

The generation and peculiarity of monochromatic internal waves

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Under monochromatic excitation of frequency ω smaller than the buoyancy frequency N , each direction of a stratified fluid, inclined at angle $\theta = \arccos(\omega/N)$ to the vertical, acts as an independent oscillator along which fluid motion and energy propagation take place. For an actual wave field to emerge some additional coupling mechanism is required, which transmits phase information to the neighbouring fluid directions. Relevant mechanisms are the boundary conditions at the surface of the source of the waves on the one hand, and viscosity on the other hand. The present paper applies the Green's function formalism to this problem, taking both possibilities into account. In this way a spatial picture of how monochromatic internal wave fields build up is obtained, complementing the spectral picture proposed in [1].

Proceeding from the Green's function of the inviscid internal wave equation [2], a superposition procedure leads to the expression of the waves generated by any source of finite spatial extent. Lighthill's result [1] is recovered, with the same interpretation: waves are confined to a conical shell of the same thickness as the source and angle θ to the vertical; there, they exhibit a longitudinal decrease inversely proportional to the square root of the distance from the source, and transverse phase variations. Two particular cases are considered: a Gaussian mass source, and a pulsating sphere. Waves are not quasi-plane, in that the variations of their amplitude are not slow compared with the variations of their phase, and not even a complete wavelength is observed. For the pulsating sphere, moreover, singularities arise along the edges of the shell, the total radiated energy remaining finite and of the same order as for the Gaussian source. Such singularities, consistent with direct calculations [2–5], are an artifact of the inviscid world and characterize any source with well-defined boundaries. Upon incorporation of viscosity into the theory, the singularities are replaced by “boundary layers”, which grow up in size with increasing distance from the source and eventually fill the whole of the shell, resulting in a faster decrease of the amplitude and a progressive widening of the shell. Then the viscous self-similar region investigated in two dimensions e.g. in [6–8] is reached.

- [1] LIGHTHILL, M. J. (1978) *Waves in Fluids*. Cambridge University Press.
- [2] VOISIN, B. (1991) *J. Fluid Mech.* **231**, 439–480.
- [3] HENDERSHOTT, M. C. (1969) *J. Fluid Mech.* **36**, 513–527.
- [4] GORODTSOV, V. A. & TEODOROVICH, E. V. (1986) *J. Appl. Mech. Tech. Phys.* **27**, 523–529.
- [5] APPLEBY, J. C. & CRIGHTON, D. G. (1987) *J. Fluid Mech.* **183**, 439–450.
- [6] THOMAS, N. H. & STEVENSON, T. N. (1972) *J. Fluid Mech.* **54**, 495–506.
- [7] PETERS, F. (1985) *Exp. Fluids* **3**, 261–269.
- [8] IVANOV, A. V. (1989) *Izv. Atmos. Ocean. Phys.* **25**, 61–64.