# Peer-to-Peer Electricity Market Analysis: From Variational to Generalized Nash Equilibrium

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# Peer-to-Peer Electricity Market Analysis: From Variational to Generalized Nash Equilibrium*

Hélène Le Cadre† Paulin Jacquot‡ Cheng Wan§ Clémence Alasseur¶

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**Abstract**

We consider a network of prosumers involved in peer-to-peer energy exchanges, with differentiation price preferences on the trades with their neighbors, and we analyze two market designs: **(i)** a centralized market, used as a benchmark, where a global market operator optimizes the flows (trades) between the nodes, local demand and flexibility activation to maximize the system overall social welfare; **(ii)** a distributed peer-to-peer market design where prosumers in local energy communities optimize selfishly their trades, demand, and flexibility activation. We first characterize the solution of the peer-to-peer market as a Variational Equilibrium and prove that the set of Variational Equilibria coincides with the set of social welfare optimal solutions of market design **(i)**. We give several results that help understanding the structure of the trades at an equilibrium or at the optimum. We characterize the impact of preferences on the network line congestion and renewable energy surplus under both designs. We provide a reduced example for which we give the set of all possible generalized equilibria, which enables to give an approximation of the price of anarchy. We provide a more realistic example which relies on the IEEE 14-bus network, for which we can simulate the trades under different preference prices. Our analysis shows in particular that the preferences have a large impact on the structure of the trades, but that one equilibrium (variational) is optimal. Finally, the learning mechanism needed to reach an equilibrium state in the peer-to-peer market design is discussed together with privacy issues.

Keywords: OR in Energy, Peer-to-Peer Energy Trading, Preferences, Variational Equilibrium, Generalized Nash Equilibrium.

## 1 Introduction

New regulations are restructuring electricity markets in order to build the grid of the future. Instead of a centralized market design where all the operations have been managed by a global central market operator [23; 38; 42], new *decentralized* models emerge. These models involve local energy communities which can trade energy, either by the intermediate of a global market operator [18], or in a peer-to-peer setting [32; 41]. Peer-to-peer energy trading allows flexible energy trades between peers, where, for instance, local prosumers exchange between them energy surplus from multiple small-scale distributed energy resources (DERs) [21; 22].

Significant value is brought to the power system by coordinating local renewable energy source (RES)-based generators and DERs to satisfy the demand of local energy communities, since it decreases the need for investment in conventional generations and transmission networks. Also, thanks to the decreasing feed-in-tariffs, using RES-based generations on site (e.g., at household level, within the

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†VITO/EnergyVille research center, Thor scientific park, Genk, Belgium, email: helene.lecadre@energyville.be

‡EDF R&D, OSIRIS, Inria, CMAP Ecole polytechnique, CNRS, France, email: paulin.jacquot@polytechnique.edu

§EDF R&D, OSIRIS, France, email: cheng.wan.05@polytechnique.org

¶EDF R&D and FIME, Energy Markets research center, France, email: clemence.alasseur@edf.fr, the author's research is part of the ANR project PACMAN (ANR-16-CE05-0027)

microgrid) is more attractive than feeding it into the grid, because of the difference between electricity selling and buying prices [22]. Peer-to-peer energy trading encourages the use of surplus energy within local energy communities, resulting in significant cost savings even for communities with moderate penetration of RES [22].

In practice, the radial structure of the distribution grid calls for hierarchical market designs, involving transmission and distribution network operators [19]. Nevertheless, various degrees of coordination can be envisaged: full coordination organized by a global market operator (transmission system operator), bilateral contract networks [28], fully decentralized market designs allowing peer-to-peer energy trading between the prosumers in a distributed fashion [26; 40] or, still, within and between coalitions of prosumers, called community or hybrid peer-to-peer [25]. A community-based organization involves a community manager which organizes trades among the community and is in charge of the interactions with the rest of the market. A distributed market structure exists when the decentralized elements explicitly share, in a peer-to-peer fashion, local information, resulting in a system in which all the elements may not have access to the same level of information. This information asymmetry might create differences in valuations of the traded resource (e.g., price arbitrage) and result in market imperfections, implying that the prices associated with the bilateral trading of resource allocation between couples of agents do not coincide. This price gap can be interpreted as a bid-ask spread due to a lack of liquidity in the market [30].

Energy exchange between production units and local demand of energy communities are formulated as a symmetric assignment problem. Its solution relies on two main streams in the literature. The first stream deals with matching models which put in relation RES-based generators and consumers by the intermediate of a platform, with various consumers classes and different possible objective functions for the platform operator [21]. The second stream combines multi-agent modeling, as well as classical distributed optimization algorithms which are applied to solve the assignment problem in real-time [26; 27; 40]. Auctions theory can be used, in addition, to schedule the DER commitment in day-ahead.

## 1.1 Matching Models for Peer-to-Peer Energy Trading

In the energy sector, peer-to-peer energy trading is a novel paradigm of power system operation. There, prosumers provide their own energy from solar panels, storage technologies, demand response mechanisms, and they exchange energy with one another in a distributed fashion. Zhang et al. provide in [46] an exhaustive list of projects and trails all around the world, which build on new innovative approaches for peer-to-peer energy trading. A large part of these projects rely on platforms, understood as two-sided markets, that match RES-based generators and consumers according to their preferences and locality aspects (e.g. Piclo in the UK, TransActive Grid in Brooklyn, US, Vandebron in the Netherlands, etc.). In the same vein, cloud-based virtual market places, which deal with excess generation within microgrids, are developed by PeerEnergyCloud and Smart Watts in Germany. Some other projects rely on local community building for investment sharing in batteries, solar PV panels, etc., in exchange for bill reduction or a certain level of autonomy with respect to the global grid (e.g. Yeloha and Mosaic in the US, SonnenCommunity in Germany, etc.). How other components of the platform's design can influence the nature and the preference of the prosumers involved is also studied in the literature. Typical elements of the platform's design are: the impact of pricing mechanism (e.g. setting one common market price versus individual prices per transaction set – for instance through auction design – or per class of prosumers), the platform's objective (e.g. maximizing the social welfare versus maximizing the platform's benefit), the influence of the platform's commission per transaction. For example, in [3], the authors study the impact of the price of the goods exchanged on the level of collaboration and also on the level of ownership among participants. In [8], the impact of different platform's objective functions is analyzed considering a set of heterogeneous renters and owners. Dynamic pricing for operations of the platform based on supply and demand ratio of shared RES-based generation is investigated in [21]. Peer-to-peer organizations are also a way to enable small and flexible actors to enter markets by lowering the entrance barrier [4].

Platform design constitutes an active area of research in the literature on two-sided markets [4; 8]. Three needs are identified for platform deployment. Firstly, it should help buyers and sellers find each other, while taking into account the heterogeneity in their preferences. This requires the platform

to find a trade-off between low entry cost and information retrieval from big, heterogeneous and dynamic information flows. Buyers' and sellers' search can be performed in a *centralized* (e.g. Amazon, Uber), effective *decentralized* (e.g. Airbnb, eBay), or even *fully distributed* (OpenBazaar, Arcade City) manner. Secondly, the platform must set prices that balance demand and supply, and ensure that prices are set competitively in a decentralized fashion. Finally, the platform ought to maintain trust in the market, relying on reputation and feedback mechanisms [9]. Sometimes, supply might be insufficient so that subsidies need to be designed to encourage sharing on the platform [8].

## 1.2 Distributed Optimization Approaches

Computational and communication bottlenecks have largely been alleviated by recent work on distributed and peer-to-peer optimization of large-scale optimal power flow [6; 17; 33]. Mechanisms for the optimization of a common objective function by a decentralized system are known as decomposition-coordination methods [31]. In such methods, a centralized (large-scale) optimization problem is typically split into small-size local optimization problems whose outputs are coordinated dynamically by a central agent (called "master") so that the overall objective of the system becomes aligned (after a certain number of iterations) with the (large-scale) centralized optimization problem outcome. Following this stream, a consensus-based Alternating Direction Method of Multipliers (ADMM) algorithm is implemented in [27; 40; 14] to approximate the optimal solution which maximizes the prosumers social welfare, in a peer-to-peer electricity market. Similar approaches relying on dual decomposition, which iteratively solves the problem in a distributed manner with limited information exchange, were implemented for energy trading between islanded microgrids in [13; 24]. Two main drawbacks of these algorithmic approaches are listed in [40]: first, they do not take into account the strategic behaviors of the prosumers; second, they are computationally limited, which might constitute a blocking point when studying large-scale peer-to-peer networks. The latter issue is overcome in [26] with an improved consensus algorithm.

In addition, these distributed-optimization approaches enable incorporating heterogeneous energy preferences of individual prosumers in network management. The added value of multi-class prosumer energy management is evaluated in [27] for a distribution network that has a "green prosumer", a "philanthropic prosumer" and a "low-income household". Three energy classes are introduced to account for the prosumers' preferences: "green energy", "subsidized energy" and "grid energy". A platform agent is introduced to act as an auctioneer, allowing energy trading between the prosumers and the wholesale electricity market. The platform agent sets the price of each energy class in the distribution network. The tool of receding-horizon model predictive control is used to provide a real-time implementation. Consumer preferences are also introduced in [40] in the form of product differentiation prices. They can either be pushed centrally as dynamic and specific tax payments, or be used to better describe the utility of the consumers who are willing to pay for certain characteristics of trades.

## 1.3 Privacy Issues

From the perspective of information and communication technology (ICT), a fully decentralized market design provides a robust framework since, if one node in a local market is attacked or in case of failures, the whole architecture should remain in place, while information could find other paths to circulate from one point to another, avoiding malicious nodes and corrupted paths.

From an algorithmic point of view, the implementation of a fully distributed market design might be challenging, since it has to deal with far more complex communication mechanisms than the centralized market design. Efficient communication will allow collaboration among prosumers, so that energy produced by one can be utilized by another in the network. Multiple peer-to-peer communication architectures exist in the literature, including structured, unstructured and hybrid ones. They are all based on common standards for the communication network operation, which are measured through latency, throughput, reliability and security [16]. In addition to the large size of the communication problem, privacy issues may also directly impact the market outcome. Indeed, if prosumers are allowed to keep some private information, then they might not have access to the same level of information,

i.e. information asymmetry appears. Since the prosumers make decisions based on the information at their disposal, such asymmetry can introduce bias in the market outcome. To avoid or, at least, to limit bias introduced in the market outcome while guaranteeing the optimum of the social welfare, various algorithms that preserve local market agents' privacy have been discussed in the literature. For example, the algorithms can require the agents to update no more than their dual variables – e.g., local prices [6; 40]. Of course, the efficiency of these algorithms depends on the level of privacy defined by the agents as well as which private information could be inferred from the released values.

## 1.4 Contributions

The peer-to-peer structure adopted in this paper is different from the approaches involving decomposition-coordination methods. Works relying on a decomposition-coordination method require for example to exchange Lagrangian multipliers updated at each iteration of the decentralized clearing [24; 26; 25; 40], that can be used by the coordinator to infer some information about the preferences of the peers. Such an approach has therefore two main drawbacks at the market level: for each market clearing, it requires in general a large number of iterations to reach an optimum - such latency in the clearing price computation might be difficult to allow from the point of view of market operators; it offers limited privacy guarantees as the market operator can infer private information from the peers under repeated interactions. In this paper, we assume that there is no central authority coordinating the exchanges (in quantity, price and information) between the nodes. The mechanism - involving the learning of the private information of the peers - needed to reach an equilibrium state in the peer-to-peer market design is discussed in Section 6 together with privacy issues. Within this framework, strategic communication mechanisms can appear, and nodes have the possibility to self-organize into coalitions or local energy communities, as reviewed in [43]. With such strategic behaviors, the equilibrium of the peer-to-peer market design might not coincide with the social welfare global optimum achieved with full coordination of the nodes by a "master" controlling all the information and decisions, as in [45] where the authors consider a noncooperative game involving storage units.

In this paper, we first characterize the solution of a peer-to-peer electricity market as a Variational Equilibrium, assuming that all the agents have equal valuation of the price associated with the traded resource. We prove that the set of Variational Equilibria coincides with the set of social welfare optima. However, in a fully-distributed setting, it is very unlikely that each couple of agents coordinate on their valuations of the trading price. As a result, imperfections appear in the market, which we capture by considering Generalized Nash Equilibrium solutions as possible outcomes. We characterize analytically the impact of preferences on the network line congestion and energy surplus, both under centralized and peer-to-peer market designs. Our results are illustrated in two test cases (a three node network with arbitrage opportunity and the standard IEEE-14 bus network). We evaluate the loss of efficiency caused by peer-to-peer market imperfections in the three nodes network, with the Price of Anarchy as a performance measure. We also evaluate numerically the impact of the differentiation prices by computing the equilibria of our 14 nodes network under different price configurations. Last, we quantify the impact of privacy on the prosumers' utilities at equilibrium by providing a closed form expression for the privacy cost in the nodes together with an upper-bound, and evaluating it in a three nodes toy network calibrated with real data from the Australian grid.

For ease of reading, we also reference the link between the main results we obtain (summarized through propositions in the course of the text) below:

- Under *centralized market design*, we derive in Proposition 1 analytical expressions for the demand, flexibility activation and net import at the optimum, as linear functions of the nodal prices, at each node of the network.
- By substitution of these results at the optimum in the balancing equation, we observe that there might be energy surplus in the local energy community. We derive in Proposition 2 a necessary condition on technologies and RES generation to avoid energy surplus.
- This condition being not sufficient, we identify conditions on the nodal prices and preferences such that no congestion, and then no energy surplus, appears in the local distribution network in Proposition 3. This proposition is extended by highlighting the link between the occurrence

of strictly positive or negative cycles in the matrix of the preference reciprocity gaps between any couple of nodes and the line congestion in Proposition 5.

- In Proposition 4, we obtain analytical expressions of the nodal prices at the root node and at each node of the distribution grid.
- Under *decentralized market design*, assuming a complete market, we prove in Proposition 6 that the set of Variational Equilibrium, whose definition is recalled in Definition 2 coincides with the set of social welfare optima solutions of the centralized market clearing.
- However, there is no guarantee that there exists a market to determine the price system associated with the bilateral trade reciprocity constraints. In case it does not exist, the peer-to-peer market would become incomplete and the bilateral trade prices between any couple of nodes might diverge. We reformulate the Generalized Nash Equilibrium problem (Generalized Nash Equilibrium being defined in Definition 1) as an optimization problem applying a parameterized variational inequality approach, enabling the computation of Generalized Nash Equilibria via a sampling method and a standard optimization algorithm, in Proposition 7.
- Making the parallel with the centralized market design results, we capture the impact of the capacity of the lines, preferences and structure of the matrix of the preference reciprocity gaps, on line congestion in Propositions 8, 9.
- Finally, the mechanism - involving learning of the private information of the peers - needed to reach an equilibrium state in the peer-to-peer market design is discussed in Section 6 together with privacy issues. A closed form expression of the privacy cost and upper-bound are derived in Propositions 10 and 11.

The paper is organized as follows. In Section 2, we introduce the model of the generalized noncooperative game we consider in this work, and we give our main assumptions. In Section 3, the centralized market design **(i)** is formulated and its solutions characterized. We introduce the peer-to-peer market design **(ii)** in Section 4; its solutions are characterized in terms of Variational Equilibrium and Generalized Nash Equilibria in the presence of market incompleteness. Congestion analysis and performance measure based on the Price of Anarchy are also introduced. These solutions concepts are then applied to two test cases in Section 5: a three node toy network and the IEEE 14-bus network. The impact of privacy is quantified in Section 6, and illustrated on the three node toy network.

