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# Application of linear functional observers for the thermal estimation in power modules

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## Keywords

«Thermal modeling», «State-space representation», «Linear functional observer».

## Abstract

High integrated power electronic modules are more and more designed with the emergence of new semiconductor technologies. Thus, increase of reliability of power modules induces the precise knowledge of the local temperature, even if it can not be measured at any location. In this paper, the application of an observer is proposed. It allows to estimate the temperature at any location using measurements provided from thermal sensors located at a few precise points. The aim is to design a reduced size observer that could be implemented on a real-time embedded target such as Digital Signal Processor. Consequently, it is necessary to obtain a minimal order observer to limit the computation complexity.

## Introduction

The joint emergence of Wide Band Gap materials (SiC, GaN, C) and new generation hybrid integration techniques significantly enhance performances of power electronic modules. Such modules should operate in severe environment and constraints: high temperature and high power density, fast switching, etc. Consequently of high temperature, new constraints appear and become critical for power electronics assemblies. Several studies aim at identifying failure modes or critical interfaces [1], [2]. Thus, estimation of local temperatures becomes a real challenge in new generation of power modules to increase their lifetime. Indeed, it has been shown in [3], [4] that the evolution of local constraints in a power electronic module, which can be thermal or thermo-mechanical, have a negative effect on the lifetime of the module. These constraints increase the occurrence of potentially critical defects and failures on the module. Consequently, it becomes necessary to have a precise knowledge of the temperatures at specific locations in the module, such as the temperature of semi-conductor chips or wire bondings. However, due to the size of sensors and possible electromagnetic field disturbances close to measurement points, the use of thermal sensors may be difficult at some locations inside of the power module. For these reasons the objective of the following work is to estimate this physical variable in a specific non measured location, using measured data by few sensors.

As a case study, a simple two-dimensions (2D) thermal system is considered in this paper and then modeled using an analogy between thermal and electrical domains. Then, equations of thermal evolution of

the system with respect to time and space can be rewritten using a linear state-space representation. Using this representation, the temperature can be estimated at any location with a linear functional observer or a partial state observer.

The first section deals with the construction of a thermal model and its representation in state space. In this work, the thermal behavior of a  $(30 \times 30\text{mm})$  2D heated plate which may represent a section of a power electronics module is considered as a test benchmark of our technics. The matrix representation of previous model is established and aims to design a reduced size observer. We propose in the second section a way to design an observer, based on the use of successive derivatives of the measured outputs. The interest of the observer design lies in the possibility to observe the temperature at any location in the system. Finally, through simulation results, the application of a reduced size observer on the 2D heated plate is validated in the last section.

## Thermal model 2D

### Thermal model

The thermal evolution of a 2D heated plate is given by the heat equation (1), [5], [6] with:

- $T$  the local temperature in  $K$ ,
- $t$  the time in  $s$ ,
- $\rho$  the mass density of the material in  $kg.m^{-3}$ ,
- $C_p$  the thermal capacity in  $J.kg^{-1}.K^{-1}$ ,
- $\lambda$  the thermal conductivity in  $W.m^{-1}.K^{-1}$ ,
- $S$  the heat source in the system in  $W.m^{-3}$

$$\rho C_p \frac{\partial T(x, y, t)}{\partial t} = -\lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + S \quad (1)$$

The heat equation (1) reflects linear transfer phenomena such as conductive and convective transfers induced by the presence of the temperature gradient represented by (2), [7], [8], [9]:

$$\vec{\phi} = -\lambda \cdot \overrightarrow{\text{grad}} T \quad (2)$$

where  $\vec{\phi}$  stands for the heat flux density.

Radiative transfers are non considered in the heat equation. However, in this work, this kind of transfers are neglected. As (2) is similar to Ohm's law in electrical domain, a thermo-electrical analogy between the different domains can be defined and summarized in Table I, [10].

Table I: Electro-thermal analogy

Thermal domain	Electrical domain
Parameter (unit), Notation	Parameter (unit), Notation
Temperature ( $K$ ), $T$	Electric potential ( $V$ ), $V$
Thermal flux ( $W$ ), $Q$	Current ( $A$ ), $I$
Thermal resistance ( $m^2 K/W$ ), $R_{th}$	Resistance ( $\Omega$ ), $R$
Heat capacity ( $J/K$ ), $C_{th}$	Capacity ( $F$ ), $C$

This analogy leads to obtain an equivalent electrical model which represents the thermal behavior of a system.

In order to design a continuous-time observer, the first step is to discretize the obtained model with respect to the two dimensions of space (see Fig. 1).

