

The trapping of lee waves

G. HUERRE AND B. VOISIN¹

LEGI, Institut de Mécanique de Grenoble, CNRS–UJF–INPG,

BP 53, 38041 Grenoble Cedex 9, France,

Phone: +33 4 76 82 70 57, Fax: +33 4 76 82 52 71, E-mail: Gael.Huerre@hmg.inpg.fr

In most natural systems internal gravity waves do not propagate freely but are trapped, inside regions of strong density gradients called thermoclines. Trapping takes place as a consequence of refraction by the buoyancy frequency profile, and reflection on fluid boundaries. In particular, in the oceans and in the atmosphere, lee waves generated by flows over obstacles change structure as trapping occurs: free lee waves have hyperbolic crests and troughs [1], [2], while trapped lee waves have divergent and transverse crests and troughs located within a wedge similar to the Kelvin wake of a ship [3], [4].

This paper adapts the approach elaborated in [5] for the generation of internal waves by bodies in arbitrary motion (or, equivalently, by flows with arbitrary time dependence) to situations where trapping is significant. First the Green's function of trapped internal waves is calculated, and from it the wave field generated by a moving source is deduced, generalizing [6], [7]. Waves are expressed in terms of the retarded time of their emission; the dispersion relationship being known, at each point and time the retarded time and the wavenumber satisfy a system of two equations stating propagation at the group velocity and stationarity of the wavecrests with respect to the source.

Lee waves, corresponding to uniform horizontal translation, are then considered, for two stratifications: a fluid of limited depth and constant buoyancy frequency [4], [8], and a thermocline associated with a hyperbolic tangent density profile [9], [10]. For translation at velocity U of a body of radius a in a fluid of maximum buoyancy frequency N_0 where internal waves are trapped in a layer of thickness ϵ , two internal Froude numbers can be defined: U/N_0a , which measures the effect of the dimension of the source, and $U/N_0\epsilon$, which measures the intensity of trapping. Detailed study of their influence is performed. Mechanisms leading to vertical mode selection are discussed, and theoretical results are compared with experiments [11].

- [1] Wurtele, M. G. (1957). *Beitr. Phys. Atmos.* **29**, 242–252.
- [2] Umeki, M. & Kambe, T. (1989). *Fluid Dyn. Res.* **5**, 91–109.
- [3] Gjevik, B. & Marthinsen, T. (1978). *Q. J. R. Met. Soc.* **104**, 947–957.
- [4] Sharman, R. D. & Wurtele, M. G. (1983). *J. Atmos. Sci.* **40**, 396–427.
- [5] Voisin, B. (1994). *J. Fluid Mech.* **261**, 333–374.
- [6] Keller, J. B. & Munk, W. H. (1970). *Phys. Fluids* **13**, 1425–1431.
- [7] Sturova, I. V. (1985). *Fluid Dyn.* **20**, 600–606.
- [8] Aksenov, A. V., Mozhaev, V. V., Skorovarov, V. E. & Sheronov, A. A. (1989). *Fluid Dyn.* **24**, 107–113.
- [9] Gray, E. P., Hart, R. W. & Farrell, R. A. (1983). *Phys. Fluids* **26**, 2919–2931.
- [10] Aksenov, A. V., Mozhaev, V. V., Skorovarov, V. E. & Sheronov, A. A. (1989). *J. Appl. Mech. Tech. Phys.* **30**, 101–105.
- [11] Laffly, F. (1996). Etude expérimentale du confinement des ondes internes. *Météo–France Report*.

¹Present address: DAMTP, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK, Phone: +44 1223 337031, Fax: +44 1223 337918, E-mail: B.Voisin@damtp.cam.ac.uk