# Notation

We summarize the main notations used throughout the paper. Vectors and matrices are denoted by **bold** letters.

**Sets**

| | |
|---|---|
| $\mathcal{N}$ | Set of $N$ nodes, each one of them being made of an agent (prosumer) |
| $\Omega_n$ | Set of neighbors of $n$ |
| $\mathcal{D}_n$ | Agent $n$'s demand set |
| $\mathcal{G}_n$ | Agent $n$'s flexibility activation set |
| $\mathrm{SOL}^{\mathrm{GNEP}}$ | Set of GNE solutions of the peer-to-peer non-cooperative game |

**Variables**

| | |
|---|---|
| $D_n$ | Agent $n$'s demand |
| $G_n$ | Agent $n$'s flexibility activation (micro-CHP, storage facilities, etc.) |
| $\Delta G_n$ | Agent $n$'s random self-generation obtained from RES (solar PV panels) |

| | |
|---|---|
| $q_{mn}$ | Quantity exchanged between $n$ and $m$ in the direction from $m$ to $n$ |
| $Q_n$ | Net import at node $n$ |
| $\zeta_{nm}$ | Bilateral trade price between agent $n$ and $m$ |
| $\lambda_n$ | Nodal price at node $n$ |
| $\xi_{nm}$ | Congestion price between nodes $n$ and $m$ |
| $\underline{\mu}_n, \overline{\mu}_n$ | Demand capacity constraint dual variables at node $n$ |
| $\underline{\nu}_n, \overline{\nu}_n$ | Flexibility activation capacity constraint dual variables at node $n$ |
| $\epsilon^D_{nm}, \epsilon^G_{nm}$ | Agent $n$'s biases in the estimation of $m$ demand and RES-based generation |

**Parameters**

| | |
|---|---|
| $\underline{D}_n$ | Lower-bound on demand capacity |
| $\overline{D}_n$ | Upper-bound on demand capacity |
| $\underline{G}_n$ | Lower-bound on flexibility activation capacity |
| $\overline{G}_n$ | Upper-bound on flexibility activation capacity |
| $D^\star_n$ | Agent $n$'s target demand |
| $\kappa_{nm}$ | Equivalent interconnection capacity between node $n$ and node $m$ |
| $a_n, b_n, d_n$ | Flexibility activation cost parameters |
| $\tilde{a}_n, \tilde{b}_n$ | Consumer utility parameters |
| $c_{mn}$ | Product differentiation price capturing agent $n$'s trading cost preferences |
| $\delta_{nm}$ | Agent $m$ valuation of $\zeta_{nm}$ |
| $\sigma^D_{nm}, \sigma^G_{nm}$ | Standard deviation of agent $n$'s error in demand and RES forecasts |

**Functions**

| | |
|---|---|
| $C_n$ | Agent $n$'s flexibility activation (production) cost |
| $U_n$ | Agent $n$'s usage benefit |
| $\tilde{C}_n$ | Agent $n$'s total trading cost |
| $\Pi_n$ | Agent $n$'s utility function |
| $SW$ | Social welfare |
| $F_n$ | Agent $n$'s forecast |
| $\mathcal{A}$ | Absolute risk aversion |

# 2 Prosumers and Local Communities

In this section, we define the generic framework of agent (prosumer) interactions, and a stylized representation of the underlying (distribution) graph. We formulate the local supply and demand balancing constraint that holds in each node. To formalize the two market designs **(i)** and **(ii)**, we introduce the costs, utility functions, social welfare, private information and main assumptions on which our model relies.

## 2.1 Generic Framework

Let $\mathcal{N}$ be a set of $N$ nodes, each of them representing an agent (prosumer), except the root node 0 which is assumed to contain only conventional generation. The root node belongs to the set $\mathcal{N}$. It can trade energy with any other node in $\mathcal{N}$. Under this assumption, the distribution network is a radial graph, with the root node being the interface between the local energy communities and the transmission network. Figure 1 illustrates such a graph structure.

Let $\Omega_n$ be the set of neighbors of $n$, with the structure of a communication network (local energy community). It does not necessarily reflect the grid constraints. As usual, we assume that $n \in \Omega_n$, for all $n \in \mathcal{N}$. In particular, $\Omega_0 := \mathcal{N} \setminus \{0\}$.

In each node $n$, we introduce $\mathcal{D}_n := \{D_n \in \mathbb{R}_+ | \underline{D}_n \leq D_n \leq \overline{D}_n\}$ as agent $n$'s demand set, with $\underline{D}_n$ and $\overline{D}_n$ being the lower and upper-bounds on demand capacity.

In parallel to the demand-side, we define the self-generation-side by letting $\mathcal{G}_n := \{G_n \in \mathbb{R}_+ | \underline{G}_n \leq G_n \leq \overline{G}_n\}$ be agent $n$'s flexibility activation set, where $\underline{G}_n$ and $\overline{G}_n$ are the lower and upper-bounds

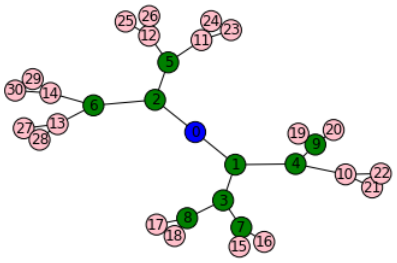



Figure 1: Example of a radial network. The root node at the interface of the distribution and transmission networks, can trade energy with any other node in the distribution network. In the distribution network, prosumer nodes organize in local energy communities, trading energy with neighbors inside their local community.

on flexibility activation capacity.

The decision variables of each prosumer $n$ are her demand $D_n$, flexibility activation $G_n$, and the quantity exchanged between $n$ and $m$ in the direction from $m$ to $n$, $q_{mn}$, for all $m \in \Omega_n \setminus \{n\}$. If $q_{mn} \geq 0$, then $n$ buys $q_{mn}$ from $m$, otherwise ($q_{mn} < 0$) $n$ sells $-q_{mn}$ to $m$. We impose an inequality on the trading reciprocity:

$$q_{mn} + q_{nm} \leq 0 \ , \tag{1}$$

which means that, in the case where $q_{mn} > 0$, the quantity that $n$ buys from $m$ can not be larger than the quantity $q_{nm}$ that $m$ is willing to offer to $n$.

**Remark 2.1.** *In this paper, we model the trade reciprocity constraint as the inequality* (1)*. Other works, as [40], consider a different model with an equality $q_{mn} = -q_{nm}$, meaning that the quantity proposed by agent $n$ should be equal to the quantity the agent $m$ wants. In our model, with* (1)*, those quantity do not necessarily correspond: $n$ can be willing to offer more than the quantity wanted by $m$. If the inequality is strict (for instance, $n$ has too much to offer), then part of her energy is produced in excess. Considering a model with an equality means that energy surplus is not allowed. Another important point is that, although considering an equality constraint is intuitive and does not raise any problem when studying centralized solutions as in [40], the model becomes degenerated when studying GNEs, which is one of the main objective of this paper. Indeed, a profile is a GNE if, by definition, it is optimal for each agent when considering the actions of the other agents fixed. Thus, if we impose an equality in* (1)*, any feasible solution $(\boldsymbol{q}_n)_n$ is a GNE as, for each player $n$, the quantities $(q_{mn})_n$ are fixed by the others. This degenerated situation does not appear when considering an inequality, as each agent $n$ has a degree of freedom in her trade with other agents.*

The difference between the sum of imports and the sum of exports in node $n$ is defined as the net import in that node: $Q_n := \sum_{m \in \Omega_n} q_{mn}$. Furthermore, each line is constrained in capacity. Let $\kappa_{nm} \in [0, +\infty[$ be the equivalent interconnection capacity between node $n$ and node $m$, such that $q_{nm} \leq \kappa_{nm}$, $\kappa_{nm} = \kappa_{mn}$.

RES-based (solar PV panels) self-generation at each node $n$ is modeled as a random variable $\Delta G_n$. Its realization is exogenous to our model.

## 2.2 Local Supply and Demand Balancing

Local supply and demand equilibrium leads to the following equality in each node $n$ in $\mathcal{N}$:

$$\begin{aligned} D_n &= G_n + \Delta G_n + \sum_{m \in \Omega_n} q_{mn}, \\ &= G_n + \Delta G_n + Q_n. \end{aligned} \tag{2}$$

Assuming perfect competition, a Market Operator (MO) maximizes the system social welfare, defined as the sum of the utilities of all the agents in the system, under a set of operational and power-flow constraints, while checking that supply and demand balance each other at each node of the network. In nodal markets, allocative market efficiency can be achieved by setting (locational marginal) nodal price, $\lambda_n$, equal to the dual variable of the local supply and demand balancing equation [39].

In this paper, we consider an *innovative decentralized* market clearing, by comparison with the classical centralized approach, which is used for example in nodal markets. For that purpose, we introduce decentralization in agents' decision-making. This decentralization results firstly from the fact that demands, flexibility activation and trades are defined selfishly by each prosumer in the nodes; secondly from the fact that all the information regarding preferences and private information on target demands and RES-based generations is not available to all the nodes. The decentralized market clearing relies on a peer-to-peer market design, where each agent $n$ computes the Lagrangian variable associated with her (local) supply and demand balancing equation, using the information at her disposal. Dual variables $\lambda_n$ are kept private to agent $n$ and used to compute her bilateral trading prices.

## 2.3 Cost and Usage Benefit Functions

Flexibility activation (production) cost in node $n$ is modeled as a quadratic function of local activated flexibility, using three positive parameters $a_n$, $b_n$ and $d_n$:

$$C_n(G_n) = \frac{1}{2}a_n G_n^2 + b_n G_n + d_n, \tag{3}$$

with $-\frac{b_n}{a_n} \geq \underline{G}_n$.

We make the standard assumption that self-generation occurs at zero marginal cost.

The usage benefit perceived by agent $n$ is modeled as a strictly concave function of node $n$ demand [8], using two positive parameters $\tilde{a}_n$, $\tilde{b}_n$ and a target demand defined exogenously by agent $n$:

$$U_n(D_n) = -\tilde{a}_n(D_n - D_n^\star)^2 + \tilde{b}_n. \tag{4}$$

The quantity $-U_n(.)$ can also be considered as the consumption cost of agent $n$ [40]. As $U_n(.)$ captures a usage benefit, which is interpreted as the comfort perceived by agent $n$, we impose that it always remains non-negative, i.e., $\overline{D}_n - \sqrt{\frac{\tilde{b}_n}{\tilde{a}_n}} \leq D_n^\star \leq \underline{D}_n + \sqrt{\frac{\tilde{b}_n}{\tilde{a}_n}}$. The rational beneath this definition of usage benefit relates to the expected-utility theory [34]: $U_n(D_n)$ represents the perceived comfort resulting from demand $D_n$ satisfaction. The utility function is defined up to a positive affine transformation, and could be multiplied by a positive constant factor without changing the interpretation. The concavity of the function captures the (hyperbolic absolute) risk aversion (HARA) of agent $n$. This is the most general class of utility functions that are often used because of their mathematical tractability. It admits an upward slope for $D_n \leq D_n^\star$ – meaning that larger $D_n$s lead to higher usage benefits up to the maximum usage benefit, and a downward slope for $D_n > D_n^\star$ – meaning that lower $D_n$s are better once the maximum usage benefit has been reached.

Derivating agent $n$ usage benefit with respect to $D_n$, we observe that her maximum usage benefit is reached in $D_n = D_n^\star$ and, in that point, $U_n(D_n^\star) = \tilde{b}_n$. We consider that usage benefit vanishes in case of zero demand, i.e., $U_n(0) = 0 \Leftrightarrow \tilde{a}_n = \frac{\tilde{b}_n}{(D_n^\star)^2}, \forall n \in \mathcal{N}$. This means that under the assumption that zero demand implies zero usage benefit, an explicit relationship exists between the parameter $\tilde{a}_n$, the maximum usage benefit $\tilde{b}_n$, and the target demand $D_n^\star$.

In this work, we consider that prosumers have preferences on the possible trades with their neighbors. The preferences are modeled with (product) differentiation prices [40]: each agent $n$ has a positive price $c_{nm} > 0$ to buy energy from an agent $m$ in her neighborhood $\Omega_n$. The total trading cost function of agent $n$ is denoted by:

$$\tilde{C}_n(\boldsymbol{q}_n) = \sum_{m \in \Omega_n, m \neq n} c_{nm} q_{mn}. \tag{5}$$

Parameters $c_{nm}$ can model taxes to encourage/refrain the development of certain technologies (micro-CHPs, storage, solar panels) in some nodes. They can also capture agents' preferences to pay regarding certain characteristics of trades (RES-based generation, location of the prosumer, transport distance, size of the prosumer, etc.). If $q_{mn} > 0$ (i.e., $n$ buys $q_{mn}$ from $m$) then $n$ has to pay the cost $c_{nm}q_{mn} > 0$. Thus, the higher $c_{nm}$ is, the less interesting it is for $n$ to buy energy from $m$ but the more interesting it is for $n$ to sell energy to $m$. On the other side, if $q_{mn} < 0$, then $n$ sends the energy $-q_{mn}$ and receives the value $-c_{nm}q_{mn} > 0$ even if $m$ does not accept all this energy (i.e. $q_{nm} + q_{mn} < 0$). In that case the energy surplus is bought by an aggregator and sold on the wholesale electricity market in exchange for a compensation intended for the prosumers with energy surpluses. This mechanism will be discussed in detail in Section 3.

### 2.4 Utility Function and Social Welfare

Agent $n$'s utility function is defined as the difference between the usage benefit resulting from the consumption of $D_n$ energy unit and the sum of the flexibility activation and trading costs. Formally, it takes the form:

$$\Pi_n(D_n, G_n, \boldsymbol{q}_n) = U_n(D_n) - C_n(G_n) - \tilde{C}_n(\boldsymbol{q}_n), \tag{6}$$

where $\boldsymbol{q}_n = (q_{mn})_{m\in\Omega_n, m\neq n}$.

We introduce the social welfare as the sum of the utility functions of all the agents in $\mathcal{N}$:

$$SW(\boldsymbol{D}, \boldsymbol{G}, \boldsymbol{q}) = \sum_{n\in\mathcal{N}} \Pi_n(D_n, G_n, \boldsymbol{q}_n). \tag{7}$$

### 2.5 Private Information at the Nodes

There is private information at each node $n$ that can be associated with:

- $\Delta G_n$, local RES-based generation;
- $D_n^\star$, target demand;
- $C_n(.)$, flexibility activation cost function, more specifically parameters $a_n$, $b_n$, $d_n$;
- $U_n(.)$, usage benefit function, more specifically parameters $\tilde{a}_n$, $\tilde{b}_n$;
- $\tilde{C}_n(.)$, bilateral trade cost function, more specifically parameters $(c_{nm})_{m\in\mathcal{N}\setminus\{n\}}$ .

In a centralized market design, all the private information is reported to the Market Operator (MO). This means that the local target demands $(D_n^\star)_{n\in\mathcal{N}}$ and RES-based generations $(\Delta G_n)_{n\in\mathcal{N}}$, are known by the MO. In contrast, in a peer-to-peer market design, $D_n^\star$ and $\Delta G_n$ are known only by agent $n$. In Section 6, the impact of information asymmetry will be formally quantified.