Using a spatial finite difference discretization of heat equation and the previously defined thermo-electrical analogy, thermal behavior of the 2D plate is modeled as a network composed of resistors for spatial thermal conductivity and convection, capacitors for heat storage, voltage sources for temperature sources

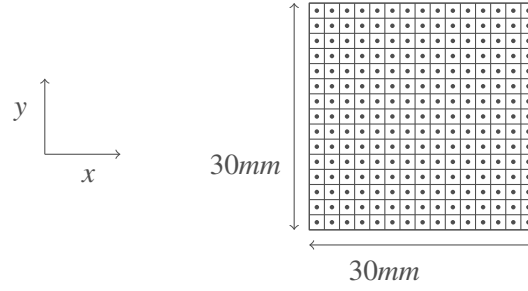


Fig. 1: Surface discretization of the heated plate into elementary surfaces (illustrative example)

and current sources for heat sources, [11]. On the one hand, the conductive transfer (resp. convective) is characterized by a conduction resistance  $R_{cd}$  (resp. convection resistance  $R_{cv}$ ) defined by :

$$R_{cd} = \frac{e}{\lambda S} \quad (\text{resp. } R_{cv} = \frac{1}{hS}) \quad (3)$$

where  $e$  is the distance between two nodes,  $S$  is the exchange surface between elementary surfaces and  $h$  is the convection coefficient, [12].

On the other hand, the storage of thermal energy in an elementary surface is modeled by a thermal capacity  $C_{th}$  connected between the center and the mass (thermal reference) and given by:

$$C_{th} = \rho C_p V$$

where  $V$  is the volume of the material in  $m^3$ .

The ambient temperature is represented by thermal sources of values  $T_a$  on each border of the plate. Finally, heat sources  $P_{th}$  may be inserted into some elements to induce the dynamic thermal response of the system.

### State space representation of the heated plate

From the spatial discretization, the temperature  $T$  is defined on the centers of the elementary surfaces (denoted nodes in the following) of the plate. It depends on the temperatures of its neighbors and thermal impedances connected to the considered element. Depending on the position of nodes on the plate, two kinds of impedances connected to the nodes must be considered as shown in Fig. 2. Thus, the heated plate is represented by a network of impedances that translates the conduction between center and edges of this surface.

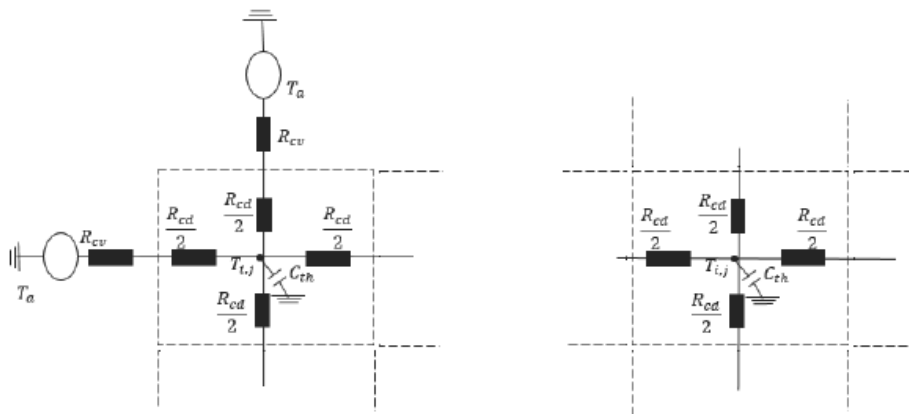


Fig. 2: Representation of an impedance network of an elementary surface at the edge (left) and inside (right) of the plate, [13]

These elementary schemes must be combined to build the complete network described in Fig. 3. Millman's

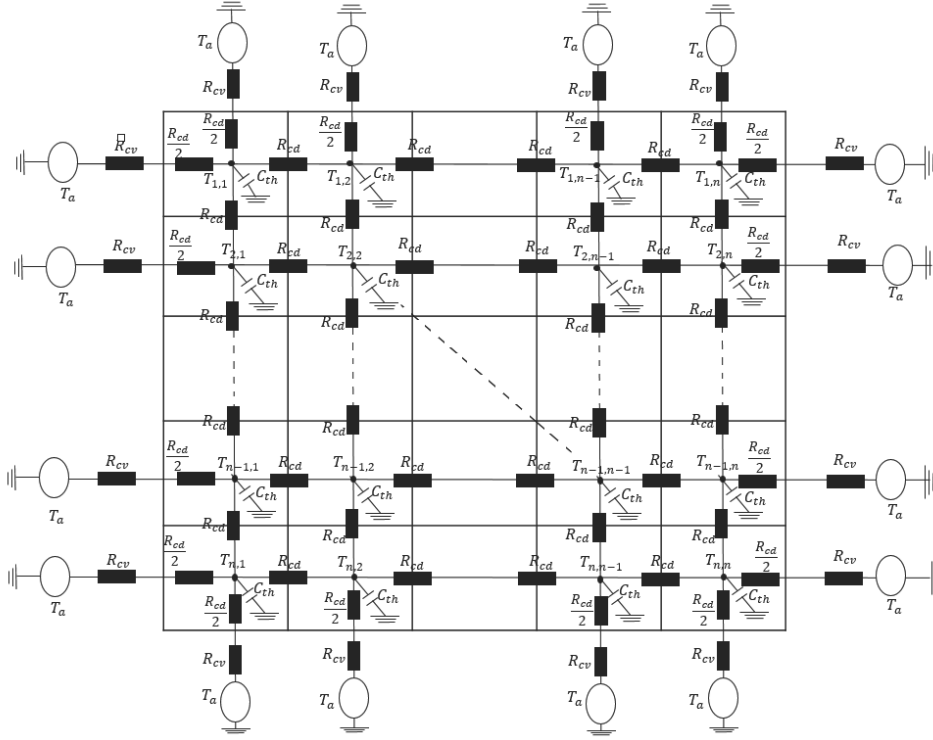


Fig. 3: Electro-thermal nodal model of the heated plate [13]

