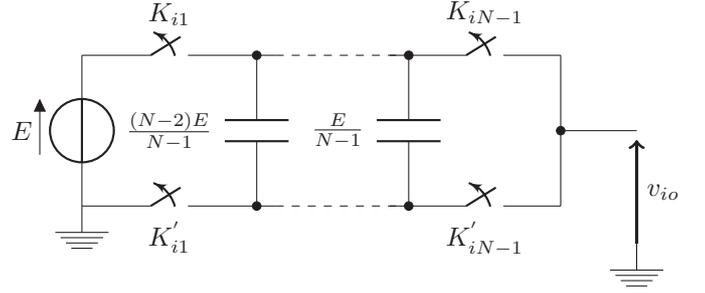With such constrain, the ith leg voltage is

Fig. 6. Isolated leg of a N-level FC inverter.

$$V_{io} = \sum_{j=1}^{N-1} V_{K_j} = \sum_{j=1}^{N-1} \frac{E}{N-1}\alpha_{i_j}. \qquad (24)$$

The α_i vector is defined such as:

$$\alpha_i = \begin{bmatrix} \alpha_{i_1} & \dots & \alpha_{i_j} & \dots & \alpha_{i(N-1)} \end{bmatrix}^T. \qquad (25)$$

This duty cycle vector is combined with the $(1 \times (N-1))$ matrix $S_{NPC} = [1 \ \dots \ 1]$ to simplify (24)

$$V_{io} = \frac{E}{N-1}S_{NPC}\alpha_i \qquad (26)$$

As in (10), the three-phase line voltages are then obtained by duplicating the leg model thanks to the Kronecker product

$$V_{lo} = \frac{E}{N-1}(I_3 \otimes S_{NPC})\alpha \qquad (27)$$

where α aggregates the α_i vectors of the 3 legs within a single column vector.

Finally, the model of the line voltages involves the M matrix as

$$V_{ln} = MV_{lo} = \frac{E}{N-1}M(I_3 \otimes S_{NPC})\alpha. \qquad (28)$$

4.3 FC

The N-level FC is composed of nested elementary switching cells, with per level capacitor as illustrated figure 6 . $p = N - j$ stands for the ratio value of the voltage source E applied to the concerned capacitor. Within the jth switching cell, made of K_{ij} and K'_{ij} and two capacitors, the difference between the two capacity voltage is $\frac{E}{N-1}$. For a given leg, the line voltage V_{io} is

$$V_{io} = \sum_{j=1}^{N-1} V_{K_{ij}} = \sum_{j=1}^{N-1} \frac{E}{N-1}\alpha_{ij}. \qquad (29)$$

Where α_{ij} is the duty cycle defined for K_{ij}. Finally an overall duty cycle column vector α_i is defined such as

$$\alpha_i = \begin{bmatrix} \alpha_{i1} & \dots & \alpha_{ij} & \dots & \alpha_{i(N-1)} \end{bmatrix}^T. \qquad (30)$$

Furthermore, for each leg i, a $(1 \times (N-1))$ matrix S_{FC} is defined as $S_{FC} = [1 \ \dots \ 1]$ to allow simplifying (29) as

$$V_{io} = \frac{E}{N-1}S_{FC}\alpha_i. \qquad (31)$$

Finally an equivalent expression of (7) is obtained using the Kronecker product.

$$V_{lo} = \frac{E}{N-1}\left(I_3 \otimes S_{FC}\right)\alpha. \tag{32}$$

The line voltages are finally computed so that

$$V_{ln} = MV_{lo} = \frac{E}{N-1}M\left(I_3 \otimes S_{FC}\right)\alpha. \tag{33}$$

5. SOLUTION SET EXPRESSION

All of linear systems obtained, (28), (33) and (22), express the line voltages V_{ln} as linear functions of the duty cycles α. The M matrix is also used as it corresponds to three-phase inverters. The use of the Kronecker product $\otimes$, combined with the same sized $(1 \times (N-1))$ S_H, S_{NPC} or S_{FC} matrices, implies that the linear system is singular.