## 3 Centralized Market Design

The centralized market design is inspired from the existing pool-based markets. The global Market Operator (MO) maximizes the social welfare defined in Equation (7) under demand capacity constraints (8a) and flexibility activation capacity constraints (8b) in each node, capacity trading flow constraints for each couple of nodes (8c), trading reciprocity constraint (8d) and supply-demand balancing (8e) in each node:

$$\begin{aligned}
\max_{\boldsymbol{D},\boldsymbol{G},\boldsymbol{q}} \quad & SW(\boldsymbol{D}, \boldsymbol{G}, \boldsymbol{q}), & & \\
s.t. \quad & \underline{D}_n \leq D_n \leq \overline{D}_n, \forall n \in \mathcal{N}, & (\underline{\mu}_n, \overline{\mu}_n) \quad & (8a)\\
& \underline{G}_n \leq G_n \leq \overline{G}_n, \forall n \in \mathcal{N}, & (\underline{\nu}_n, \overline{\nu}_n) \quad & (8b)\\
& q_{mn} \leq \kappa_{mn}, \forall m \in \Omega_n, m \neq n, \forall n \in \mathcal{N}, & (\xi_{nm}) \quad & (8c)\\
& q_{mn} \leq -q_{nm}, \forall m \in \Omega_n, m > n, \forall n \in \mathcal{N}, & (\zeta_{nm}) \quad & (8d)\\
& D_n = G_n + \Delta G_n + Q_n, \forall n \in \mathcal{N}. & (\lambda_n) \quad & (8e)
\end{aligned}$$

**Remark 3.1.** *The constraint* (8d) *is indexed by $m > n$ so that the constraint is considered only once.*

Dual variables are denoted in blue font between brackets at the right of the corresponding constraints. Some of the dual variables can be interpreted as shadow prices, with classical interpretations in the energy economics literature. In the remainder, $\xi_{nm}$ will be interpreted as the shadow price (congestion price) associated with capacity trading flow constraint (8c) between nodes $n$ and $m$; $\zeta_{nm}$ will be understood as the bilateral trade price offered by $n$ to $m$ associated with the trading reciprocity constraint (8d); while $\lambda_n$ is the nodal price associated with the supply and demand balancing constraint in node $n$ (8e), as discussed in Subsection 2.2.

The Social Welfare function is concave as the sum of concave functions defined on a convex feasibility set. Indeed, the feasibility set is obtained as Cartesian product of convex sets. We can compute the Lagrangian function associated with the standard constrained optimization problem of social welfare maximization under constraints (8a)-(8e):

$$\begin{aligned}
\mathcal{L}(\boldsymbol{D}, \boldsymbol{G}, \boldsymbol{Q}, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\xi}, \boldsymbol{\zeta}, \boldsymbol{\lambda}) &= \sum_{n\in\mathcal{N}} \mathcal{L}_n(D_n, G_n, \boldsymbol{q}_n, \boldsymbol{\mu}_n, \boldsymbol{\nu}_n, \boldsymbol{\xi}_n, \boldsymbol{\zeta}_n, \lambda_n) \\
&= -\sum_{n\in\mathcal{N}} \Pi_n(D_n, G_n, \boldsymbol{q}_n) + \sum_{n\in\mathcal{N}} \underline{\mu}_n(\underline{D}_n - D_n) \\
&+ \sum_{n\in\mathcal{N}} \overline{\mu}_n(D_n - \overline{D}_n) + \sum_{n\in\mathcal{N}} \underline{\nu}_n(\underline{G}_n - G_n) + \sum_{n\in\mathcal{N}} \overline{\nu}_n(G_n - \overline{G}_n) \\
&+ \sum_{n\in\mathcal{N}} \sum_{m\in\Omega_n, m\neq n} \xi_{nm}(q_{mn} - \kappa_{mn}) + \sum_{n\in\mathcal{N}} \sum_{m\in\Omega_n, m>n} \zeta_{nm}(q_{mn} + q_{nm}) \\
&+ \sum_{n\in\mathcal{N}} \lambda_n\Big(D_n - G_n - \Delta G_n - Q_n\Big).
\end{aligned} \tag{9}$$

To determine the solution of the centralized market design optimization problem, we compute KKT conditions associated with Lagrangian function (9). Taking the derivative of the Lagrangian function (9) with respect to $D_n$, $G_n$, $q_{mn}$, for all $n$ in $\mathcal{N}$ and all $m \in \Omega_n, m \neq n$, the stationarity conditions write down as follows:

$$\frac{\partial \mathcal{L}}{\partial D_n} = 0 \Leftrightarrow 2\tilde{a}_n(D_n - D_n^\star) - \underline{\mu}_n + \overline{\mu}_n + \lambda_n = 0\,, \quad \forall n \in \mathcal{N}\,, \tag{10a}$$

$$\frac{\partial \mathcal{L}}{\partial G_n} = 0 \Leftrightarrow a_n G_n + b_n - \underline{\nu}_n + \overline{\nu}_n - \lambda_n = 0\,, \quad \forall n \in \mathcal{N}\,, \tag{10b}$$

$$\frac{\partial \mathcal{L}}{\partial q_{mn}} = 0 \Leftrightarrow c_{nm} + \xi_{nm} + \zeta_{nm} - \lambda_n = 0, \quad \forall m \in \Omega_n, m \neq n, \forall n \in \mathcal{N}\,, \tag{10c}$$

where, for $m < n$, $\zeta_{nm}$ is defined as equal to $\zeta_{mn}$.

From Equation (10c), we infer that the nodal price at $n$ can be expressed analytically as the sum of the node product differentiation prices regarding the other prosumers in her neighborhood, the congestion constraint dual variable from Equation (8c) and the bilateral trade prices:

$$\lambda_n = c_{nm} + \xi_{nm} + \zeta_{nm}, \quad \forall m \in \Omega_n, m \neq n, \quad \forall n \in \mathcal{N}\,. \tag{11}$$

The complementarity constraints[1] take the following form:

$$0 \leq \underline{\mu}_n \perp D_n - \underline{D}_n \geq 0, \quad \forall n \in \mathcal{N}\,, \tag{12a}$$

$$0 \leq \overline{\mu}_n \perp \overline{D}_n - D_n \geq 0, \quad \forall n \in \mathcal{N}\,, \tag{12b}$$

$$0 \leq \underline{\nu}_n \perp G_n - \underline{G}_n \geq 0, \quad \forall n \in \mathcal{N}\,, \tag{12c}$$

$$0 \leq \overline{\nu}_n \perp \overline{G}_n - G_n \geq 0, \quad \forall n \in \mathcal{N}\,, \tag{12d}$$

$$0 \leq \xi_{nm} \perp \kappa_{mn} - q_{mn} \geq 0, \quad \forall m \in \Omega_n, m \neq n, \forall n \in \mathcal{N}\,, \tag{12e}$$

$$0 \leq \zeta_{nm} \perp -q_{mn} - q_{nm} \geq 0, \quad \forall m \in \Omega_n, m > n, \forall n \in \mathcal{N}\,. \tag{12f}$$

[1] A complementarity constraint enforces that two variables are complementary to each other, i.e., for two scalar variables $x, y$: $xy = 0$, $x \geq 0$, $y \geq 0$. This condition is often expressed more compactly as: $0 \leq x \perp y \geq 0$.

From Equation (10c), we infer, for any couple of nodes $n \in \mathcal{N}, m \in \Omega_n, m > n$, that:

$$\zeta_{nm} = \lambda_n - c_{nm} - \xi_{nm} = \lambda_m - c_{mn} - \xi_{mn}\,, \tag{13}$$

Subtracting those two last members in (13), we infer that:

$$c_{nm} - c_{mn} + \xi_{nm} - \xi_{mn} = \lambda_n - \lambda_m, \forall m \in \Omega_n, m \neq n, \forall n \in \mathcal{N}\,. \tag{14}$$

From Equations (10a) and (10b), we infer that, at the optimum, for each node $n$:

$$D_n = D_n^\star - \frac{1}{2\tilde{a}_n}\Big(\lambda_n + (\overline{\mu}_n - \underline{\mu}_n)\Big)\,, \tag{15}$$

$$G_n = -\frac{b_n}{a_n} + \frac{1}{a_n}\Big(\lambda_n - (\overline{\nu}_n - \underline{\nu}_n)\Big)\,. \tag{16}$$

Substituting Equations (15) and (16) in the local demand and supply balance Equation (8e), we infer that the net import at node $n$ can be expressed as a linear function of the nodal price:

$$Q_n = \Big(D_n^\star - \frac{1}{2\tilde{a}_n}(\overline{\mu}_n - \underline{\mu}_n) + \frac{b_n}{a_n} + \frac{1}{a_n}(\overline{\nu}_n - \underline{\nu}_n)\Big) - \left(\frac{1}{2\tilde{a}_n} + \frac{1}{a_n}\right)\lambda_n - \Delta G_n\,. \tag{17}$$

The results are summarized in the following proposition.

**Proposition 1.** *In the quadratic model defined by equations (3-6), the optimal demands, flexibility activations and net imports at each node n can be expressed as linear functions of the nodal price at that node, given by Equations* (15)*,* (16)*, and* (17)*.*

The total sum of the net imports at all nodes should be negative or null, i.e., $\sum_{n\in\mathcal{N}} Q_n \leq 0$. From the supply-demand balancing (8e), this is equivalent to $\sum_{n\in\mathcal{N}}(D_n - G_n) \leq \sum_{n\in\mathcal{N}} \Delta G_n$. A strict inequality would lead to a situation of energy surplus, i.e., the total energy generation is in excess compared to the total demand of the prosumers.

To deal with that energy surplus, we assume that a feed-in-tariff or feed-in-premium applies. The root node (node 0) who makes the link between the transmission and the distribution network could be a good candidate to manage the excess of generation. Indeed, she should be able to inject it in the transmission network. However, due to the radial structure of our network, all the distribution nodes are not directly connected to the root node. Relying on (26), this means that the bilateral trading prices between 0 and a node $n \in \mathcal{N}\backslash\{0\}$ cannot be the same for all the nodes in the distribution network because the trade price also depends on $c_{0n}$ and $\xi_{0n}$ which captures the congestion state of the path between 0 and $n$. As a result, node 0 cannot apply a feed-in-tariff in case of energy surplus. However, it might be possible to introduce another agent, such as an aggregator, having a very large demand and no generation capacity, that would be connected to any nodes of the distribution network. This aggregator would take care of the forecasting and bidding of the renewable generation and self-generation surpluses, while paying to prosumers the amount of energy they actually produced in excess at a price defined in advance (for example, the feed-in-tariff price or a premium). This compensation mechanism for the agents is similar to the *purchase obligations* or feed-in tariffs mechanism for renewable energy sources set in the European Union [10].

Constraints on the technologies could also be applied at the prosumer level, to limit the RES-based generation and to choose large enough demand capacities. Note that the sizing of the prosumers' capacities and RES-based generation possible clipping strategies are out of the scope of this work. This result is formalized in the proposition below.

**Proposition 2.** *A necessary condition for no energy surplus is that there is at least one prosumer n in $\mathcal{N}$ whose capacities and RES-based generation are such that $\overline{D}_n - \underline{G}_n \geq \Delta G_n$.*

*Proof.* By combining (8a) and (8b), we obtain $\underline{D}_n - \overline{G}_n \leq D_n - G_n \leq \overline{D}_n - \underline{G}_n$. Subtracting $\Delta G_n$ in each part of the inequalities and applying (8e), we get $\underline{D}_n - \overline{G}_n - \Delta G_n \leq Q_n \leq \overline{D}_n - \underline{G}_n - \Delta G_n$. Then, $\overline{D}_n - \underline{G}_n - \Delta G_n < 0$ implies that $Q_n < 0$, i.e., there are more exports than imports from $n$. If $\overline{D}_n - \underline{G}_n - \Delta G_n < 0$, for all $n \in \mathcal{N}$ then, $\sum_{n\in\mathcal{N}} Q_n < 0$. No energy surplus is equivalent to $\sum_{n\in\mathcal{N}} Q_n = 0$. For this equality to hold, it is necessary that there exists at least one prosumer $n$ in $\mathcal{N}$ such that $\overline{D}_n - \underline{G}_n \geq \Delta G_n$. □

In practice, this means that the prosumer should size their capacities such that the difference between their upper-bound on demand capacity and lower-bound on flexibility activation capacity is larger than their RES-based generation. However, the previous proposition is a necessary condition.

The following proposition gives a sufficient condition on the locational marginal prices $(\lambda_n)_n$ for having no energy surplus at optimality:

**Proposition 3.** *At the optimum, if for any prosumer node $m$, for any node $n_0$ such that there exists a non congested path $(n_0, n_1, \dots, n_p = m)$ from $n_0$ to $m$ such that $\lambda_m > c_{n_0,m_0} + \sum_{k=0}^{p-1} c_{n_k} - c_{n_{k+1}}$, where $m_0 \in \Omega_{n_0}$, then there is no energy surplus at $n_0$ in the trade with $m_0$ (that is: $q_{n_0,m_0} + q_{m_0,n_0} = 0$). In particular:*

- *if users have symmetric preferences $c_{nm} = c_{mn}$, there is no congestion and there exists $m$ such that $\lambda_m > c_{n_0,m_0}$, then there is no energy surplus at $n_0$ in the trade with $m_0$ ;*
- *for $m = n_0$, if $\lambda_{n_0} > c_{n_0,m_0}$, then there is no energy surplus at $n_0$ in the trade with $m_0$, which can be directly inferred by the complementarity condition* (12f) *and* (10c).

*Proof.* Suppose on the contrary that there is some energy surplus at $n_0$: there exists some $m_0$ such that $q_{n_0,m_0} + q_{m_0,n_0} < 0$ and $q_{m_0,n_0} < 0$ (i.e. $n_0$ rejects energy). In the case where $G_m > \underline{G}_m$, Consider the infinitesimal transformation to the trades and production:

$$\begin{aligned} q_{n_i,n_{i+1}} &\leftarrow q_{n_i,n_{i+1}} + \varepsilon, \qquad & q_{n_{i+1},n_i} &\leftarrow q_{n_{i+1},n_i} - \varepsilon, \quad \forall i \in \{0, \dots, p-1\}, \\ q_{m_0,n_0} &\leftarrow q_{m_0,n_0} + \varepsilon \ , & G_m &\leftarrow G_m - \varepsilon \ . \end{aligned} \tag{18}$$

Then, for $\varepsilon$ small enough, all constraints are still satisfied and the variations in SW has the same sign as:

$$\lambda_m - c_{n_0,m_0} + \sum_{i=0}^{p-1} (c_{n_i,n_{i+1}} - c_{n_{i+1},n_i}) > 0 \ .$$

Hence, we can strictly increase SW , which contradicts the optimality. In the case where $G_m = \underline{G}_m$, then we necessarily have $D_m < \overline{D}_m$ (otherwise $\lambda_n = -2\tilde{a}_n(\overline{D}_n - D_n^*) - \overline{\mu}_n < 0$ which is impossible from (10c)), and we can strictly increase $D_m$ instead of decreasing $G_m$ in (18), leading to the same contradiction. □

**Remark 3.2.** *From the previous proposition, we see that even if there is no excess in the renewable production, i.e. $\sum_n \Delta G_n < \sum_n D_n^*$, we can still have some energy surplus if the trades preference prices $(c_{nm})_{n,m}$ are large enough.*

**Remark 3.3.** *In some rare cases, energy surpluses might lead to a strictly positive social welfare and create some missing money issues that can be interpreted as being caused by the irrational behaviors of the consumers. As discussed earlier, a first possibility to deal with these issues would be to introduce an external aggregator who would compensate the consumers for the energy surpluses. Another possibility would be to formally integrate the subjective perceptions of the consumers in the noncooperative game, relying on the broader notion of prospect theory for prosumers' centric energy trading [5]. This extension could be an avenue for further work.*

Hence, assuming no energy surplus, the total sum of the net imports in all nodes should vanish, which implies the following relation:

$$\begin{aligned} &\sum_{n \in \mathcal{N}} Q_n = 0 \\ &\Leftrightarrow \sum_{n \in \mathcal{N}} \left( \frac{1}{2\tilde{a}_n} + \frac{1}{a_n} \right) \lambda_n = \sum_{n \in \mathcal{N}} \left( D_n^\star - \frac{1}{2\tilde{a}_n} (\overline{\mu}_n - \underline{\mu}_n) + \frac{b_n}{a_n} + \frac{1}{a_n} (\overline{\nu}_n - \underline{\nu}_n) - \Delta G_n \right), \end{aligned} \tag{19}$$

using Equation (17).