theorem [14] allows to express the temperature for each node with a first order differential equation. By combining all node's equations, a state space representation is obtained (4).

$$\begin{cases} C_{th}\dot{\mathbf{T}}(t) = A\mathbf{T}(t) + B \begin{pmatrix} T_a(t) \\ P_{th}(t) \end{pmatrix} \\ \mathbf{y}(t) = C\mathbf{T}(t) \end{cases} \quad (4)$$

where  $\mathbf{T}(t)$  is the vector of local temperatures,  $C_{th}$  is the diagonal matrix of thermal capacities,  $A$  is the thermal resistances matrix,  $\mathbf{y}(t)$  is the measurements vector,  $C$  is the sensors position, and  $B$  the influence of boundary conditions for temperatures and heat sources on the plate.

Simulation of local temperatures is then obtained through the state space model (4). As a contrary to experiments, simulation of (4) allows the knowledge of all temperatures. Consequently, the estimation of the temperature in a specific non measured location is necessary. We propose in this paper to achieved this objective through linear functional observers.

## Estimation of non-measured variables

### Functional linear observers

Let us consider a system described by the linear state space equations:

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) \\ \mathbf{y}(t) = C\mathbf{x}(t) \end{cases} \quad (5)$$

where, for every  $t$  in  $\mathbb{R}^+$ ,  $\mathbf{x}(t)$  is a  $n$ -dimensional state vector,  $\mathbf{u}(t)$  is a  $p$ -dimensional control vector supposed to be known,  $\mathbf{y}(t)$  is a  $m$ -dimensional measured vector and,  $A(n \times n)$ ,  $B(n \times p)$  and  $C(m \times n)$  are constants matrices.

The aim of a functional observer is to estimate state variables, at least asymptotically, from the measurements on the system. Estimated state variables are defined by :

$$\mathbf{v}(t) = L\mathbf{x}(t) \quad (6)$$

where  $L$  is a constant full row rank ( $l \times n$ ) matrix selecting estimated components.

The observation of  $\mathbf{v}(t)$  can be carried out by the designed linear functional observer or Luenberger observer, [15], [16], which is described by the state equations:

$$\begin{cases} \dot{\mathbf{z}}(t) = F\mathbf{z}(t) + G\mathbf{u}(t) + H\mathbf{y}(t) \\ \hat{\mathbf{v}}(t) = P\mathbf{z}(t) + V\mathbf{y}(t) \end{cases} \quad (7)$$

where  $\mathbf{z}(t)$  is a  $q$ -dimensional state vector and  $\hat{\mathbf{v}}(t)$  is a  $l$ -dimensional vector. The constants matrices  $F, G, H, P, V$  and the order  $q$  are determined such that  $\lim_{t \rightarrow +\infty} (\mathbf{v}(t) - \hat{\mathbf{v}}(t)) = 0$ . Moreover, it must be kept in mind that we look for a minimal order observer. The asymptotic tracking is ensured if  $F$  is a Hurwitz matrix, i.e. all the real part of its eigenvalues are negative. The Fig. 4 expresses the structure of an estimator: A reduced  $(n-l)^{th}$  order observer introduced in [15], [16] can be designed. Nevertheless,

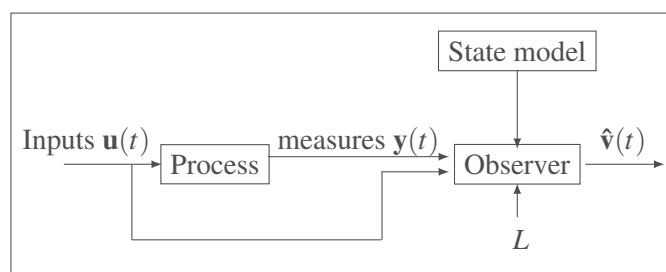


Fig. 4: Estimator principle

the main drawback lies in estimating the whole state and the observer has a too high order as well. To overcome this point, a linear functional observer induces a relevant reduction in the observer order.

### Design of a Luenberger observer

This section deals with the search for a minimum order of a functional observer. This point is achieved in order to obtain a fast and implementable observer.