5.1 The Generalized inverse notion

It is well known that each square and non-singular matrix A has a unique inverse, named A^{-1}, which satisfies

$$AA^{-1}A = A. \tag{34}$$

At the beginning of the XXth century, needs for some kind of generalized inverse were pointed out for differential operator (Fredholm, 1903). Then, for every matrix A, a generalized inverses of A, denoted $A^{[1]}$, fulfills

$$AA^{[1]}A = A. \tag{35}$$

Some features of generalized inverses are mentioned in (Ben-Israel, 1974): they are not unique; they exist for every matrices; they have some of the properties of the usual inverse. When A is non-singular, A^{-1} is the generalized inverse of A.

Based on work of Moore, (Moore, 1920), (Ben-Israel, 2002) and Penrose (Penrose, 1955),a particular generalized inverse, the Moore-Penrose inverse or pseudo-inverse of every matrix A, denoted, $A^{\dagger}$, is defined, satisfying the four Penrose properties

$$AA^{\dagger}A = A, \tag{P1}$$
$$A^{\dagger}AA^{\dagger} = A^{\dagger}, \tag{P2}$$
$$(AA^{\dagger})^* = A^{\dagger}A, \tag{P3}$$
$$(A^{\dagger}A)^* = A^{\dagger}A, \tag{P4}$$

where A^* denotes the conjugate transpose of A. The main property of pseudo-inverse is that $A^{\dagger}$ always exists and is unique. It is obvious that $A^{\dagger}$ is a particular generalized inverse of A and equals the usual inverse for non-singular matrices. Its main interest is that stable numeric algorithms have been defined to obtain the pseudo-inverse (Golub, 1996).

The application of generalized inverse theory is to get the solution set of a linear system (Lovass-Nagy, 1978). Let us consider a linear system described by

$$AX = B \tag{36}$$

where A is a $(n \times m)$ matrix and B is a $(n \times p)$ matrix. Then, if (36) is consistent, the solution set of (36) is

$$\left\{A^{[1]}B + (I_m - A^{[1]}A)Y , Y \in \mathbb{R}^{m \times p}\right\}, \tag{37}$$

where $A^{[1]}$ is a generalized inverse of A. It is obvious that Y can be chosen in order to satisfy at the outset, some fixed

constraints. The $A^{[1]}B$ part in (37) stands for the fixed solution, namely obtained with $Y = 0$. It depends on the particular choice for $A^{[1]}$. As $A^{\dagger}$ is a particular generalized inverse, a solution can be generated as

$$X = A^{\dagger}B + (I_m - A^{\dagger}A)Z , \tag{38}$$

where Z is an arbitrary $(m \times p)$ matrix.

The Sylvester theorem states that the number of d.o.f. is related to the size of the kernel of A as,

$$n_{\text{d.o.f.}} = dim\left(ker\left(A\right)\right) . \tag{39}$$

For further simplification of the solution expression, Z will be later developed to best highlight the number of d.o.f.

5.2 Generalized inverse applied to N-level three-phases inverter model

From the problem description highlighted in (22) (28), and (33), it is obvious that the N-level VSI model is a linear system. It implies several singular matrices: M, and $(I_3 \otimes S_H)$, $(I_3 \otimes S_{NPC})$, and $(I_3 \otimes S_{FC})$, where usual inverses can not be defined. Nevertheless, as stated previously a generalized inverse can be obtained. As S_H, S_{NPC}, and S_{FC} have similar size : $(1 \times (N-1))$, the solution set is expressed for the H-bridge structure. A similar resolution may be used for other inverter topologies. A Kronecker product property, highlighted in (Van-Loan, 2000), states that the line voltages can be simplified as follows

$$V_{ln} = \frac{2E}{N-1}M(I_3 \otimes S_H)\alpha = \frac{2E}{N-1}(M \otimes S_H)\alpha. \tag{40}$$

The linear system described in Eq. (40) is consistent and admits an infinity of solution. Indeed,

$$\begin{cases} rank(M \otimes S_H) = rank(M \otimes S_H \quad V_{ln}) \\ rank(M \otimes S_H) < \text{Number of row of } V_{ln}. \end{cases} \tag{41}$$

