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# Teachers designing e-books to foster creative mathematical thinking: the case of curvature

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*This study focuses on the design of a novel genre of e-books incorporating dynamic constructionist artefacts-widgets that aim to induce mathematical creativity allowing students to interact with their content in significant ways (called 'c-books', c for creativity). The design of the c-books is addressed through collectives of educational professionals with a diversity of expertise. The analysis of the design process of a c-book on Curvature shows that the interactions fostered by the socio-technical environment allowed diverse practitioners to learn from, work with and collaborate across their boundaries supported by collectively evolving artefacts as boundary objects. The end-product of this social creative process, the c-book on Curvature, evolved through the constant versioning of a narrative intertwined with malleable dynamic constructionist artefacts.*

*Keywords: Curvature, design of digital resources, social creativity, documentational approach of didactics, theory networking.*

## Introduction

Curvature is weakly addressed as a conceptual field (Vergnaud, 2009) with respect to its potential to generate environments rich in opportunities for mathematical meaning making by students. In traditional curricula it lies disparately in Euclidean Geometry sections, in Algebra and Calculus depicting systematic co-variation, in 3D Geometry but in simple applications like conic sections. The dynamic and diverse representational repertoire provided by digital media allows us to approach curvature anew with a disposition to re-structure (to use Wilensky & Papert's term, 2010) the ways in which mathematics is conceptualized in education in the quest to make it more attractive to students, affording meaning making, creative mathematical thinking (through problem-solving/posing and constructionist activity) and engagement. Previous research has shown that dynamic digital environments and especially 3D spatial environments support students in constructing meanings about challenging conceptual fields (Kynigos & Psycharis, 2003; Zantzou & Kynigos 2012). In this study, we look at a digital medium affording the meshing of narrative with malleable constructionist artefacts such as *half-baked microworlds* (Kynigos, 2007) and study the ways in which a diverse community of professionals jointly design narratives around curvature with an explicit interest to afford creative thinking in their prospective students. We perceived of the medium which we called the 'c-book', as potentially affording thinking out of the box, allowing for restructuration of mathematical conceptual fields. We also began from the premise that collaborative design by educational professionals with diverse expertise would generate a socio-technical environment more likely to produce creative ways to enhance creative mathematical thinking (CMT) for students. In this study, the designed c-book eventually focused on a story involving comparisons between Archimedes' and exponential spirals. In addition, by involving in-service teachers in all stages of the design process, we aimed to induce their reflection on the affordances and pedagogies incorporated in the tools, the tasks and narratives under

development, as well as the changes to the mathematical content and the classroom practices that the presence of technology brings about.

## Theoretical background

The design of digital educational resources for fostering mathematical creativity and meaning generating in a mathematically rich conceptual field like curvature is a complex task, which brings the issue and study of creativity in the design to the fore. In the design, process and product are inextricably linked in the sense that the former draws its very existence on the pursuit of the creation of some novel products, and that a creative product acquires its substance as the end-result of some design processes. However, the role of creativity in instructional design, or specifically in educational resource design, has been only recently acknowledged (Clinton & Hokanson, 2012). In this study, we looked at the process of designing such a resource collaboratively, focusing on the emergence of Social Creativity (e.g., Fischer 2005; 2014) which we employed as a theoretical frame to both understand and foster creativity in collective design. Our designer communities were engineered to include diverse expertise and personal histories of educational professionals (we borrowed Fischer's term 'communities of interest' or CoI) working together and using digital tools specially designed to amplify the outcome of their collaborative efforts. We hypothesized that social creativity builds on the wealth of diverse individual perspectives brought in by different stakeholders in addressing a design problem of common concern and focuses on the interactions occurring in socio-technical environments (Fischer, 2014) i.e., among the individual members of a community and between them and particular technologies and artefacts.