From Equation (14), we infer that the nodal price at node $n$ is a linear function of the nodal price at the root node, product differentiation and congestion prices with all the other nodes in $\mathcal{N}$:

$$\lambda_n = c_{n0} - c_{0n} + \xi_{n0} - \xi_{0n} + \lambda_0, \quad \forall n \in \Omega_0 \, . \tag{20}$$

Substituting Equation (20) in Equation (19), we infer the closed form expression of the nodal price at the root node:

$$\lambda_0 \sum_{n\in\mathcal{N}} \left(\frac{1}{2\tilde{a}_n}+\frac{1}{a_n}\right) = \sum_{n\in\mathcal{N}} \left(D_n^\star - \frac{1}{2\tilde{a}_n}(\overline{\mu}_n - \underline{\mu}_n) + \frac{b_n}{a_n} + \frac{1}{a_n}(\overline{\nu}_n - \underline{\nu}_n) - \Delta G_n\right) - \sum_{n\in\Omega_0} \left(\frac{1}{2\tilde{a}_n}+\frac{1}{a_n}\right)\left(c_{n0} - c_{0n} + \xi_{n0} - \xi_{0n}\right) . \tag{21}$$

From Equations (20) and (21), assuming that $(c_{n0})_n$, $(c_{0n})_n$, $(\xi_{n0})_n$, $(\xi_{0n})_n$ are known, the MO can iteratively compute all the $(\lambda_n)_{n\in\mathcal{N}}$. Note that $\underline{\boldsymbol{\mu}}, \overline{\boldsymbol{\mu}}$ and $\underline{\boldsymbol{\nu}}, \overline{\boldsymbol{\nu}}$ are determined by the MO when optimizing $\boldsymbol{D}$ and $\boldsymbol{G}$. Once computed by the MO, the nodal prices are announced to all the agents $n \in \mathcal{N}$. Then, to determine the optimal bilateral trading prices, each agent $n$ has to refer to Equation (13), which gives the bilateral trading prices as linear functions of the nodal price and congestion price. The results are summarized in the following proposition:

**Proposition 4.** *Assuming no energy surplus and knowing $(c_{n0})_n$, $(c_{0n})_n$, $(\xi_{n0})_n$, $(\xi_{0n})_n$, the MO computes the nodal price at the root node by Equation* (21)*. The nodal prices in all the other nodes of the distribution network can be inferred from $\lambda_0$ according to Equation* (20)*. Then, for each node $n \in \mathcal{N}$, bilateral trading prices can be computed for any node $m \in \Omega_n, n \neq m$ by Equation* (13) *provided congestion price $(\xi_{nm})_{m>n, m\in\Omega_n}$ is known*[2]*.*

If all agents reveal their product differentiation prices $(c_{n0})_n$ to the MO and all the congestion prices $(\xi_{n0})_n$, $(\xi_{0n})_n$ in the lines involving the root node are known (or rationally anticipated), then the MO can compute all the nodal prices $(\lambda_n)_{n\in\mathcal{N}}$ from $\lambda_0$.

We now want to make the link between the market and the state of the distribution grid. In the following proposition, we show that the distribution grid lines become congested if there are "cycles" in the preferences as explained below.

**Proposition 5.** *Suppose that the matrix $\tilde{C} := (c_{nm} - c_{mn})_{nm}$ has a strictly negative cycle of length $k > 2$, i.e. there is a sequence of distinct indices $(n_i)_{1\leq i\leq k}$ such that $\sum_{1\leq i\leq k} \tilde{C}_{n_i,n_{i+1}} < 0$, where $n_{k+1} := n_1$. Then, at an optimal centralized solution, there is a trade opposed to the cycle made at full capacity, i.e. there exists $i \in \{1,\dots,k\}$ such that $q_{n_{i+1},n_i} = \kappa_{n_{i+1},n_i}$.*

*Symmetrically, if there is a strictly positive cycle $(n_i)_{1\leq i\leq k}$ such that $\sum_{1\leq i\leq k} \tilde{C}_{n_i,n_{i+1}} > 0$, then at an optimal centralized solution, there is a trade in the direction of the cycle made at full capacity, i.e. there exists $i \in \{1,\dots,k\}$ such that $q_{n_i,n_{i+1}} = \kappa_{n_i,n_{i+1}}$.*

*Proof of Proposition 5.* We prove the first part of the proposition as the second is symmetric.

Consider the trades $(q_{nm})_{nm}$ at an optimal solution and suppose on the contrary that there is $\epsilon > 0$ such that, for each $i \in \{1,\dots,k\}$, we have $q_{n_{i+1},n_i} \leq \kappa_{n_{i+1},n_i} - \epsilon$.

Then consider the same solution with trades $(\tilde{q}_{nm})_{nm}$ defined as follows: for each $i \in \{1,\dots,k\}$, let $\tilde{q}_{n_{i+1},n_i} := q_{n_{i+1},n_i} + \epsilon$ and $\tilde{q}_{n_i,n_{i+1}} := q_{n_i,n_{i+1}} - \epsilon$, while $\tilde{q}_{nm} = q_{nm}$ otherwise. Then all constraints are still feasible because, for each $i$, $\sum_{m\neq n_i} q_{m,n_i} = Q_n - \epsilon + \epsilon = Q_n$. Besides, by definition of $\tilde{q}$, we still have $\tilde{q}_{mn} = -\tilde{q}_{nm}$ for any $m > n$. Moreover, if we denote by SW the social welfare of the previous solution $(q_{nm})_{nm}$, the social welfare of this new solution is:

$$\begin{aligned}
\widetilde{\mathrm{SW}} &= \mathrm{SW} + \sum_n \sum_{m\neq n} c_{nm}(q_{mn} - \tilde{q}_{mn}) \\
&= \mathrm{SW} + \sum_{1\leq i\leq k} \left(c_{n_i,n_{i+1}}(q_{n_{i+1},n_i} - \tilde{q}_{n_{i+1},n_i}) + c_{n_i,n_{i-1}}(q_{n_{i-1},n_i} - \tilde{q}_{n_{i-1},n_i})\right) \\
&= \mathrm{SW} + \sum_{1\leq i\leq k} \epsilon\left(c_{n_i,n_{i-1}} - c_{n_i,n_{i+1}}\right) = \mathrm{SW} - \epsilon \sum_{1\leq i\leq k} \tilde{C}_{n_i,n_{i+1}} > \mathrm{SW} ,
\end{aligned}$$

[2]Two assumptions can be made on the determination of the congestion prices: first, they are determined exogenously while checking the complementarity constraint (12e); second, they are determined through a market for (distribution) capacity line transmission. This second assumption enables the MO to complete the market. It will be discussed later in the paper.

which contradicts the fact that SW is maximal. □

**Remark 3.4.** *The property stated by Proposition 5 shows that the lines become congested if there is a strictly positive or negative cycle in the matrix $\tilde{C}$. In practice, a central MO should try to avoid such an outcome, since the congested lines are unavailable in case of unplanned real need (outages, peak demand). The existence of a positive cycle in $\tilde{C}$ means that there is an "arbitrage" opportunity in the network. In other words, one can strictly increase the social welfare by doing an exchange of power quantities. We can make the assumption that this kind of opportunities do not exist in practice, since they should vanish quickly in a liquid market.*

*From the point of view from mechanism design, we might also prevent this kind of cycling behavior by adding a transaction fee (e.g. $\tau \times |q_{mn}|$ with $\tau > 0$) on the trades, regardless they are positive or negative.*

Section 5.1 shows an example where there is a cycling trade that is purely due to arbitrage opportunities because of the preferences.

# 4 Peer-to-Peer Market Design

The centralized market design is used, in this section, as a benchmark against which we test the performance of a fully distributed approach relying on peer-to-peer energy trading. We first start by defining in Subsection 4.1 the solution concepts that we will use to analyze the outcome of the fully distributed market design. Then, various results are introduced to characterize the relations between these sets of solutions. Congestion issues and performance measures are discussed in Subsection 4.2.

## 4.1 General Nash Equilibrium and Variational Equilibrium

In the peer-to-peer setting, each agent $n \in \mathcal{N}$ determines, by herself, her demand, flexibility activation and bilateral trades with other agents in her local energy community under constraints on demand, flexibility activation and transmission capacity so as to maximize her utility. A trade between two agents in a local energy community supposes that these two have decided on a certain quantity to be sent from one side and received by the other side. Therefore, there must be an "agreement" or trade constraint between each pair of agents in a local community, which couples their respective decisions. As a result, although the utility of a prosumer depends only on her own decisions, some of these decisions, such as the quantity she agrees to trade with all the other prosumers in her neighborhood, have an impact on the set of feasible actions of her neighbors. In the same way, her feasible actions are determined by the actions of her neighbors.

Formally, each agent in node $n \in \mathcal{N}$ solves the following optimization problem:

$$\max_{D_n, G_n, (q_{mn})_{m\in\Omega_n, m\neq n}} \quad \Pi_n\Big(D_n, G_n, \boldsymbol{q}_n\Big), \tag{22a}$$

$$s.t. \quad \underline{D}_n \leq D_n \leq \overline{D}_n, \qquad (\underline{\mu}_n, \overline{\mu}_n) \tag{22b}$$

$$\underline{G}_n \leq G_n \leq \overline{G}_n, \qquad (\underline{\nu}_n, \overline{\nu}_n) \tag{22c}$$

$$q_{mn} \leq \kappa_{mn}, \forall m \in \Omega_n, m \neq n, \qquad (\xi_{nm}) \tag{22d}$$

$$q_{mn} \leq -q_{nm}, \forall m \in \Omega_n, m \neq n, \qquad (\zeta_{nm}) \tag{22e}$$

$$D_n = G_n + \Delta G_n + Q_n, \qquad (\lambda_n) \tag{22f}$$

where $\boldsymbol{q}_n = (q_{mn})_{m\in\Omega_n}$ are the trading decisions of agent $n$.

Hence, the peer-to-peer setting leads to $N$ optimization problems, one for each agent $n \in \mathcal{N}$, with *individual constraints* on demand (22b), flexibility activation (22c), trade capacity (22d), supply and demand balancing (22f); as well as *coupling constraints* (22e) that ensure the reciprocity of the trades.

The Lagrangian function associated with optimization problem (22a) under constraints (22b)-(22f), writes down as $\mathcal{L}_n$ defined in equation (9).

For each agent $n$, the first order stationarity conditions are the same as (10a)-(10c), and the complementarity constraints are the same as (12a)-(12f), except that (12f) is indexed by all $(m, n)$

with $m \neq n$ and that $\zeta_{nm}$ is not necessarily equal to $\zeta_{mn}$. Let this condition system be denoted by $KKT_n$ for each $n \in \mathcal{N}$.

As the problem given by (22) is convex, $KKT_n$ are necessary and sufficient conditions for a vector $(D_n, G_n, \boldsymbol{q}_n)$ to be an optimal solution of (22).

**Remark 4.1.** *In Equation* (22), $\Pi_n$ *depends on the variables of player* $n$ *only, and not on the variables of the other players. A consequence is that the social welfare function is decomposable:* $SW(D, G, \boldsymbol{q}) = \sum_n \Pi_n\big(D_n, G_n, \boldsymbol{q}_n\big)$. *Therefore, without the existence of the coupling transaction constraint* (22e), *the minimization of SW is equivalent to the minimization of each individual objective function* $\Pi_n$. *We will see that this equivalence between social optimizer and equilibria also happens for the so-called Variational Equilibria.*

A common adopted equilibrium notion that generalizes Nash Equilibria in the presence of coupling constraints is the notion of Generalized Nash Equilibrium (GNE) [15]

**Definition 1** (Generalized Nash Equilibrium [7])**.** *A Generalized Nash Equilibrium of the game defined by the maximization problems* (22) *with coupling constraints, is a vector* $(D_n, G_n, \boldsymbol{q}_n)_n$ *that solves the maximization problems* (22) *or, equivalently, a vector* $(D_n, G_n, \boldsymbol{q}_n)_n$ *such that* $(D_n, G_n, \boldsymbol{q}_n)$ *solves the system* $KKT_n$ *for each* $n$.

The constraint (22e), $q_{mn} \leq -q_{nm}$, written both in the problem of $n$ and in that of $m \neq n$ leads to the same inequality, but is associated to the multiplier $\zeta_{nm}$ in the problem of $n$ and to $\zeta_{mn}$ in the problem of $m$. In this paper, we consider two scenarios for the allocation of the resources represented in these coupling constraints:

**Scenario (i)** A market allocates the resources associated with (22e) through a single price system, therefore leading to the determination of one price for each constraint: $\zeta_{nm} = \zeta_{mn}$.

**Scenario (ii)** There does not exist any market to determine the price system associated with (22e). Hence, two prosumers $n$, $m$ might attribute different evaluations of the same transaction $q_{mn} \leq -q_{nm}$ or, equivalently, the same dual variables to the trade constraint (22e) between $n$ and $m$. This *can* lead to different prices $\zeta_{nm} \neq \zeta_{mn}$ for agents $n$ and $m$.

The two scenarios have implications on the market organization. Let us discuss them one after another.

**Scenario (i)** corresponds to a complete market, where the common resources are shared in an efficient way. It suggests that all constraints are traded at a single price, which reflects the common valuation of each product from all agents. The associated solution concept is that of Variational Equilibrium [15], a refinement of Generalized Nash Equilibrium, where we ask for more symmetry: the Lagrangian multipliers associated to a constraint shared by several players have to be equal from one player to another. Note that a natural way to complete the market would be to introduce a market for (distribution) capacity line transmission, enabling the determination of congestion prices $(\xi_{nm})_{n,m}$. A similar idea was proposed by Oggioni et al. in [30] at the transmission level for a subproblem of market coupling.