Let  $q$  be the smallest integer such that,

$$\text{rank } \Sigma_q = \text{rank} \begin{pmatrix} \Sigma_q \\ LA^q \end{pmatrix} \quad (8)$$

with:

$$\Sigma_q = \begin{pmatrix} C \\ L \\ CA \\ LA \\ \vdots \\ CA^{q-1} \\ LA^{q-1} \\ CA^q \end{pmatrix}$$

### First step

The design of the observer uses the successive derivations of  $\mathbf{v}(t)$ . After  $q$  derivations of  $\mathbf{v}(t) = L\mathbf{x}(t)$ , we obtain:

$$\mathbf{v}^{(q)}(t) = LA^q \mathbf{x}(t) + \sum_{i=0}^{q-1} LA^i B \mathbf{u}^{(q-1-i)}(t) \quad (9)$$

It can be noticed from (8) that it exists matrices,  $\Gamma_i, i \in \llbracket 0 ; q \rrbracket$  and  $\Lambda_i, i \in \llbracket 0 ; q-1 \rrbracket$  such that:

$$LA^q = \sum_{i=0}^q \Gamma_i CA^i + \sum_{i=0}^{q-1} \Lambda_i LA^i \quad (10)$$

Using (10), (9) can be written as:

$$\mathbf{v}^{(q)}(t) = \sum_{i=0}^q \Gamma_i CA^i \mathbf{x}(t) + \sum_{i=0}^{q-1} \Lambda_i LA^i \mathbf{x}(t) + \sum_{i=0}^{q-1} LA^i B \mathbf{u}^{(q-1-i)}(t) \quad (11)$$

### Second step

The second step is to eliminate the state  $\mathbf{x}(t)$  from (11) so that  $\mathbf{v}^{(q)}(t)$  will be expressed only with  $\mathbf{v}(t)$ ,  $\mathbf{y}(t)$ ,  $\mathbf{u}(t)$  and their successive derivatives. To do so, the state equation (5) is used after each derivation of  $\mathbf{v}(t) = L\mathbf{x}(t)$  and  $\mathbf{y}(t) = C\mathbf{x}(t)$ , [17]. Thus, we get:

$$\mathbf{v}^{(q)}(t) = \sum_{i=0}^q \Gamma_i \mathbf{y}^{(i)}(t) + \sum_{i=0}^{q-1} \Lambda_i \mathbf{v}^{(i)}(t) + \sum_{i=0}^{q-1} \Phi_i \mathbf{u}^{(i)}(t) \quad (12)$$

where, for  $i \in \llbracket 0 ; q-2 \rrbracket$ :

$$\Phi_i = \left[ LA^{q-1-i} - \sum_{j=i+1}^q \Gamma_j CA^{j-i-1} - \sum_{j=i+1}^{q-1} \Lambda_j LA^{j-i-1} \right] B$$

and

$$\Phi_{q-1} = [L - \Gamma_q C] B$$

### Third step

The third step consists in realizing the input-output differential equation (12) [17], [18], as:

$$\begin{cases} \dot{\mathbf{z}}(t) = \begin{pmatrix} 0 & & \Lambda_0 \\ 1 & \ddots & \Lambda_1 \\ & \ddots & 0 & \vdots \\ & & 1 & \Lambda_{q-1} \end{pmatrix} \mathbf{z}(t) + \begin{pmatrix} \Phi_0 \\ \Phi_1 \\ \vdots \\ \Phi_{q-1} \end{pmatrix} \mathbf{u}(t) + \begin{pmatrix} \Gamma_0 + \Lambda_0 \Gamma_q \\ \Gamma_1 + \Lambda_1 \Gamma_q \\ \vdots \\ \Gamma_{q-1} + \Lambda_{q-1} \Gamma_q \end{pmatrix} \mathbf{y}(t) \\ \hat{\mathbf{v}}(t) = [0 \ 0 \ \dots \ 0 \ 1] \mathbf{z}(t) + \Gamma_q \mathbf{y}(t) \end{cases} \quad (13)$$

When  $F$  is a Hurwitz matrix, it is demonstrated that (13) is an asymptotic observer of the functional linear  $L\mathbf{x}(t)$ . Otherwise, it becomes necessary to increase the order  $q$  and to do again the building procedure with a higher order, [19], [20].



## Application to the 2D heated plate

### Design of a minimal-order observer

In this section, the functional observer is applied on temperature estimation of a heated plate. In order to closely follow the design of the observer, the plate is spatially discretized into 9 elementary surfaces (Fig. 5) leading to a 9-order state space model.  $Ca$ ,  $S$  and  $O$  denotes respectively the sensor, the heat source and the estimated temperature locations. Let us remark that from symmetrical reasons, for iden-

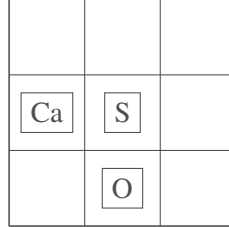


Fig. 5: Discretization of the heated plate

tical initial conditions, the temperature of cell  $Ca$  shall be equal to the one of cell  $O$ .

