

The build-up of internal wave fields

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A major difficulty of internal wave theory is, on the Boussinesq approximation, the absence of any length scale. Shorter waves accumulate at particular places and times, leading to a divergence of the amplitude and indefinite phase variations. For example, for a point monochromatic source, waves are confined on a characteristic cone of vertical axis and apex at the source; there, however, their amplitude diverges and their phase jumps, while no indication is obtained on their variations inside the cone.

Three mechanisms, at least, are responsible for eliminating the contribution of the smaller wavelengths: the finite time elapsed since the beginning of the motion, the finite size of any real source, and viscous attenuation. Similarly, each direction of a stratified fluid, inclined at an angle θ to the vertical, is an independent oscillator resonating at its natural frequency $\omega = N \cos \theta$, with N the buoyancy frequency; for an actual wave field to emerge some additional coupling mechanism is required, which transmits phase information to the neighbouring directions. And three such mechanisms are, as before, the dispersion of the initial impulse, the boundary condition at the surface of the wave generator, and viscous momentum transfer.

In this paper we examine how internal wave fields build up, taking all three possibilities into account (figure 1). Specifically, using results of [1,2], we apply the Green's function formalism to a monochromatic source of finite size switched on at some instant of time in a fluid of low viscosity. A transient is first observed. Soon it begins to decrease except in a narrowing vicinity of the characteristic cone [3], and there it becomes a monochromatic oscillation of increasing amplitude [4]. Then the size of the source and the viscosity of the fluid come into play [5]. Near the source, the size effect is dominant and the waves are confined inside a conical shell defined by the two characteristic cones tangent to the source above and below [6]. On these cones the velocity field is singular [7], inducing the development of boundary layers as energy propagates away from the source (figure 2). Ultimately the layers fill the whole of the shell [8], and the viscous self-similar region of [9] is reached.

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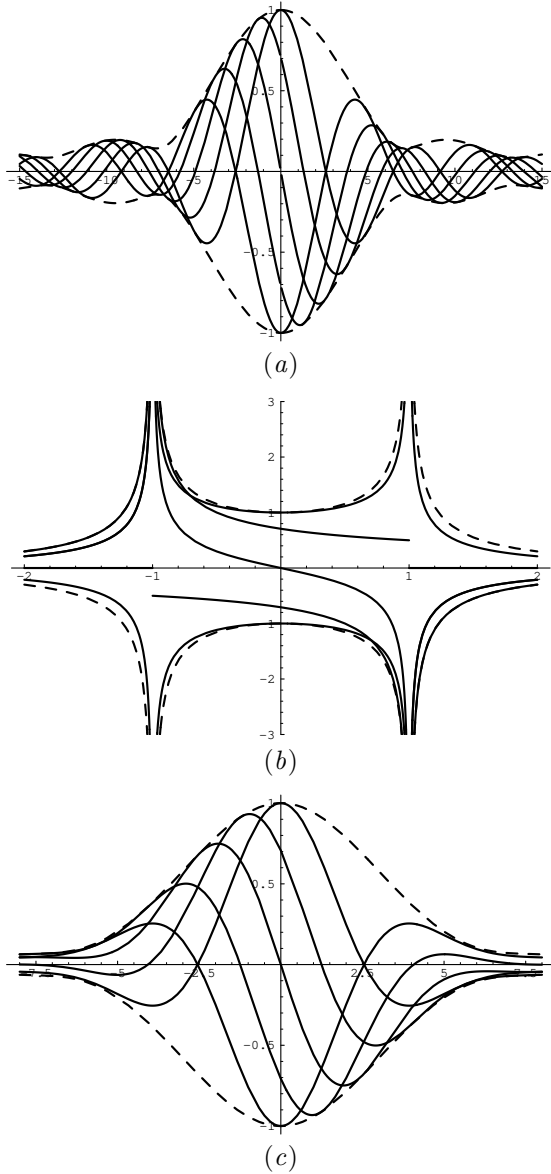