**Definition 2** (Variational Equilibrium [7])**.** *A Variational Equilibrium of the game defined by* (22) *is a solution* $(D_n, G_n, \boldsymbol{q}_n)_n$ *that solves the maximization problems* (22) *or, equivalently, a vector* $(D_n, G_n, \boldsymbol{q}_n)_n$ *such that* $(D_n, G_n, \boldsymbol{q}_n)$ *solves the system* $KKT_n$ *for each* $n$ *and, in addition, such that the Lagrangian multipliers associated to the coupling constraints* (22e) *are equal, i.e.:*

$$\zeta_{nm} = \zeta_{mn}, \ \forall n \in \mathcal{N}, \forall m \in \Omega_n, m \neq n \ . \tag{23}$$

The term "variational" refers to the variational inequality problem associated to such an equilibrium: indeed, if we define the set of admissible solutions as:

$$\mathcal{R} := \{\boldsymbol{x} = (D_n, G_n, \boldsymbol{q}_n)_n \ | (22\text{b}) - (22\text{f}) \text{ hold for each } n \in \mathcal{N}\} \ . \tag{24}$$

then $\hat{\boldsymbol{x}} \in \mathcal{R}$ is a Variational Equilibrium if, and only if, it is a solution of (cf. [7]):

$$\left\langle \sum_n \nabla \Pi_n(\hat{\boldsymbol{x}}_n),\ \boldsymbol{x} - \hat{\boldsymbol{x}} \right\rangle \leq 0,\ \forall \boldsymbol{x} \in \mathcal{R}\ . \tag{25}$$

A remarkable fact is that Variational Equilibria exist under mild conditions [15; 37], even if the additional equality conditions on the multipliers seem restrictive.

We can observe, following Remark 4.1, that Variational Equilibria are defined by exactly the same KKT system than the social welfare maximizer (or equivalently as the solution of the same variational inequality (25)). Therefore, we obtain the following result:

**Proposition 6.** *The set of Variational Equilibria (such that $\zeta_{nm} = \zeta_{mn}$ for all $n \in \mathcal{N}$ and all $m \neq n \in \Omega_n$) coincides with the set of social welfare optima.*

**Scenario (ii)** corresponds to the case of partial price coordination or a completely missing market for some products. Agents with different willingness to pay for a certain resource face a price gap due to the lack of arbitrage opportunities that prevent price convergence. This imperfect coordination among agents relates to the notion of Generalized Nash Equilibrium (GNE), where nothing prevents the multipliers $\zeta_{nm}$ and $\zeta_{mn}$ to be different.

**Remark 4.2.** *A particular class of GNE is called restricted GNE [11]. It assumes that the dual variables of the shared constraint* (22e) *belongs to a non empty cone of $\mathbb{R}^{N(N-1)}$.*

*A particular class of restricted GNE is called normalized equilibrium, introduced by Rosen [37]. There, the dual variables of the shared constraint* (22e) *are equal up to a constant endogenously given factor $r_n$ that depends on prosumer $n$, but not on constraints. Mathematically, it means $r_n\zeta_{nm} = r_m\zeta_{mn}$, for all $n \in \mathcal{N}$ and all $m \in \Omega_n, m \neq n$.*

From $KKT_n$, we see that, as in the centralized case, $\lambda_n = \zeta_{nm} + c_{nm} + \xi_{nm}$, i.e., the per-unit nodal price at $n$ is the sum of the transaction price, the preference price and the congestion price, all for getting one unit from $m$ to $n$, for each neighbor of $m$.

Besides,

$$\zeta_{nm} = \lambda_n - c_{nm} - \xi_{nm}, \forall m \in \Omega_n, m \neq n\ , \tag{26}$$

which gives the transaction price for agent $n$ or, in other words, her evaluation of the trade $q_{mn}$.

In order to derive some results on GNE and simplify notations, let us introduce the coefficient $r_n$ as:

$$\zeta_{0n} r_n = \zeta_{n0}, \quad \forall n \in \mathcal{N}\ . \tag{27}$$

**Remark 4.3.** *We interpret this situation as one where there is an imperfect market for determining the bilateral trade prices obtained as dual variables of the shared constraint* (22e)*. Between any couple of prosumer nodes, bilateral trade prices do tend to equalize (i.e., $r_n$ is close to 1 for any $n \in \Omega_0$ — meaning that the GNE approaches the Variational Equilibrium), but there remains a gap due to insufficient liquidity or differences in the price bids for the asked quantity [30]. To some extent, $r_n$ can be interpreted as a measure of the efficiency loss introduced by the GNE in comparison with the Variational Equilibrium.*

Using Equation (26) for the node 0 and an arbitrary node $n \in \Omega_0$ and for an arbitrary node $n \in \Omega_0$ and the node 0, and summing up both relations, we get:

$$\lambda_n = r_n \lambda_0 + \big(c_{n0} - r_n c_{0n}\big) + \big(\xi_{n0} - r_n \xi_{0n}\big),\ \forall n \in \Omega_0. \tag{28}$$

Similarly to the centralized market design case, since the total sum of the net imports in all nodes should vanish under no RES-based generation surplus, i.e., $\sum_{n\in\mathcal{N}} Q_n = 0$, we infer the closed form expression of the nodal price at the root node, similar to the centralized case:

$$\begin{aligned}\lambda_0 \sum_{n\in\mathcal{N}} \left(\frac{1}{2\tilde{a}_n} + \frac{1}{a_n}\right) r_n = &\sum_{n\in\mathcal{N}} \Big(D_n^\star - \frac{1}{2\tilde{a}_n}(\overline{\mu}_n - \underline{\mu}_n) + \frac{b_n}{a_n} + \frac{1}{a_n}(\overline{\nu}_n - \underline{\nu}_n) - \Delta G_n\Big) \\ &- \sum_{n\in\Omega_0} \left(\frac{1}{2\tilde{a}_n} + \frac{1}{a_n}\right) \Big[\big(c_{n0} - c_{0n} r_n\big) + \big(\xi_{n0} - r_n \xi_{0n}\big)\Big].\end{aligned} \tag{29}$$

We introduce $\text{SOL}^{\text{GNEP}}$ as the set of GNE solutions of the peer-to-peer non-cooperative game.

GNEs are not unique in general.

It is relevant to study how efficient those different outcomes can be in comparison to the Variational Equilibrium outcome (where the bilateral trades would be settled down by a MO).

Although there exist several standard methods to compute numerically a variational equilibrium in a generalized game (e.g., with variational inequalities methods), it is in general harder to compute numerically other GNEs or even the complete set of GNEs.

A possible method to evaluate the set of GNEs is to apply the parameterized variational inequality approach [29; 30] which enables to characterize each GNE as the solution of an optimization problem. Results based on a similar approach were also presented by Gabriel et al. [12] through an extensive study of Nash-Cournot and other energy market models (some integrating market clearing conditions) that use mixed complementarity problems. However, peer-to-peer market design was not considered in this book, and we would like to highlight in this paper how such results also apply in fully distributed markets. In our specific case, this leads to the optimization problem $\mathcal{P}_{\boldsymbol{\omega}}^{\text{GNE}}$, parameterized by the coefficients $\omega_{nm} > 0$ corresponding to an additional value for user $n$ for its trading constraint with $m$:

$$\mathcal{P}_{\boldsymbol{\omega}}^{\text{GNE}} \qquad \max_{\boldsymbol{D},\boldsymbol{G},\boldsymbol{q}} \sum_{n\in\mathcal{N}} \left[ \Pi_n(D_n, G_n, \boldsymbol{q}_n) - \sum_{m\in\Omega_n, m\neq n} \omega_{nm} q_{mn} \right], \tag{30a}$$

$$s.t. \quad \underline{D}_n \leq D_n \leq \overline{D}_n, \forall n \in \mathcal{N}, \qquad (\underline{\mu}_n, \overline{\mu}_n) \tag{30b}$$

$$\underline{G}_n \leq G_n \leq \overline{G}_n, \forall n \in \mathcal{N}, \qquad (\underline{\nu}_n, \overline{\nu}_n) \tag{30c}$$

$$q_{nm} \leq \kappa_{nm} \qquad (\xi_{nm}) \tag{30d}$$

$$q_{nm} + q_{mn} \leq 0, \forall m \in \Omega_n, m > n, \forall n \in \mathcal{N}. \qquad (\zeta_{nm}) \tag{30e}$$

$$D_n = G_n + \Delta G_n + Q_n, \forall n \in \mathcal{N}, \qquad (\lambda_n)\ . \tag{30f}$$

From [29, Cor 3.1] and [29, Thm. 3.3], we can make a link between the set of GNEs and the solutions of problem (30), as given in the following proposition:

**Proposition 7.** *(i) All GNEs can be found from problem* (30)*, that is:*

$$SOL^{GNEP} \subset \bigcup_{(\omega_{nm})\in\mathbb{R}_+^{*N(N-1)}} SOL\left(\mathcal{P}_{\boldsymbol{\omega}}^{GNE}\right)\ ;$$

*(ii) reciprocally, if* $(\boldsymbol{D}, \boldsymbol{G}, \boldsymbol{q}, \boldsymbol{\zeta})$ *is a solution of* $\mathcal{P}_{\boldsymbol{\omega}}^{GNE}$ *(where* $\boldsymbol{\zeta}$ *are multipliers associated to* (30e)*), then*

$$(\boldsymbol{D}, \boldsymbol{G}, \boldsymbol{q}, \boldsymbol{\zeta}) \text{ is a GNE } \iff \omega_{nm}(q_{nm} + q_{mn}) = 0,\ \forall n \neq m\ , \tag{31}$$

*and in that case the multipliers associated to* (22e) *in the GNE problem are defined by* $\hat{\zeta}_{nm} = \zeta_{nm} + \omega_{nm}$ *.*

*Proof.* For (i), writing the KKT conditions verified by a solution $(\boldsymbol{D}, \boldsymbol{G}, \boldsymbol{q})$ of the GNE problem (22) with Lagrangian multipliers $(\hat{\zeta}_{nm})_{n\neq m}$, it is easy to verify that $(\boldsymbol{D}, \boldsymbol{G}, \boldsymbol{q})$ verifies the KKT conditions of (30) $\mathcal{P}_{\hat{\boldsymbol{\zeta}}}^{\text{GNE}}$, where the parameters are taken to $\boldsymbol{\omega} := \hat{\boldsymbol{\zeta}}$.

For (ii), we use the fact that problem (30) has linearly independent constraints, and apply [29, Thm. 3.3] directly. □

Proposition 7 gives us a characterization of GNEs which enables their computation via a sampling method on $\boldsymbol{\omega}$ and the optimization of parameterized problems (30) (see Section 5).

## 4.2 Dealing with Congestion

Let us first explicit the following fact on congested lines:

**Lemma 1.** *For any couple of nodes $n \in \mathcal{N}, m \in \Omega_n, m \neq n$, such that $\kappa_{nm} > 0$, $\kappa_{mn} > 0$, $q_{nm} = \kappa_{nm}$ and $q_{mn} = \kappa_{mn}$ cannot hold simultaneously.*

The proof is direct from the capacity and transaction constraints. Then, we obtain the following sufficient condition for a line to be saturated:

**Proposition 8.** *Suppose $\xi_{n0} = \xi_{0n} = 0, \forall n \in \Omega_0$, i.e., there are large line capacities from and to node 0, $c_{n0} = c_{m0}$, i.e., the nodes have the same preferences for node 0, and the node 0 has the same preferences for any node, i.e., $c_{0n} = c_{0m}$. For any couple of nodes $n \in \mathcal{N}, m \in \Omega_n, m \neq n$, asymmetric preferences (such as $c_{mn} > c_{nm}$ or $c_{mn} < c_{nm}$) imply that the node with the smaller preference for the other saturates the line.*

*Proof.* For any $n \in \mathcal{N}, m \in \Omega_n, m \neq n$, applying Equation (14) for the three couples of nodes: $(n, 0)$, $(0, m)$, $(m, n)$, we obtain:

$$\begin{aligned} c_{n0} - c_{0n} + \xi_{n0} - \xi_{0n} &= \lambda_n - \lambda_0, \\ c_{0m} - c_{m0} + \xi_{0m} - \xi_{m0} &= \lambda_0 - \lambda_m, \\ c_{mn} - c_{nm} + \xi_{mn} - \xi_{nm} &= \lambda_m - \lambda_n. \end{aligned}$$

Summing up the three equations, we get:

$$\xi_{nm} - \xi_{mn} = (c_{0m} - c_{0n}) + (c_{n0} - c_{m0}) + (\xi_{n0} - \xi_{0n}) + (\xi_{0m} - \xi_{m0}) + (c_{mn} - c_{nm}).$$

Under the assumptions of the proposition, the equation can be simplified to give:

$$\xi_{nm} - \xi_{mn} = c_{mn} - c_{nm}.$$

Then, two cases arise depending on the order of $(n, m)$ preferences:

**(i)** If $c_{mn} > c_{nm}$ (meaning that $m$ wants to sell to (buy from) $n$ more (less) than $n$ wants to sell to (buy from) $m$), $\xi_{nm} - \xi_{mn} > 0$, which implies from Lemma 1 that $\xi_{nm} > 0$. Then, for the complementarity constraint (12e) to hold we need to have $q_{nm} = \kappa_{nm}$, i.e., $m$ saturates the line from $m$ to $n$;

**(ii)** If $c_{nm} > c_{mn}$ (meaning that $n$ wants to sell to (buy from) $m$ more (less) than $m$ wants to sell(buy) to $n$), $\xi_{nm} - \xi_{mn} < 0$, which implies from Lemma 1 that $\xi_{mn} > 0$. Then, for the complementarity constraint (12e) to hold we need to have $q_{mn} = \kappa_{mn}$, i.e., $n$ saturates the line from $n$ to $m$.

□

The following proposition gives a sufficient condition for the distribution grid lines become congested along a cycle, analog to Proposition 5. The proof is similar and is omitted.

**Proposition 9.** *Suppose that there is a sequence of distinct indices $(n_i)_{1 \leq i \leq k}$ such that $\tilde{C}_{n_i,n_{i+1}} - \tilde{C}_{n_i,n_{i-1}} < 0$ for all $i = 1, \ldots, k$, where $n_{k+1} := n_1$. Then, at an equilibrium, there is a trade opposed to the cycle made at full capacity i.e. there exists $i \in \{1, \ldots, k\}$ such that $q_{n_{i+1},n_i} = \kappa_{n_{i+1},n_i}$.*

**Remark 4.4.** *Classically, the Price of Anarchy (PoA) is introduced as a performance measure to assess the performance of the peer-to-peer market design by comparison to the centralized market design. The PoA is defined as the ratio of the social welfare evaluated in the social welfare optimum to the social welfare evaluated in the worst GNE in the set $SOL^{GNEP}$. Formally, it is defined as follows:*

$$PoA := \frac{\max_{\mathbf{D},\mathbf{G},\mathbf{q}} SW(\mathbf{D}, \mathbf{G}, \mathbf{q})}{\min_{\mathbf{D},\mathbf{G},\mathbf{q} \in SOL^{GNEP}} SW(\mathbf{D}, \mathbf{G}, \mathbf{q})}. \tag{32}$$

*From Proposition 6, in a Variational Equilibrium, $PoA = 1$, because a Variational Equilibrium coincides with the optimum of the centralized social welfare optimization problem. However, the GNE set might contain equilibria that do not coincide with the (social welfare) optimum solution of the centralized optimization problem.*

# 5 Test Cases

## 5.1 A Three Nodes Network with Arbitrage Opportunity

In this section, we first present a toy model with only three nodes indexed by $\{0, 1, 2\}$, as illustrated in Figure 2. The root node 0 has only conventional generation ($\Delta G_0 = 0$) with cost $(a_0, b_0) = (4, 30)$ and $(\underline{G}, \overline{G}) = (0, 10)$. Nodes 1 and 2 are prosumers with RES-based generators ($\overline{G}_n = \underline{G}_n = 0$ and $\Delta G_n > 0$ for $n \in \{1, 2\}$). Each node is a consumer (with $(\underline{D}, \overline{D}) = (0, 10)$) and generator (RES or conventional), therefore producing energy that can be consumed locally to meet demand $D_n$ and exported to the other nodes to meet the unsatisfied demand.