Considering the modeling method in the previous section, the following state space representation is obtained:

$$\begin{aligned}
 A &= \begin{pmatrix} a & b & 0 & b & 0 & 0 & 0 & 0 & 0 \\ b & c & b & 0 & b & 0 & 0 & 0 & 0 \\ 0 & b & a & 0 & 0 & b & 0 & 0 & 0 \\ b & 0 & 0 & c & b & 0 & b & 0 & 0 \\ 0 & b & 0 & b & d & b & 0 & b & 0 \\ 0 & 0 & b & 0 & b & c & 0 & 0 & b \\ 0 & 0 & 0 & b & 0 & 0 & a & b & 0 \\ 0 & 0 & 0 & 0 & b & 0 & b & c & b \\ 0 & 0 & 0 & 0 & 0 & b & 0 & b & a \end{pmatrix} \\
 B &= 10^{-3} \begin{pmatrix} g & 0 \\ f & 0 \\ g & 0 \\ f & 0 \\ 0 & f \\ f & 0 \\ g & 0 \\ f & 0 \\ g & 0 \end{pmatrix} \\
 C &= \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\
 L &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}
 \end{aligned} \tag{14}$$

where  $a = -1.4, b = 0.0069, c = -0.71, d = -0.028, f = 0.69$  and  $g = 1.39$ . These values are obtained using the thermal characteristics of the material of the heated plate.

Obviously, the structure of the matrices  $C, L$  and  $B$  are linked to the locations of  $Ca, O$  and  $S$  in the plate.

The following steps illustrate how to obtain a minimal observer:

- **Search of the minimal order:**

**Iteration 1:** Test for  $q = 1$ .

As:

$$CA = 10^{-3} \times (6.9 \ 0 \ 0 \ -715 \ 6.9 \ 0 \ 6.9 \ 0 \ 0)$$

$$LA = 10^{-3} \times (0 \ 0 \ 0 \ 0 \ 6.9 \ 0 \ 6.9 \ -715 \ 6.9)$$

we obtain  $\text{rank}(\Sigma_1) = 3$  and  $\text{rank} \begin{pmatrix} \Sigma_1 \\ LA \end{pmatrix} = 4$ .

Thus, a first-order minimum observer cannot be designed.

**Iteration 2:** Test for  $q = 2$ .

As:

$$CA^2 = 10^{-3} \times (-14.7 \ 0.1 \ 0 \ 512 \ -5.2 \ 0 \ -14.7 \ 0.1 \ 0)$$

$$LA^2 = 10^{-3} \times (0 \ 0 \ 0 \ 0.1 \ -5.2 \ 0.1 \ -14.7 \ 512 \ -14.7)$$

we get  $\text{rank}(\Sigma_2) = 5$  and  $\text{rank} \begin{pmatrix} \Sigma_2 \\ LA^2 \end{pmatrix} = 6$ .

Thus, a second-order minimum observer cannot be designed.

**Iteration 3:** Test for  $q = 3$ .

$$CA^3 = 10^{-3} \times (24.2 \ -0.2 \ 0 \ -366 \ 3.7 \ -0.1 \ 24.2 \ -0.2 \ 0)$$

$$LA^3 = 10^{-3} \times (0 \ -0.1 \ 0 \ -0.2 \ 3.7 \ -0.2 \ 24.2 \ -366 \ 24.2)$$

we get  $\text{rank}(\Sigma_3) = 7$  and  $\text{rank} \begin{pmatrix} \Sigma_3 \\ LA^3 \end{pmatrix} = 7$

Thus, a minimal third-order candidate observer can be envisaged.

It has to be verified that  $F$  is a Hurwitz matrix:

$$LA^3 \Sigma_3^\dagger = (0.72 \ -0.72 \ 2.52 \ -2.52 \ 2.83 \ -2.83 \ 1)$$

Then,  $\Lambda_0 = -0.72, \Lambda_1 = -2.52, \Lambda_2 = -2.83$ . It yields to:

$$F = \begin{pmatrix} 0 & 0 & -0.72 \\ 1 & 0 & -2.52 \\ 0 & 1 & -2.83 \end{pmatrix}$$

The eigenvalues of  $F$  are  $(-0.715, -0.715, -1.4)$  and  $F$  is effectively a Hurwitz matrix. Consequently, the observer is asymptotically convergent.

- **Design of the minimal third-order observer:**

From  $LA^3 \Sigma_3^\dagger$ , it gets  $\Gamma_0 = 0.72, \Gamma_1 = 2.52, \Gamma_2 = 2.83$  and  $\Gamma_3 = 1$ .

