

Internal wave generation by extended sources

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Internal gravity waves in density stratified fluids differ strikingly from ‘classical’ waves, which, as sound or light, satisfy the wave equation. For instance, internal waves can only propagate with frequencies ω smaller than the buoyancy frequency N . Then the angle θ of their planes of constant phase to the vertical is fixed, as $\theta = \arccos(\omega/N)$, but their wavelength λ remains arbitrary; moreover their group velocity \mathbf{c}_g , with which energy propagates, is parallel to those planes and perpendicular to the phase velocity \mathbf{c}_ϕ .

Qualitative insight into the consequences of those properties on wave generation is provided by application of the group velocity theory [1]. In this way most experimental results are recovered. These include, for a monochromatic source $e^{i\omega t}$, as well the confinement of the waves on a characteristic cone of vertical axis, apex at the source and semi-angle $\arccos(\omega/N)$, as the motion of the surfaces of constant phase at right angle to this cone [2]–[3]. Similarly, for an impulsive source $\delta(t)$, both the filtering in each direction θ of the frequency $N \cos \theta$ which alone can propagate in this direction, and the conical shape of the surfaces of constant phase, are recovered, giving to the internal wave field a ‘fan-like’ appearance [3]–[4].

A tentative way of putting those conclusions into quantitative form is, in a linear formulation, to investigate the response of the stratified medium to a point mass source, and accordingly to calculate monochromatic and impulsive Green’s functions for internal waves [5]. Unfortunately, in both cases, the Green’s function gives useful indications on the waves but fails to describe them completely. Necessity arises then to consider extended sources. (Another regularising process involving viscosity was introduced in [6], and is discussed more thoroughly in [7]–[8].) Waves, expressed as the convolution of the Green’s function with the source function [9], are evaluated asymptotically at distances large compared with the source radius a and, for transient excitation, at times large compared with both the buoyancy period and the source duration Θ . Results of the evaluation are depicted in [10], and generalise those of [1] and [11].

Outside the conical shell defined by the two characteristic cones tangent to the source above and below, monochromatic waves reduce to those for a point source. However, they are of constant phase, implying no energy radiation. Inside the shell this expansion breaks down and must be replaced by one, identical to that in ([1], §4.10), involving coordinates Z and X tangent and normal to the characteristic cones, respectively, with $Z \gg a$ and $X \lesssim a$. There, waves exhibit an amplitude decay as $Z^{-\frac{1}{2}}$ conformable to energy conservation, and a phase variation with X consistent with the group velocity theory. However, not a single complete wavelength is identifiable at any time and the waves, whose amplitude varies as rapidly as the phase, are not quasi-plane. This is illustrated for a pulsating sphere, modelled as a surface distribution of mass sources, in which case the results of [5] and [12]–[13] are recovered.

Transient waves have the same wave packet form as for a point source, the amplitude varying slowly, but they are modulated by the spectrum of the source, in agreement with ([1], §4.8), [9], [11] and [14]. Far from an expanding torus of vertical axis, center at the

source and radius Nta , the wavelength is large, the source is small, and waves satisfy the group velocity theory. Near to the torus they are blurred by destructive interference, and well inside it they give way to non-propagating oscillations at the buoyancy frequency. Applying again the analysis to a pulsating sphere illustrates this, and confirms the results of [5] and [12].

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