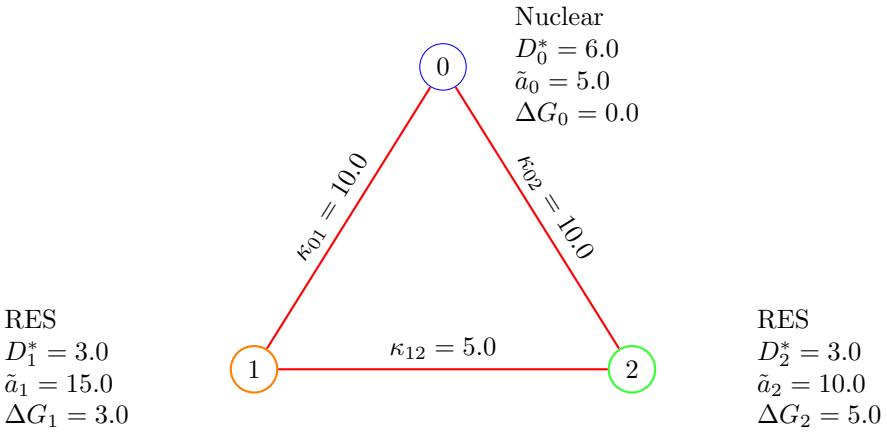



Figure 2: Three node network example.

Regarding the preferences $(c_{nm})_{nm}$, nodes 1 and 2 both prefer to buy local and to RES-based generators. Node 0 is assumed to be indifferent between buying energy from node 1 or node 2. Capacities are also defined larger from the source node 0 ($\kappa_{0n} = 10$) than between the prosumers nodes ($\kappa_{nm} = 5$).

| $c_{nm}$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | – | 1.0 | 1.0 |
| 1 | 3.0 | – | 1.0 |
| 2 | 2.0 | 1.0 | – |

| $c_{nm} - c_{mn}$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | – | -2.0 | -1.0 |
| 1 | 2.0 | – | 0.0 |
| 2 | 1.0 | 0.0 | – |

Table 1: Price differentiation parameters and matrix of differences.

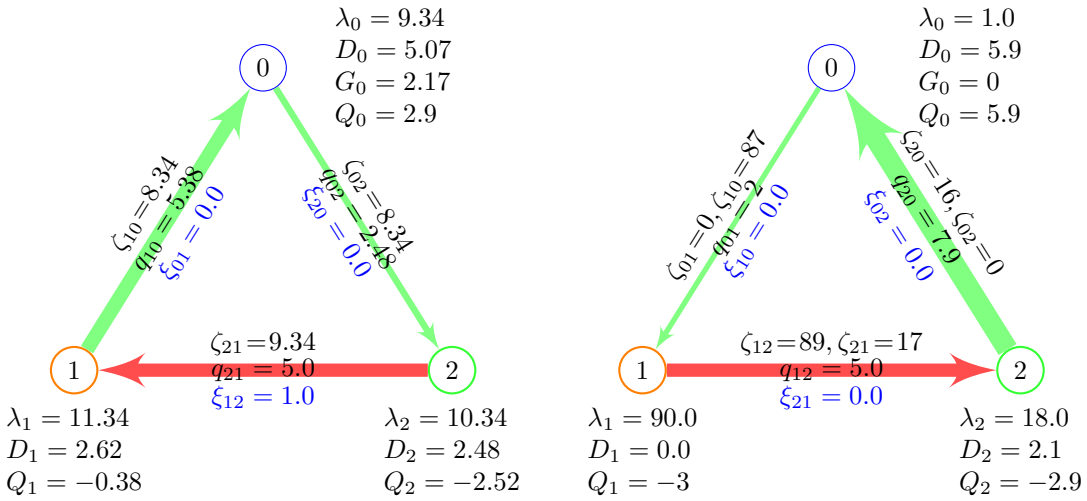
In Figure 3 (a), we illustrate the optimal solution of the centralized market design problem in which the global MO maximizes the social welfare under operational and power-flow constraints (8a)-(8e).

We remark on this figure that the trade from node 1 to node 2 is at full capacity, which is explained by Proposition 5. Indeed, we see from Table 1 that there is a "cycle" in preferences $\tilde{C}_{01}+\tilde{C}_{12}+\tilde{C}_{20} = -1$ which explains why we obtain $q_{10} = \kappa_{10}$ and $q_{21} = \kappa_{21}$ in the centralized solution (Figure 3).

On the contrary, we remark that, in the GNE solution depicted in Figure 3b, the same edge is congested in the reverse way: Proposition 5 only applies in the case of a centralized solution.

In the example above, the cycle comes from the fact that it is easier for node 2 to buy from 0 than node 1 to buy from 0: thus, the social welfare can be increased if 1 buys from 2 who buys from 0. Changing the parameters to $c_{10} = c_{20} = 3$ removes the cycle in the optimal solution of $(q_{nm})_{nm}$.

In Figure 4, we show the different GNEs existing for this reduced problem in the three-dimensional space of transactions. As one can see on this figure, an interesting property is that, for any GNE, the edge from node 1 to node 2 is saturated in one way or the other.




(a) Centralized solution ($SW = 378.3$)

(b) One GNE ($SW = 255.5$)

Figure 3: Comparison of the optimal centralized solution (a) and a GNE solution with low social welfare (b).

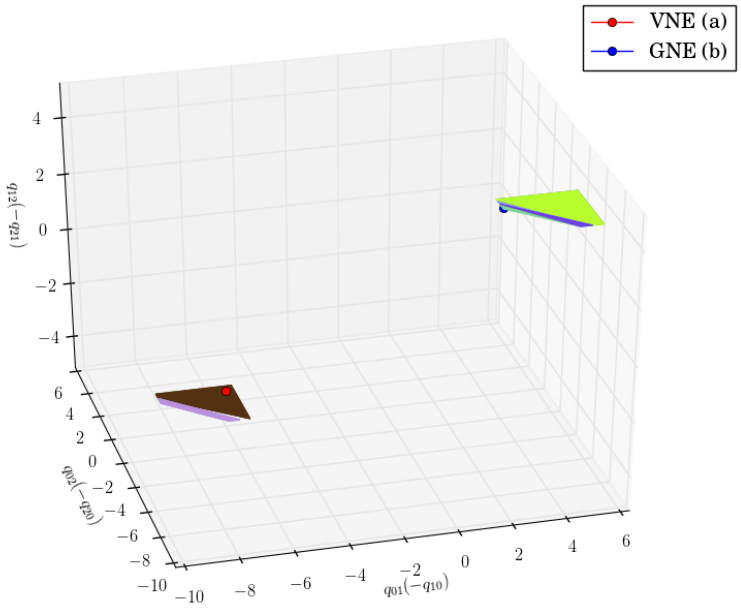



Figure 4: All existing GNEs in $q$-space. *The set of GNEs is given as two connected components, corresponding to the edge (1,2) saturated in one way and the other.*

Evaluating the GNE with the lowest social welfare is difficult because this task does not correspond to a convex problem (in particular, the SW is a concave function). However, the GNE depicted in Figure 3b is the worst GNE that we found with the sampling method given by Proposition 7, using a sampling $(\omega_{nm})_{n>m} \in \{0, \dots, 100\}^3$. Therefore, we can have the following bound on the PoA:

$$\text{PoA} = \frac{\max_{\mathbf{D},\mathbf{G},\mathbf{q}} SW(\mathbf{D},\mathbf{G},\mathbf{q})}{\min_{\mathbf{D},\mathbf{G},\mathbf{q}\in \text{SOL}^{\text{GNEP}}} SW(\mathbf{D},\mathbf{G},\mathbf{q})} \geq \frac{378.3}{255.5} \simeq 1.48 \ , \tag{33}$$

which means that, in the peer-to-peer market, in the presence of market imperfections, the resulting social welfare can be more than 50% smaller than the optimal social welfare (or, the VE obtained in the absence of market imperfections).

## 5.2 IEEE 14-bus Network

In this example, we consider the IEEE 14-bus network system introduced in [41]. Each bus of the network corresponds to a prosumer in our model as described on Figure 5. The busses 3, 4, 5 and 9 to 14 contain only consumers without any production. Nodes 2 and 3 are prosumers node (consumption and RES production) and also contain thermal production plants. The bus 6 is a prosumer with only intermittent solar energy production. Last, the bus 8 contains only production, renewable and thermal.

The bus 1 corresponding to the grid connection is also able to provide power to the buses linked to it.

Each pair of busses is able to trade with its neighboring busses, up to the capacity of the edge linking the pair of busses.

For simplicity, we compute the trades and optimal productions and consumptions for a particular unique time period. The renewable energy productions $(\Delta G_n)_n$ and the objective consumptions $(D_n^\star)_n$ for this time period are provided in Figure 5. Note that in this particular example, we have the inequality:

$$28.39 = \sum_{n\in\mathcal{N}} \Delta G_n < \sum_{n\in\mathcal{N}} D_n^\star = 69.94 \text{ [GWh]} \ ,$$

which explains partly why we do not have any energy surplus in the solutions depicted on Figure 6.

For the trade differentiation prices $(c_{nm})_{n,m}$, we consider four cases:

(a) uniform prices: $c_{nm} = 1$ for each $n$ and $m$, so that we ensure that there does not exist any cycle in the matrix of price differences as described in Proposition 5;

(b) heterogeneous prices: for $n \neq 1$ and $m \neq 1$, $c_{nm}$ is chosen uniformly in $[0, 1]$. We assume that agents have a preference for local trades so the price with the grid connection bus $c_{n1}$ is larger and chosen uniformly in $[1, 2]$. The grid connection bus has no preferences so that $c_{1n} = 1$ for each $n$ neighboring bus 1.

(c) symmetric prices: $(c_{nm})_{nm}$ random and symmetric (for $n < m$, $c_{nm}$ is taken as in (b)).

(d) preferences for local trades with uniform prices: $(c_{nm})_{nm} = 1$ if $m \neq 1$ and $c_{n1} = 3$.

For each of this case, we compute the centralized solution (also corresponding to the VNE). The solutions are illustrated in Figure 5: directions of trades are represented by arrows, the wideness of each arrow is proportional to the quantity traded. Trades made at full capacity ($q_{nm} = \kappa_{nm}$) are represented by red arrows, while the others are represented by green arrows. We observe that cases (c) and (d) give the same trade solutions $(q_{nm})_{nm}$ at VNE as case (a).

We see in Figure 6 that the differentiation prices $(c_{nm})_{nm}$ modify completely the solution. We observe that the quantities traded in case ($b$) are much larger. While some edges are almost unused in case (a) and no edge is congested, ten of the twenty-two edges become congested in case (b) with heterogeneous prices. This effect can be explained by Propositions 5 and 8.

Also, we observe that marginal prices $(\zeta_{nm})_{n,m}$ are all equal to 2.16 \$/MWh in case (a), while they are heterogeneous in the case (b). In case (a), the equality is explained by both the absence of congestion ($\xi_{nm} = 0$) and the equality of $(c_{nm})_{nm}$ among users (absence of preferences).

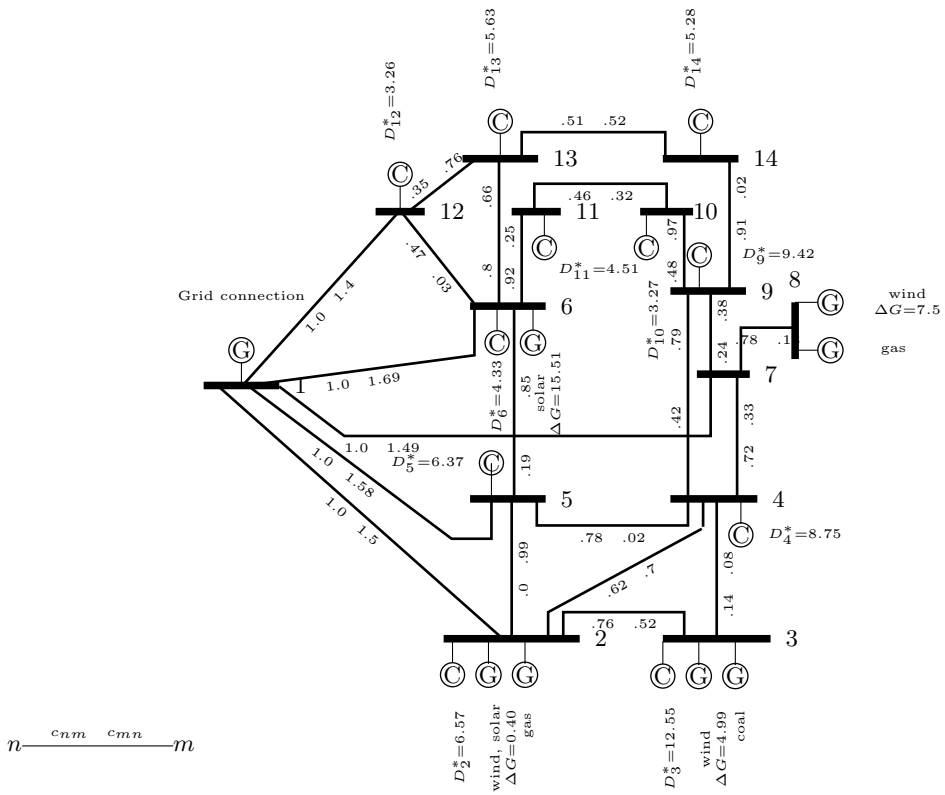
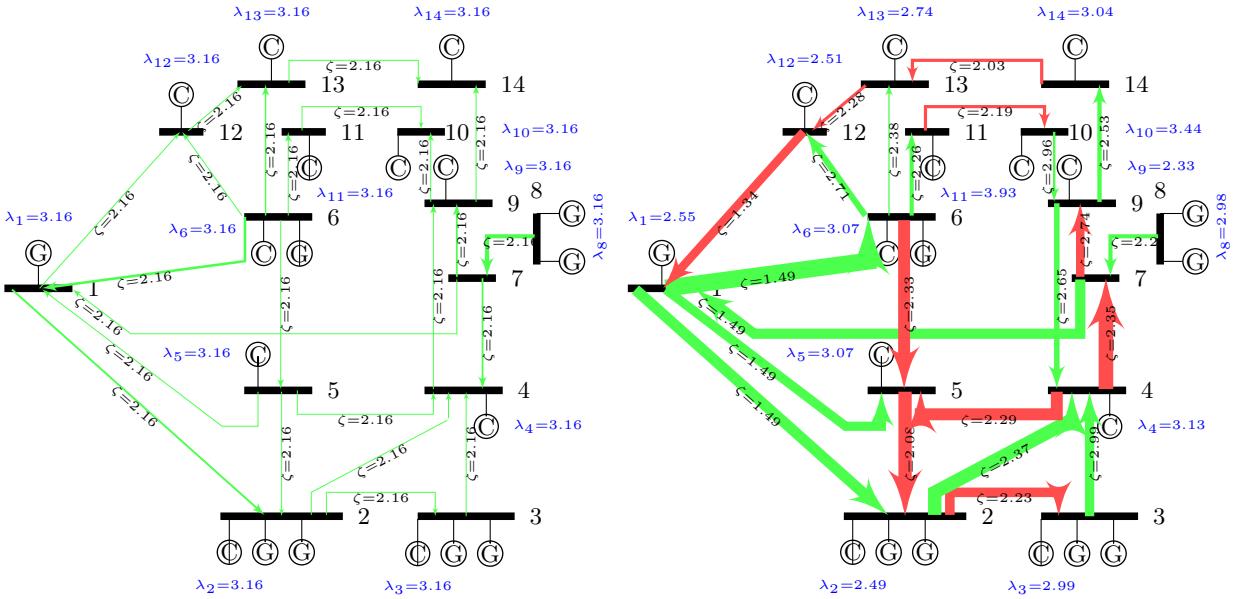



Figure 5: IEEE 14-bus network system




(a) with $(c_{nm})_{nm} = (1)_{nm}$ uniform, SW = 395.28 M$

(b) with $(c_{nm})_{nm}$ random, SW = 560.51 M$

Figure 6: Trades [$/MWh] at the VNE of the IEEE 14-bus network with homogeneous differentiation prices (left) and heterogeneous differentiation prices (right). *With heterogeneous prices, the quantities traded are larger, and some links become congested. In the homogeneous case, marginal trade prices* $(\zeta_{nm})_{n,m}$ *are all equal. In the case of heterogeneous prices* $(c_{nm})_{nm}$*, marginal prices* $(\zeta_{nm})_{n,m}$ *are also heterogeneous.*

As opposed to the reduced example with three nodes given in Section 5.1, it was not possible to compute a GNE different from the VE for this 14 nodes network. The approach of Nabetani et al. [29] that we used for the three node network is not possible here because of the dimension: to search for another GNE, we have to look on a space of dimension 22, e.g., the number of lines in the network. This observation also calls for the development of algorithms not based on brute force approach, enabling an efficient approximation of the GNEs. This could be the topic of future research.