Effectively, $rank(M \otimes S_H) = rank(M) * rank(S_H) = 2 * 1$, as stated in (Feng, 2011). $(M \otimes S_H)$ is singular thus and its pseudo inverse can be established

$$(M \otimes S_H)^{\dagger} = M^{\dagger} \otimes S_H^{\dagger}, \tag{42}$$

Finally, the solution is expressed as

$$\alpha = \frac{N-1}{2E}(M \otimes S_H)^{\dagger}V_{ref} \\ + \left(I_{3(N-1)} - (M \otimes S_H)^{\dagger}(M \otimes S_H)\right)z, \tag{43}$$

where I_{N-1} is the identity matrix and z an arbitrary $((N-1) \times 1)$ vector. The use of the following property

$$(A \otimes B)(C \otimes D) = AC \otimes BD, \tag{44}$$

allows a simplified expression of (43)

$$\alpha = \frac{N-1}{2E}(M^{\dagger} \otimes S_H^{\dagger})V_{ref} \\ + \left(I_{3(N-1)} - M^{\dagger}M \otimes S_H^{\dagger}S_H\right)z. \tag{45}$$

Indeed, the solution set of every N-level VSI is established. The Sylvester theorem establishes the number of degree of freedom (d.o.f.) associated with the solution established in Eq. (45):

$$n_{d.o.f.} = dim\left(ker(M \otimes S_H)\right) \tag{46}$$

where $ker(A)$ is the kernel of the linear application A. (46) is also expressed as

$$\begin{aligned} n_{d.o.f.} =& \text{Number of row of } (M \otimes S_H) - rank(M \otimes S_H) \\ =& 3(N-1) - 2 = 3N - 5. \end{aligned} \tag{47}$$

Finally, it is concluded that the $3(N-1)$ components of α will be expressed following $3N-5$ d.o.f. To highlight these d.o.f, a maximal rank factorization is used. Firstly, the d.o.f. are grouped into a column vector denoted λ such as

$$\lambda = [\lambda_1, \ldots, \lambda_{3N-5}]^T. \qquad (48)$$

Secondly, two matrices $F_{(3(N-1)\times(3N-5))}$ and $G_{((3N-5)\times 3(N-1))}$ are chosen as

$$\left(I_{3(N-1)} - M^\dagger M \otimes S_H^\dagger S_H\right) = FG. \qquad (49)$$

Then, as $Gz = \lambda$ the solution is

$$\alpha = \frac{N-1}{2E}(M^\dagger \otimes S_H^\dagger)V_{ref} + F\lambda. \qquad (50)$$

The solution expressed is composed of

- a fixed solution denoted α_f:

$$\alpha_f = \frac{N-1}{2E}(M^\dagger \otimes S_H^\dagger)V_{ref}; \qquad (51)$$

- a variable part:

$$F\lambda. \qquad (52)$$

6. SOLUTION OF A 3-LEVEL H-BRIDGE VSI

The model of the 3-level H-bridge VSI is given in (22) where the matrix of the inverter topology $S_H = [1 \; -1]$.

Once the pseudo-inverse of $M(I_{3(N-1)} \otimes S_H)$ computed, the fixed solution (51) associated with the assumption of a three-phase balanced load leads to the duty cycles

$$\begin{cases} \alpha_{f_{i1}} = \dfrac{1}{2}\dfrac{V_{ref_i}}{E} \\ \alpha_{f_{i2}} = -\dfrac{1}{2}\dfrac{V_{ref_i}}{E} \end{cases}. \qquad (53)$$

Considering now the variable part, the maximal rank factorization of $(I_{N-1} - M^\dagger M \otimes S_H^\dagger S_H)$ chosen leads to

$$F = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}. \qquad (54)$$

Such a full rank factorization points out that one d.o.f (λ_4) sets the gap between the duty cycles of the legs.