FIGURE 1. Velocity profile across the characteristic cone, for (a) a point source beginning to pulsate at time $t = 0$, (b) a pulsating sphere of radius a , (c) a point source pulsating in a fluid of kinematic viscosity ν . Velocity is represented in arbitrary units at successive times separated by one eighth of a period, as a function of (a) $(N^2 - \omega^2)^{1/2}t(\theta - \theta_0)$, (b) x/a , (c) $x/(\beta z)^{1/3}$; the dashed line is the velocity envelope. The direction $\theta = \theta_0 \equiv \arccos(\omega/N)$ or $x = 0$ corresponds to the characteristic cone with apex at the centre of the source, with ω the frequency, θ the angle to the vertical and z and x the distances along and perpendicular to the cone, respectively; $\beta = \nu/[2(N^2 - \omega^2)^{1/2}]$.

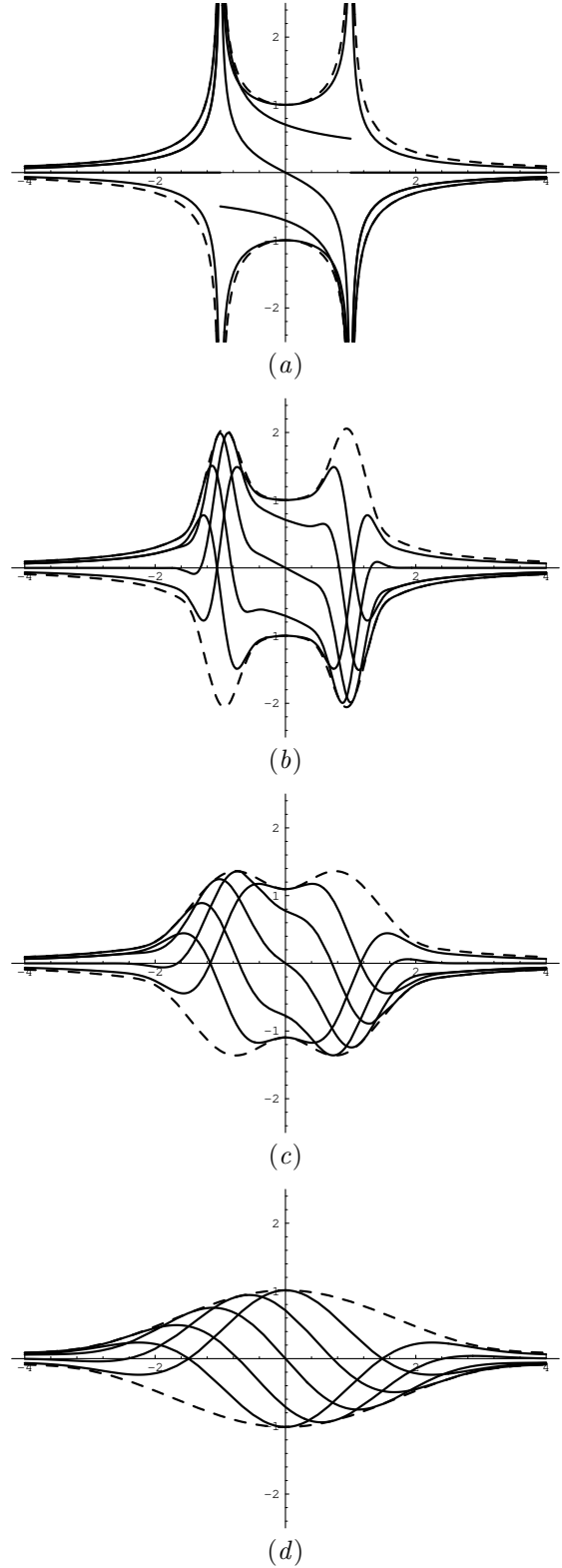


FIGURE 2. Transition, for a sphere pulsating in a viscous fluid, from the inviscid régime of figure 1b to the viscous régime of figure 1c. The velocity profiles are drawn at several dimensionless distances $\beta z/a^3$, as functions of x/a . (a) $\beta z/a^3 = 0$, (b) $\beta z/a^3 = 10^{-3}$, (c) $\beta z/a^3 = 10^{-2}$, (d) $\beta z/a^3 = 10^{-1}$.