Using (13), the matrices of the observer are deduced:

$$G = \begin{pmatrix} 1.43 \times 10^{-14} & 7.76 \times 10^{-16} \\ 1.16 \times 10^{-13} & 4.88 \times 10^{-16} \\ 7.03 \times 10^{-14} & 0 \end{pmatrix}, H = \begin{pmatrix} 5.66 \times 10^{-14} \\ 1.18 \times 10^{-13} \\ 5.06 \times 10^{-14} \end{pmatrix}$$

$$P = (0 \ 0 \ 1), \text{ and } V = 1$$

Considering orders of magnitude of the coefficients in  $G$  and  $H$ , they can be approximated by:

$$G = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, H = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, P = (0 \ 0 \ 1), V = 1$$

Then, the estimated temperature  $\hat{T}$  is calculated with (15).

$$\begin{cases} \dot{\mathbf{z}}(t) = F\mathbf{z}(t) \\ \hat{T}(t) = P\mathbf{z}(t) + T_{Ca}(t) \end{cases} \quad (15)$$

It can be noticed that the result obtained verifies the physical assumption done previously. From symmetry, the observer output leads to an estimated temperature in position  $O$  equal to the one in position  $Ca$ , up to the transient depending on initial conditions.

The observer design is ended. However, when the obtained poles  $(-0.715, -0.715, -1.4)$  are not suitable to ensure a sufficiently fast dynamic of the observer error, the observer order has to be increased.

## Simulation results

### Identical initial conditions

The simulation results of the designed observer for point  $O$ , and the initial system for points  $O$  and  $Ca$  is given in Fig. 6. It can be noted that the temperature is a relative evolution regarding initial conditions of equilibrium. It is also assumed that  $P_{th}(t) = 0$ , for  $t < 0$ . Thus, only a temperature variation study of the system around the equilibrium point is performed. Results displayed in Fig. 6 confirm the design procedure.

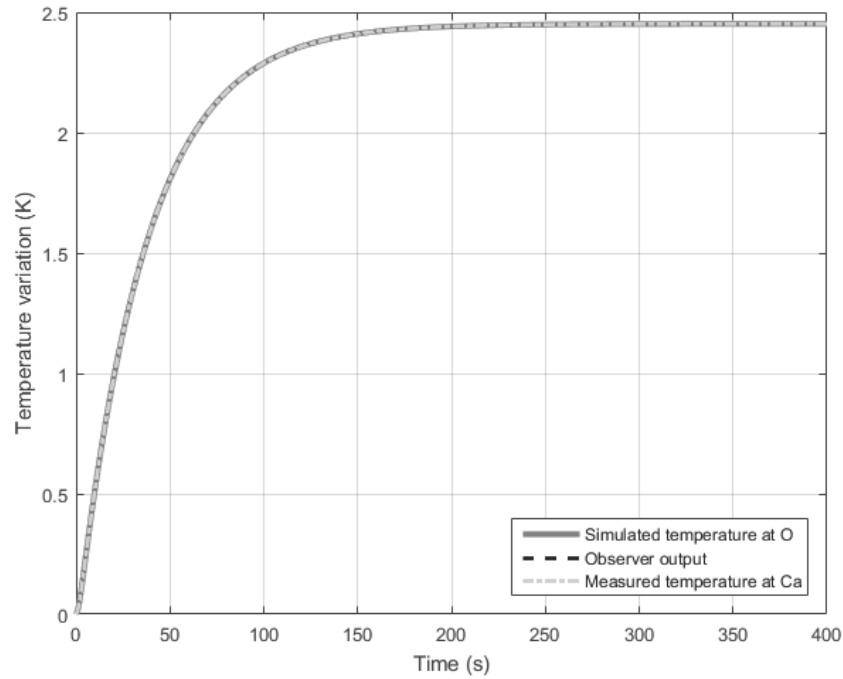


Fig. 6: Simulation result of the third order observer for (14) with identical initial conditions

### Different initial conditions

In order to evaluate observer efficiency, an other simulation is displayed in Fig. 7 with different initial conditions. In practice, this may correspond to a change in the heating flux or ambient temperature during the experiment. It can be seen in Fig. 7 that the observe output converges to the simulated temperature with the dynamic of the observer, namely the eigenvalues of  $F$ . It can be concluded that this observer is suitable to estimate the temperature whatever the initial conditions.