As soon as $\alpha_{ij} \in [0\,1]$, $\alpha_{i1} - \alpha_{i2} \in [-1\,1]$. Due to the values of the duty cycles (54), the maximal range of the d.o.f λ_4 is

$$-\frac{1}{2}\,max(\alpha_f) \leq \lambda_4 \leq \frac{1}{2} + max(\alpha_f). \qquad (55)$$

The three other d.o.f λ_i $i \in [1,2,3]$ are associated with the leg 1, 2 and 3 of the inverter respectively. These d.o.f λ_i $i \in [1,2,3]$ add the same offset to the corresponding duty cycles α_{ij}. The λ_i $i \in [1,2,3]$ d.o.f range is then defined according to the reference voltage V_{ref_i} and to the common d.o.f λ_4.

$$max(\alpha_f) + \lambda_4 \leq \lambda_i \leq 1 - (max(\alpha_f) + \lambda_4). \qquad (56)$$

7. CONCLUSION

The study demonstrates how to establish a generic model for pulse Width Modulation Voltage Source Inverters. The model aims to be applied to N-level topologies whatever the structure, or N are. The use of the Kronecker product combined with the average model of an elementary switching cell eases to depict a generic N-level linear relationship between the mean output voltage and the duty cycle. Obviously this model is mainly dedicated to carrier based PWM scheme, even if its ability to be adapted for time positioning, when Space Vector Modulation is concerned, should easily be done. Finally the generalized inverse is used to express the generic duty cycle solution set. Among these solutions and for a given example, the use of the detailed theory is driven and illustrated by some simulation results.

REFERENCES

B. Gemmell, J. Dorn, D. Retzmann and D. Soerangr. Prospects of Multilevel VSC Technologies for Power Transmission. *Transmission and Distribution Conference and Exposition*, DOI: 10.1109/TDC.2008.4517192, 21-24 April 2008.

J. Rodriguez, S. Bernet, B. Wu, J.O. Pontt, and S. Kouro. Multilevel Voltage-Source-Converter Topologies for Industrial Medium-Voltage Drives. *IEEE Transactions on Industrial Electronics*, Vol. 54, N. 6:2930-2946, December 2007.

U.-M. Choi, J.-S. Lee, K.-B. Lee. New Modulation Strategy to Balance the Neutral-Point Voltage for Three-Level Neutral-Clamped Inverter Systems. *IEEE Transactions on Energy Conversion*, Vol. 29, N. 1, March 2014.

P-E. Vidal, S. Cailhol, F. Rotella, K. Berkoune, A-M. Llor , M. Fadel. Generalized inverses applied to Pulse Width Modulation for static conversion : a first study. *European Conference on Power electronics and Applications* EPE'13 ECCE Europe, Lille, 2013.

I. Fredholm. Sur une classe d'équations fonctionnelles. *Acta Math.*, volume 27:365-390, 1903.

A. Ben-Israel and T.N.E. Greville. Generalized Inverses : Theory and Applications. *Springer-Verlag*, Second edition : 2003.

E. H. Moore. On the reciprocal of the general matrix. *Proc. Bull. Amer. Math. Soc.* 26:394-395, 1920.

A. Ben-Israel. The Moore of the Moore-Penrose inverse. *Proc. Electron. Jour. Lin. Algebra.* 9:150-157,2002.

R. Penrose. A generalized inverse for matrices. *Proc. Cambridge Philos. Soc.* 51:406-413, 1955.

G.H. Golub and C.F. Van Loan. Matrix computations. *The John Hopkins University Press* Third edition : 1996.

V. Lovass-Nagy, and R. J. Miller, and D. L. Powers", An introduction to the application of the simplest matrix-generalized inverse in systems science. *Proc. IEEE Trans. Circuits and Systems* 25:766-771, 1978.

C.F. Van Loan The ubiquitous Kronecker product. *Journal of Computational and Applied Mathematics* 123: 85-100, 2000.

Y. Feng, J. Wang, J. Lu. Property of Kronecker Product of Matrices over Skew Field. International Conference on *Control, Automation and Systems Engineering (CASE)*, DOI: 10.1109/ICCASE.2011.5997807, 30-31 July 2011.