# 6 Dealing with Privacy

In this section, we will make some assumptions regarding the information available to each agent. We summarize them below:

**Assumption 1** Since the technologies (conventional units, solar PV panels, micro-CHPs, storage facilities, etc.) used by the agents are standardized, we assume that each agent $n$'s production cost $C_n(.)$ and parameters $a_n$, $b_n$, $d_n$ are publicly known by all the agents $m \in \mathcal{N} \setminus \{n\}$. This assumption is in line with the work of Rasouli and Teneketzis [35] who argue that symmetric information of the bidders (prosumers in our case) is a common rationale and a reasonable assumption for oligopolistic electricity markets, because prosumers monitor perfectly each others technology and capacity.

**Assumption 2** The product differentiation prices $(c_{n0})_n$, $(c_{0n})_n$ for any $n \in \mathcal{N} \setminus \{0\}$ are publicly known by all the agents. In case of taxes, they might be designed by the regulator to impact the energy mix by investing in certain energy technologies supporting local or RES-based generation development to promote energy efficiency and innovation.

**Assumption 3** The congestion prices $(\xi_{n0})_n$, $(\xi_{0n})_n$ for any node $n \in \mathcal{N} \setminus \{0\}$ on the interface lines between transmission network and distribution networks are determined by a dedicated market

mechanism and publicly revealed to all the agents.

**Assumption 4** The prosumers' maximum usage benefit is homogeneous among all agents and public knowledge. The agents differentiate in their risk-aversion perception measured by the absolute risk aversion[3] $\mathcal{A}(D_n) := \frac{1}{D_n^\star - D_n} = \frac{1}{\sqrt{\frac{\tilde{b}_n}{\tilde{a}_n}} - D_n}$.

**Remark 6.1.** *The product differentiation prices $(c_{nm})_{n,m}$ for $n \neq 0$ or $m \neq 0$ remain private information to the prosumers. The preference parameters $\tilde{a}_n, \forall n \in \mathcal{N}$ are also private to prosumer $n \in \mathcal{N}$.*

In addition to the private information on preferences, privacy might limit the information released by the prosumers regarding their target demand and RES-based self-generation. In this section, the target demand and RES-based self-generation will be part of prosumer's private information. When privacy applies, prosumers can decide to not share their private information with the other prosumers in their neighborhood. Two reasons justify this behavior: first, they might be reluctant to install intrusive and costly monitoring systems to keep track of their RES-based self-generation; second, they can fear that other prosumers decide to sell it to aggregators that would use it for gaming on the wholesale market, impacting the prosumers' bills. In general, privacy has some impact on the utility functions of the prosumers. Indeed, based on the analytical expressions of the VNE and GNE we derived in the previous sections, each prosumer needs to know the target demands and RES-based self-generation of each prosumer in her neighborhood. In case privacy applies, these values will not be shared - even within the neighborhood - and each prosumer would have to forecast the private information of the other prosumers to optimize her own decision variables, therefore introducing bias in her own expected utility function. Under this setting, the peer-to-peer market design can be interpreted as a game with asymmetric information. In the present section, we derive a closed form expression for the privacy cost and provide an upper bound on the impact of the bias introduced by the distributed forecasts of the prosumers on their expected utility. This will give us a first quantification of the impact of privacy on the market outcome taking the point of view of the prosumers. This also constitutes a first step towards the implementation of learning methods or algorithms to approach an equilibrium (GNE) with minimum information exchange between the prosumers. These algorithmic aspects leave an interesting direction for future research.

## 6.1 Quantifying the Cost of Privacy

We now explain in more details how each prosumer builds forecasts of target demands, RES-based generations and nodal prices, and use them to compute a biased-forecast equilibrium.

The expressions of the prosumers' demand, flexibility activation, and net imports are given in Proposition 1. From these expressions, each prosumer needs to compute her nodal price, which is itself based on the nodal price at the root node $\lambda_0$. In a centralized market clearing approach, it is the MO who determines all the nodal prices while having access to all the information of the prosumers on their target demand and RES generations. In the reality, privacy preservation rules might allow the prosumers not to share all their private information. In a peer-to-peer market design, nodal price expressions are detailed in (28) and (29). Under **Assumptions 1, 2, 3, 4**, to compute her nodal price, each prosumer needs to compute the nodal price at the root node $\lambda_0$, which requires to know the target demand $D_m^\star, m \neq n$ and RES-based generations $\Delta G_m, m \neq n$ of all the other prosumers. Since this information is in general kept private by the prosumers, prosumer $n$ needs to build forecasts of the other prosumers' target demand and RES-based generations. To that purpose, for each agent $n \in \mathcal{N}$, we introduce forecasts in the form of simple linear estimates:

$$F_n(D_m^\star) = D_m^\star + \epsilon_{nm}^D, \tag{34}$$

$$F_n\Big(\Delta G_m\Big) = \Delta G_m + \epsilon_{nm}^G, \forall m \in \mathcal{N}, m \neq n, \tag{35}$$

[3]This relationship comes from the explicit relationship between the parameter $\tilde{a}_n$, the maximum usage benefit $\tilde{b}_n$, and the target demand, derived under the assumption that usage benefit vanishes in case of zero demand (see Section 2.3). A subproduct of this result is that the absolute risk aversion of any prosumer $n$ is fully parameterized by $\tilde{a}_n$.

where $\epsilon^D_{nm}$, $\epsilon^G_{nm}$ are the biases introduced by agent $n$ in the estimation of the demand and the RES-based generation of any agent $m \in \mathcal{N}, m \neq n$. To incorporate some proximity aspects, the bias might be supposed smaller for agents belonging to agent $n$ neighborhood (because of more frequent interactions) than for agents outside of her neighborhood. We assume that $\epsilon^D_{nm}$, $\epsilon^G_{nm}$ are independent and identically distributed (iid) random variables that follows Gaussian density functions centered in 0, with standard deviation $\sigma^D_{nm}$, $\sigma^G_{nm}$. We also set $\Delta\epsilon_{nm} := \epsilon^D_{nm} - \epsilon^G_{nm}$, as the difference between the biases introduced by agent $n$ in agent $m$ demand and RES-based generation estimations. To simplify the analytical expressions to come, let $\rho_n(\mathbf{r}) := \frac{r_n}{(\frac{1}{2\tilde{a}_0}+\frac{1}{a_0})+\sum_{m\in\Omega_0}(\frac{1}{2\tilde{a}_m}+\frac{1}{a_m})r_m}$ where $r_n$ is defined in (27) as the ratio of the bilateral trade prices with node 0: $\frac{\zeta 0n}{\zeta_{n0}}, \forall n \in \mathcal{N}$.

Substituting Equations (34) and (35) in (29) and (28), agent $n$ obtains the following estimate for the nodal price at the root node and at her node (i.e., on her local market):

$$\begin{aligned} F_n(\lambda_0) =& \lambda_0 + \rho_n(\mathbf{r}) r_n^{-1} \sum_{m\in\Omega_n} \Delta\epsilon_{nm}, \\ F_n(\lambda_n) =& \lambda_n + \rho_n(\mathbf{r}) \sum_{m\in\Omega_n} \Delta\epsilon_{nm}, \forall n \in \Omega_0. \end{aligned}$$

For any prosumer $n \in \mathcal{N}$, we observe that $\sum_{m\in\Omega_n} \Delta\epsilon_{nm} \to 0$ implies that $F_n(\lambda_0) \to \lambda_0$ and $F_n(\lambda_n) \to \lambda_n$. So, agent $n$ estimates of her nodal price is without bias if she makes no bias in the other agents' target demand and RES-based generation estimations, or biases in both estimates compensate each other.

Then, by substitution of the nodal price estimates in Proposition 1 output, we obtain the following expression for the biased-forecast equilibrium:

$$\begin{aligned} F_n(D_n) =& D_n - \frac{1}{2\tilde{a}_n}\rho_n(\mathbf{r}) \sum_{m\in\Omega_n} \Delta\epsilon_{nm}, \\ F_n(G_n) =& G_n + \frac{1}{a_n}\rho_n(\mathbf{r}) \sum_{m\in\Omega_n} \Delta\epsilon_{nm}, \\ F_n(Q_n) =& Q_n - (\frac{1}{2\tilde{a}_n} + \frac{1}{a_n})\rho_n(\mathbf{r}) \sum_{m\in\Omega_n} \Delta\epsilon_{nm}. \end{aligned} \tag{36}$$

Note that in general $F_n(D_n) \neq D_n$, $F_n(G_n) \neq G_n$, and $F_n(Q_n) \neq Q_n$, i.e., the equilibrium obtained under privacy (that we called biased-forecast equilibrium) is different from the equilibrium computed under full information. In case where the sum of the error differences tends to zero, i.e., $\sum_{m\in\Omega_n} \Delta\epsilon_{nm} \to 0$, then $F_n(D_n) \to D_n$, $F_n(G_n) \to G_n$, and $F_n(Q_n) \to Q_n$.

Then, by substitution of the biased-forecast equilibrium (36) in agent $n$ utility function, we obtain the following bounds for her utility bias:

$$\begin{aligned} F_n(\Pi_n) - \Pi_n \leq& -\frac{1}{2}(\frac{1}{\tilde{a}_n} - \frac{1}{a_n})\rho_n(\mathbf{r})^2(\sum_{m\in\Omega_n} \Delta\epsilon_{nm})^2 + \Big(\min_{m\neq n}\{c_{nm}\}(\frac{1}{2\tilde{a}_n} + \frac{1}{a_n}) - \frac{b_n}{a_n}\Big)\rho_n(\mathbf{r}) \sum_{m\in\Omega_n} \Delta\epsilon_{nm}, \\ F_n(\Pi_n) - \Pi_n \geq& -\frac{1}{2}(\frac{1}{\tilde{a}_n} - \frac{1}{a_n})\rho_n(\mathbf{r})^2(\sum_{m\in\Omega_n} \Delta\epsilon_{nm})^2 + \Big(\max_{m\in\Omega_n}\{c_{nm}\}(\frac{1}{2\tilde{a}_n} + \frac{1}{a_n}) - \frac{b_n}{a_n}\Big)\rho_n(\mathbf{r}) \sum_{m\in\Omega_n} \Delta\epsilon_{nm}. \end{aligned} \tag{37}$$

Taking the expectation of $F_n(\Pi_n) - \Pi_n$ and since the expectation preserves the inequalities, we obtain the following relation from Equation (37):

$$\mathbb{E}\Big[F_n(\Pi_n) - \Pi_n\Big] = -\frac{1}{2}(\frac{1}{\tilde{a}_n} - \frac{1}{a_n})\rho_n(\mathbf{r})^2 \sum_{m\in\Omega_n} \Big((\sigma^D_{nm})^2 + (\sigma^G_{nm})^2 + 2Cov(\epsilon^D_{nm}, \epsilon^G_{nm})\Big). \tag{38}$$

The cost of privacy for prosumer $n$ can directly be measured through (38).

If $\tilde{a}_n = a_n$, then $\mathbb{E}\Big[F_n(\Pi_n) - \Pi_n\Big] = 0$, i.e., there is no bias in the estimation of the prosumer's utility.

To simplify the notation, we set $\beta_n := -(\frac{1}{\tilde{a}_n} - \frac{1}{a_n}) \sum_{m\in\Omega_n} \Big((\sigma^D_{nm})^2 + (\sigma^G_{nm})^2 + 2Cov(\epsilon^D_{nm}, \epsilon^G_{nm})\Big)$.

**Proposition 10.** *Assuming $\beta_n > 0$ (resp. $\beta_n < 0$), the bias in the prosumer $n$ estimated utility is increasing in $r_n$ (resp. decreasing in $r_n$).*

*Proof.* By derivation of node $n$ profit with respect to $r_n$, we obtain: $\frac{\partial \mathbb{E}\Big[F_n(\Pi_n)-\Pi_n\Big]}{\partial r_n} = \beta_n \rho_n(\mathbf{r}) \frac{\partial \rho_n(\mathbf{r})}{\partial r_n}$.

Since $\frac{\partial \rho_n(\mathbf{r})}{\partial r_n} = \frac{\rho_n(\mathbf{r})}{r_n}\Big[\frac{1}{\delta_{n0}} - \rho_n(\mathbf{r})(\frac{1}{2\tilde{a}_n} + \frac{1}{a_n})\Big] = \frac{\rho_n(\mathbf{r})}{r_n}(1 - \frac{r_n\alpha_n}{\alpha_0+\sum_{m\in\Omega_0}\alpha_m r_m})$ with $\alpha_m := \frac{1}{2\tilde{a}_m} + \frac{1}{a_m}, \forall m \in \Omega_0$. By assumption $\zeta_{n0} \geq 0$ and $\zeta_{0n} \geq 0$, which imply that $r_n \geq 0$ and $\rho_n(\mathbf{r}) \geq 0$. Furthermore, by definition of the usage benefit and production cost parameters $\alpha_0 + \sum_{m\in\Omega_0\setminus n}\alpha_m r_m \geq 0 \Leftrightarrow 1 - \frac{r_n\alpha_n}{\alpha_0+\sum_{m\in\Omega_0}\alpha_m r_m} \geq 0$. Then, depending on the sign of $\beta_n$, the conclusion follows. □

Proposition 10 means that the prosumers may have incentives to play strategically with the valuations of the bilateral trading prices with the root node since it influences the bias in their expected utility. More precisely, $\zeta_{n0}$ smaller than $\zeta_{0n}$ will lead to small bias values; whereas $\zeta_{n0}$ larger than $\zeta_{0n}$ will lead to large bias values. In order to minimize her bias, the prosumer would choose smaller valuation for the trade with the root node than the root node would choose for similar trade. By playing strategically in order to minimize the cost of privacy, the risk is that the resulting GNE gets far away from the Variational Equilibria guaranteeing the social welfare maximization, and therefore generates substantial efficiency loss.