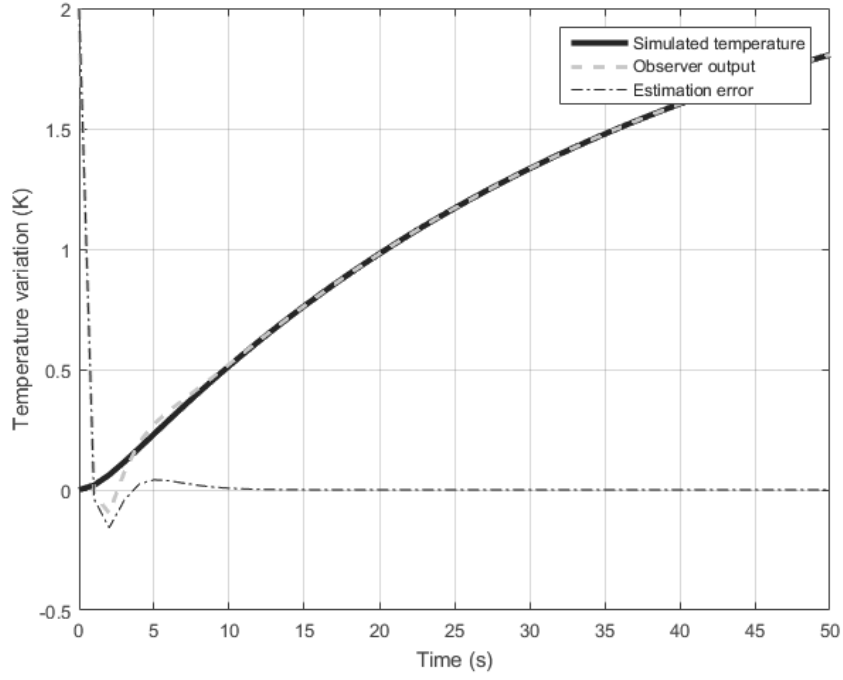


Fig. 7: Simulation result of the third order observer for (14), with non zero initial conditions.

### Variation in spatial positions of sensor, source and estimated temperature

An other example is studied here. The system with positions of sensor, source and estimated point is depicted in Fig. 8. The position of the sensor and the estimated point are chosen to ensure non symmetry in the estimation problem, as shown in Fig. 8.

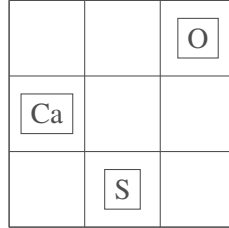


Fig. 8: Discretization of the system

Design procedure of the observer given previously is performed on the case described in Fig. 8. A linear functional observer of  $q = 4$  is obtained, defined by:

$$\begin{cases} \dot{\mathbf{z}}(t) = \begin{pmatrix} 0 & 0 & 0 & -1.01 \\ 1 & 0 & 0 & -4.25 \\ 0 & 1 & 0 & -6.49 \\ 0 & 0 & 1 & -4.24 \end{pmatrix} \mathbf{z}(t) + \begin{pmatrix} 0.997 & -2.33 \times 10^{-7} \\ 3.5 & -1.28 \times 10^{-13} \\ 3.94 & 0 \\ 1.39 & 0 \end{pmatrix} \begin{pmatrix} T_a(t) \\ P_{th}(t) \end{pmatrix} + \begin{pmatrix} 0.01 \\ 0.035 \\ 0.039 \\ 0.014 \end{pmatrix} T_{Ca}(t) \\ \hat{T}(t) = (0 \ 0 \ 0 \ 1) \mathbf{z}(t) - (1.9 \times 10^{-9}) T_{Ca}(t) \end{cases} \quad (16)$$

As in previous example, observer equations may be approximated by (17).

$$\begin{cases} \dot{\mathbf{z}}(t) = \begin{pmatrix} 0 & 0 & 0 & -1.01 \\ 1 & 0 & 0 & -4.25 \\ 0 & 1 & 0 & -6.49 \\ 0 & 0 & 1 & -4.24 \end{pmatrix} \mathbf{z}(t) + \begin{pmatrix} 0.997 & -2.33 \times 10^{-7} \\ 3.5 & 0 \\ 3.94 & 0 \\ 1.39 & 0 \end{pmatrix} \begin{pmatrix} T_a(t) \\ P_{th}(t) \end{pmatrix} + \begin{pmatrix} 0.01 \\ 0.035 \\ 0.039 \\ 0.014 \end{pmatrix} T_{Ca}(t) \\ \hat{T}(t) = (0 \ 0 \ 0 \ 1) \mathbf{z}(t) \end{cases} \quad (17)$$

The simulation result is given in Fig. 9.