**Proposition 11.** *There exists an upper-bound $\Phi_n$ such that for any $\boldsymbol{D}, \boldsymbol{G}, \boldsymbol{q} \in SOL^{GNEP}$,*

$$\Big|\mathbb{E}\Big[F_n(\Pi_n) - \Pi_n\Big]\Big| \leq \Phi_n, \forall n \in \mathcal{N}.$$

*Proof.* Taking the absolute value of the expectation of the difference between the estimated and the true utilities, we observe that $\Big|\mathbb{E}\Big[F_n(\Pi_n) - \Pi_n\Big]\Big| \leq \mathbb{E}\Big[\,|F_n(\Pi_n) - \Pi_n|\,\Big] \leq \frac{1}{2}\,|\beta_n|\,\rho_n(\mathbf{r})^2$.

Derivating $\rho_n(\mathbf{r})^2$ with respect to $r_n$, we obtain $\frac{\partial \rho_n(\mathbf{r})^2}{\partial r_n} = 2\frac{\rho_n(\mathbf{r})^2}{r_n}(1 - \frac{r_n\alpha_n}{\alpha_0+\sum_{m\in\Omega_0}\alpha_m r_m}) \geq 0$, which implies that $\rho_n(\mathbf{r})^2$ is increasing in $r_n$. Derivating $\rho_n(\mathbf{r})^2$ with respect to $r_m, m \neq n$, we obtain $\frac{\partial \rho_n(\mathbf{r})^2}{\partial r_m} = 2\rho_n(\mathbf{r})\Big[-\frac{\rho_n(\mathbf{r})}{r_n}(\frac{1}{2\tilde{a}_m} + \frac{1}{a_m})\Big] \leq 0$, which implies that $\rho_n(\mathbf{r})^2$ is decreasing in $r_m, m \neq n$.

According to the parameterized variational inequality approach of Nabetani et al. [29], the GNE set $\boldsymbol{D}, \boldsymbol{G}, \boldsymbol{q} \in \mathrm{SOL}^{\mathrm{GNEP}}$ can be described by making the valuation ratio $r_n$ span values in a certain interval, i.e., $\underline{r}_n \leq r_n \leq \overline{r}_n$, for any $n \in \Omega_0$. Based on the variational analysis of $\rho_n(\mathbf{r})^2$ - by computing the derivatives of $\rho_n(\boldsymbol{r})$ with respect to $r_n$ and $(r_m)_m \neq n$, we prove that it is increasing in $r_n$ and decreasing in $r_m, m \neq n$ - we conclude that $\Phi_n := \frac{1}{2}\,|\beta_n|\,\rho_n\Big(\overline{r}_n, (\underline{r}_m)_{m\neq n}\Big)^2$.

□

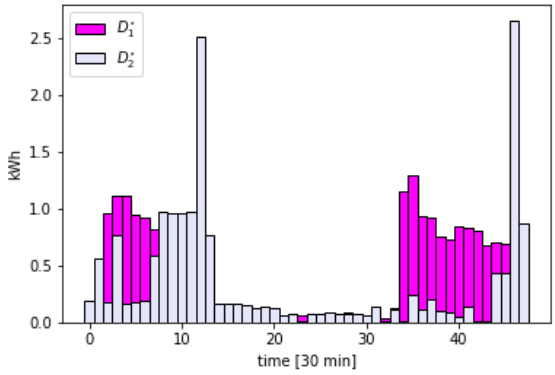
## 6.2 Sensitivity Analysis of the Privacy Cost

In this section, we focus on a sensitivity analysis of the privacy cost derived analytically in (38), considering two aspects: (a) the impact of the energy mix on the privacy cost; (b) the impact of the prosumers' absolute risk aversion on the privacy cost.

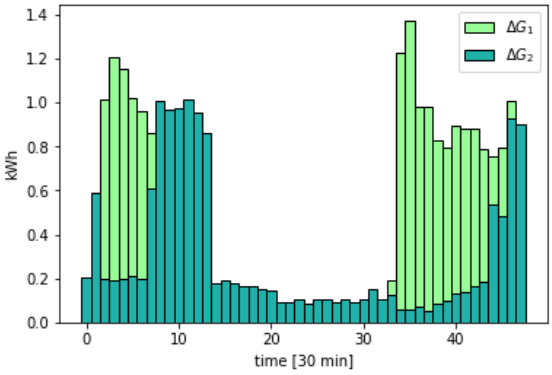
We consider a three nodes example as in Subsection 5.1, made of a root node 0 which contains conventional generation, and two other geographically distributed nodes which contain prosumers with target demands, RES generations, and possibly conventional generations. The two prosumers nodes target demands and RES generations, as well as the root node target demand, are extracted from the solar home electricity data base for Australia, shared by Ausgrid [1]. In Figures 7a and 7b, we have represented the daily target demand and RES generation schedules of the two prosumers nodes.

We report the mean and variance of the target demand and RES generation schedules in each node, in Table 2. In addition, we make the assumption that (34) and (35) take the following forms:

$$\epsilon^D_{nm} = \begin{cases} \bar{D}^\star_m - 0.2u - D^\star_m \text{if } n = 0,\ m \in \{1;2\}, \\ \bar{D}^\star_m + 0.1u - D^\star_m \text{if } n = 1,\ m \in \{0;2\}, \\ \bar{D}^\star_m - 0.1u - D^\star_m \text{if } n = 2,\ m \in \{0;1\}, \end{cases}$$




(a) Daily target demands of the two prosumers with 30 min granularity.




(b) Daily RES generations of the two prosumers with 30 min granularity.

Figure 7: Target demand and RES generation schedules of the prosumers on 07/04/2012.

| | $D_0^\star$ | $D_1^\star$ | $D_2^\star$ | $\Delta G_0$ | $\Delta G_1$ | $\Delta G_2$ |
|---|---|---|---|---|---|---|
| mean | 0.68 | 0.46 | 0.36 | 0 | 0.51 | 0.30 |
| variance | 0.54 | 0.18 | 0.30 | 0 | 0.19 | 0.09 |

Table 2: Means and variances of the nodes' daily target demand and RES generation schedules.

where $\bar{D}_m^\star$ is the mean of prosumer $m$ target demand daily schedule and $u \sim \mathcal{N}(0;1)$ is a random variable generated accorded to a normalized Gaussian density function centered in 0. Same holds for the RES generation - except that $\epsilon_{n0}^G = 0, n \in \{1,2\}$ - replacing $D_m^\star, \bar{D}_m^\star$ with $\Delta G, \bar{\Delta G}$ and $\epsilon_{nm}^D$ with $\epsilon_{nm}^G$ in the above equations. This gives us the following values for the standard deviations and covariances:

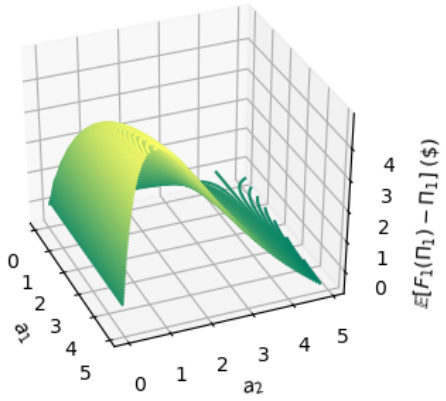
$$\boldsymbol{\sigma}^D = \begin{pmatrix} 0 & 0.43 & 0.54 \\ 0.74 & 0 & 0.55 \\ 0.74 & 0.43 & 0 \end{pmatrix}, \quad \boldsymbol{\sigma}^G = \begin{pmatrix} 0 & 0.44 & 0.31 \\ 0 & 0 & 0.31 \\ 0 & 0.44 & 0 \end{pmatrix}, \text{ and } \boldsymbol{Cov} = \begin{pmatrix} 0 & 0.19 & 0.08 \\ 0.19 & 0 & 0.14 \\ 0 & 0.19 & 0 \end{pmatrix}.$$

For the two test cases, we set $\tilde{a}_0 = 5$, $a_0 = 4.23$. With this choice of parameters, we assume that node 0 is strongly risk averse and relies on nuclear as generation technology. Furthermore, we set $r_1 = 1.05, r_2 = 0.95$.

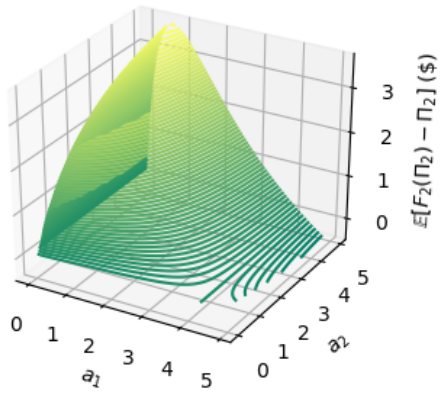
### 6.2.1 Impact of the Energy Mix on the Privacy Cost

In this part, we choose $\tilde{a}_1 = \tilde{a}_2 = 4.3$, i.e., nodes 1 and 2 have the same absolute risk aversion - which is lower than node 0 absolute risk aversion. We assume that both nodes 1 and 2 produce RES. In addition to RES, they have the possibility to invest in another technology. We want to determine the impact of the energy mix on the privacy costs for nodes 1 and 2. The impact of the technology deployed in nodes 1 and 2 is parametrized by the values of $a_1$ and $a_2$ in (38). According to the data provided by Sousa et al. [41], in node n, RES is deployed for $a_n \in [0; 0.15[$; coal is used for $a_n \in [0.15; 2.51[$; gas is chosen for $a_n \in [2.51; 4.23[$; and nuclear is preferred for $a_n \in [4.23; 5]$. In Figures 8a and 8b, we have computed the daily privacy costs at nodes 1 and 2 as functions of the technology choice made in the nodes. At both nodes, the minimum privacy cost is reached when the other node uses nuclear because it reduces the bias in the forecast of the other node generation to the single RES part of the other node production. To minimize her privacy cost, node 1 will prefer to invest in RES only (which is less expensive but more subject to forecast error for the other node) if node 2 invests in RES and in nuclear (which is more expensive but does not cause any additional forecast error) if node 2 invests in nuclear, while node 2 will prefer nuclear if node 1 chooses nuclear and RES if node 1 chooses RES,

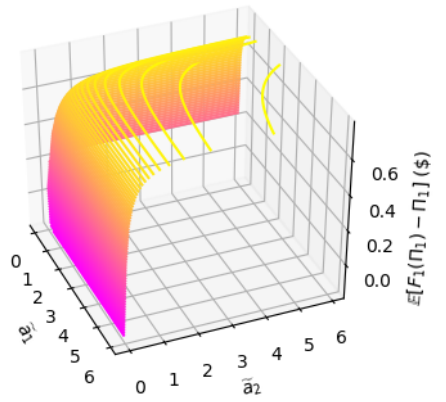
leading to a kind of tit for tat strategy for the optimal technology choice. Furthermore, the maximum daily privacy cost in node 1 (3.2 $) is lower than the maximum daily privacy cost in node 2 (3.9 $). This result can be explained by the fact that the mean and variance of RES generation in node 1 are higher than in node 2, meaning that it can be more difficult to forecast the RES generation in node 1 than in node 2.



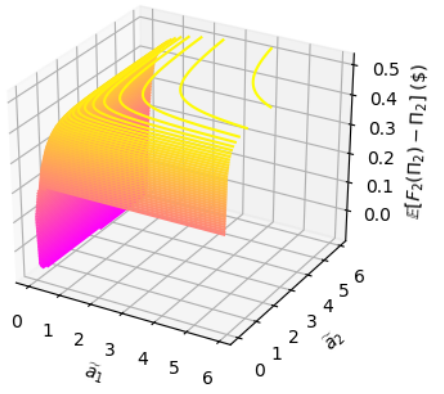

(a) Privacy cost in node 1 as a function of the prosumers quadratic production cost coefficients.




(b) Privacy cost in node 2 as a function of the prosumers quadratic production cost coefficients.




(c) Privacy cost in node 1 as a function of the prosumers usage benefit parameters.




(d) Privacy cost in node 2 as a function of the prosumers usage benefit parameters.

Figure 8: Daily loss caused by privacy in nodes 1 and 2.

### 6.2.2 Impact of the Prosumers Absolute Risk Aversion on the Privacy Cost

In this part, we choose $a_1 = 0.10, a_2 = 0.12$, meaning that both nodes use exclusively RES technology. We evaluate the impact of nodes 1 and 2 absolute risk aversion as defined in Assumption 4, on the privacy costs. Note that according to the definition of the absolute risk aversion, $\tilde{a}_n > \tilde{a}_m$ implies that n is more risk averse than m. We observe in Figures 8c and 8d that the minimum privacy cost is reached when the other node has a very small risk aversion, and maximized (0.6 $ per day for node 1 and 0.53 $ per day for node 2) in case the other node is strongly risk averse. This can be explained by

the fact that a strongly risk averse node would have more incentives to withhold information than a weakly risk averse one, resulting in higher privacy costs. So, the degree of risk aversion characterizes the willingness of the node to share information with the other nodes.

# 7 Conclusion

We considered two market designs for a network of prosumers with differentiation price preferences: **(i)** a centralized market design used as a benchmark, where a global market operator optimizes the flows (trades) and bilateral trading prices between the nodes to maximize the system overall social welfare; **(ii)** a fully distributed peer-to-peer market design where prosumers in local energy communities optimize selfishly the trades, demand, and flexibility activation. We characterized the solution of the peer-to-peer market as a Variational Equilibrium, without price arbitrage, and proved that the set of Variational Equilibria coincides with the set of social welfare optima solutions of market design **(i)**. We also discussed the fact that other solutions of the peer-to-peer market may exist, as Generalized Nash Equilibrium of the problem **(ii)**. We characterized formally the impact of preferences on the network line congestion and energy surplus under both designs. The results are illustrated in two test cases (a three nodes network and the IEEE 14-bus network). In the three nodes model, we also provided a bound on the Price of Anarchy capturing the loss of efficiency caused by market imperfections in this example. The learning mechanism needed to reach an equilibrium state in the peer-to-peer market design is discussed together with privacy issues. Based on these performance analysis and numerical results, we conclude that peer-to-peer market design gives rise to similar performance than the classical centralized market design provided market imperfections (resulting from the lack of coordination, insufficient market liquidity, information asymmetry resulting from privacy) can be corrected, and constitutes a relevant evolution for power system operation as it promises more robustness and resilience. Indeed, as the information and decisions are not optimized by a central single entity, in case of failure or if one node is attacked, the power system can still rely on the other nodes. Besides, as all prosumers are involved, they have the ability to adapt their actions to the state of grid.

Several extensions could be considered for further work. First, we could formally include the external aggregator in the study of the underlying economic system. In particular, instead of a premium or feed-in tariff—the sustainability of such mechanisms being questionable—we could get inspiration from works such as [20; 36] where the external aggregator, seen as a strategic agent, charges the consumers for their energy surplus, or [2] which deals with the design of a distribution-level electricity market. We could also extend the proposed model by adding taxes on the trades (either a constant tax or a quadratic term), designed by the market operator and in order to regulate or optimize the trades. Another interesting line of work would be to relate the notion of privacy resulting in this paper from the non-disclosure of the target demands and RES-based generations of the prosumers, to the notion of differential privacy. Indeed, the non-disclosure of the prosumers' target demands and RES-based generation introduces biases in the prosumers' strategies and equilibrium that might also be interpreted as a mean to protect sensitive information. A challenge would be to analyze the computational properties of the learning mechanism implemented by the data marketplace to reach a biased-forecast equilibrium for prosumers' given privacy levels. Last, a different point of view than the one adopted in this paper would be to consider the framework of cooperative games, in order to study the possibility of prosumers to form stable coalitions in which agents would share their locally produced energy and trade with other coalitions [44].

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