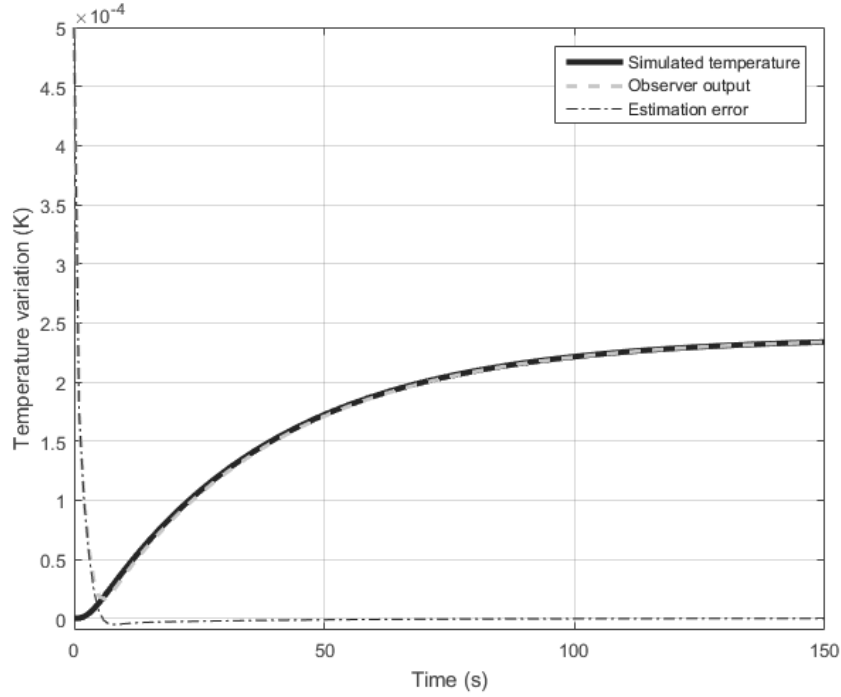


Fig. 9: Simulation result of the fourth order observer, for the case in Fig. 8, with non zero initial conditions.

From these examples, it can be concluded that the functional linear observer is able to accurately estimate the temperature of a heated plate, whatever initials conditions. Moreover, relative positions of sensor, source and estimated points have an impact on the order of the observer.

#### Increasing of the accuracy of spatial discretization

Let's consider a system model with a more accurate discretization with  $(11 \times 11)$  elementary surfaces leading to a  $121^{th}$  order state space model. The position of the sensor and the estimated point are chosen to ensure non symmetry in the estimation problem. An observer is designed according to the procedure described previously. Thus, a linear functional observer of order  $q = 13$  is obtained. The asymptotic convergence of the simulated temperature  $T$  and the estimated temperature  $\hat{T}$  is checked. This study points out the advantage of a linear functional observer compared to a reduced observer of order  $n - l = 120$ . The simulation result is given in Fig. 10.

It can be concluded that the observer accurately estimates the temperature of a desired point, whatever the initial conditions.

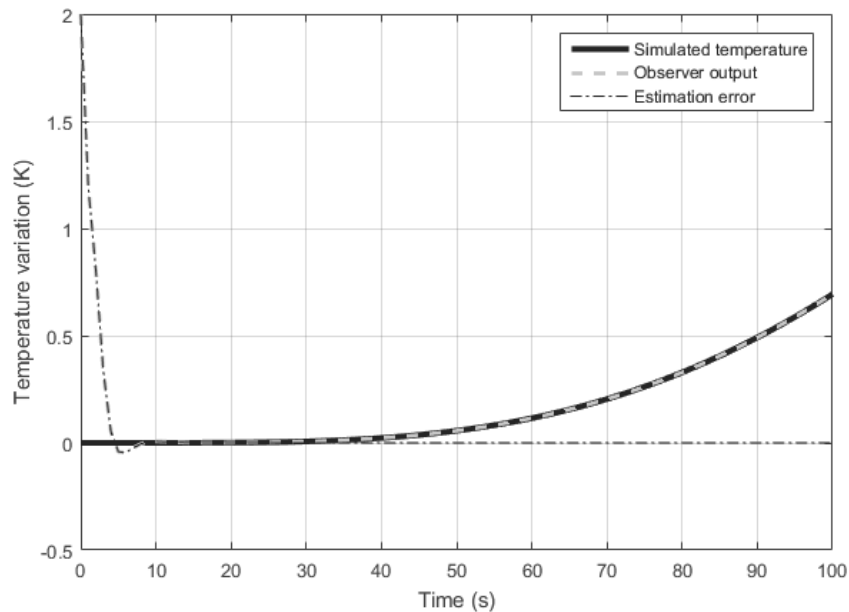


Fig. 10: Simulation result of observer of order  $q = 13$ , for a state model of order 121, with non zero initial conditions.

## Conclusion

This paper presented a thermal modeling of a heated plate, using the finite difference discretization for the  $2D$  heat equation, leading to a state space representation of the system with the thermo-electrical analogy.

From this state space representation, a linear functional observer has been designed in order to estimate the temperature in any desired point using few measurements and the knowledge of inputs. This kind of observer induces a relevant reduction in the observer order comparing to the initial system dimension. It has been demonstrated that the observer was able to accurately estimate the temperature evolution of a desired point, whatever the initial conditions. Moreover, it has been exemplified that the order of the observer was linked to the relative positions of points of interest and to the dimension of the state space representation of the system.

The main objective of this paper was to show the feasibility of our approach to set fast estimators.  $3D$  thermal models may be developed to be more representative of power electronic modules. and the proposed method for design minimal order observers may be easily applied on these models. The only difference lies in the dimension of matrices of the system and the observer. Our future work will be to experimentally validate thermal models for the observation on a  $3D$  system. However, these models will be naturally of large dimension and state space representations will be huge. Thus, it could be supposed that even the linear functional observer will not drastically reduce the order of the problem. In this case, the observer designed from experimentally identified transfers could be studied. Using small order transfers, the associated observer would have a limited dimension. This particular point will be developed in further work.

An other future research linked to the presented work will be to consider unknown perturbations and power sources acting on the heated plate. Functional observer will be design in this framework.

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