The diversity within a CoI although being a source of discontinuities and breakdowns in communication, can be also a source of new ideas, insights and artefacts. According to Akkerman and Bakker (2011) boundaries are defined as the “sociocultural differences that give rise to discontinuities in action and interaction” (p. 139), which can be overcome through boundary crossing processes, i.e., efforts made by individuals or groups ‘at boundaries’ to establish or restore continuity in action or interaction across practices, leading to learning, identity development and re-conceptualization of practice. These efforts are facilitated by boundary objects (Fischer, 2005) which are externalizations of ideas that help to establish and maintain a common ground supporting communication and shared understanding. They come in the form of artefacts (such as specially designed computer tools), discourses (as a common language), or processes that allow the coordination of actions. Thus, computational support for CoIs should enable the creation, discussion and refinement of boundary objects that allow different knowledge systems to interact (Fischer, 2005).

In our study, we worked with educational designers and thus needed to also employ a framework to help us understand the context particularly of teachers as resource designers. Thus, we adopted the documentational approach of didactics (Gueudet & Trouche, 2009), which focuses on the interactions between mathematics teachers and resources and their consequences for professional growth. Teachers ‘learn’ when selecting, transforming, implementing and revising resources in the course of their teaching. The documentational approach proposes a specific conceptualisation of this learning as a *documentational genesis*, which jointly generates a new resource and a scheme of utilisation of this resource in an ongoing process. In relation to collectives instead of individual teachers, community documentational genesis describes the process of gathering, creating and

sharing resources to achieve the teaching goals of the community. The result of this process, the community documentation, is composed of the shared repertoire of resources and shared associated knowledge (Gueudet & Trouche, 2012). A collaborative design activity, and more particularly an activity involving teachers as designers of creative educational resources, is thus a process that is expected to trigger collective documentational genesis. The present study aims to unfold social creativity, located in and nurtured by the boundary crossing encounters among the CoI members, and collective documentational genesis processes in the design of digital creative mathematical resources for curvature, which takes place in a socio-technical environment consisting of a community of diverse educational professionals and a digital environment specially designed to allow them coordinate their efforts in designing these resources.

## **Method**

### **The Community of Interest (CoI)**

A wide range of expertise was brought together in the design of the c-book “Curves in Space”. The seven CoI members participating in this joint design were practitioners in different levels of education (from primary to tertiary education) specialized in mathematics, mathematics education, creative writing, computer mediated communication and the design of digital tools for mathematics education. This diversity in knowledge domains, perspectives and cultures was meant to enhance the CoI’s creative potential.

### **The Computational environment**

The C-book environment provides the ‘CoIcode workspace’, a tool for asynchronous online discussions allowing designers to choose between a threaded forum discussion organised in a tree-like structure (see Figure 1) and a mind map view. When posting a contribution, CoI members have to state its nature (i.e., alternative, contributory, objecting, off task or management) by using a specific icon, and can attach and refer to objects like online resources, texts or widget instances that reside in the c-book under construction. In addition, the environment contains a platform which is the space for authoring (the C-book authoring tool) and the space where students interact with the c-book (the C-book player). The platform is designed to incorporate pages with dynamic and configurable widget instances accompanied by corresponding narratives (see Figure 2). In this case, MaLT+, a 3D Logo-Based Turtle Geometry tool affording dynamic manipulation of variable values was used (<http://etl.ppp.uoa.gr/malt2>). Spirals are generated by either constant or incremental curve and torsion changes to a turtle respectively repeating very small displacements.

### **Data and analytical approach**

Our data were the 124 contributions uploaded in the ‘Curves in Space’ workspace from the outset of the design process (6/4/2015) until the final version of the c-book was released (23/7/2015). The analysis of the contributions posted in CoIcode involved the selection and analysis of critical episodes, i.e., relatively brief and uninterrupted periods in CoIcode discussion, shedding light on some important aspect of the social creativity processes and/or products developed, by focusing on the interactions among the CoI members and with the C-book technology. Furthermore, we traced paths of socially creative ideas, which stretch over longer periods of time and include several critical episodes, in terms of the critical moments in their evolution from the initial to the final idea (i.e., an

idea implemented and incorporated into some part of the c-book). The emphasis was on unveiling the social nature of the processes involved in the development of ideas and in the examination of the C-book environment features which added to the formulation, elaboration and cross-fertilisation of the CoI members' ideas.

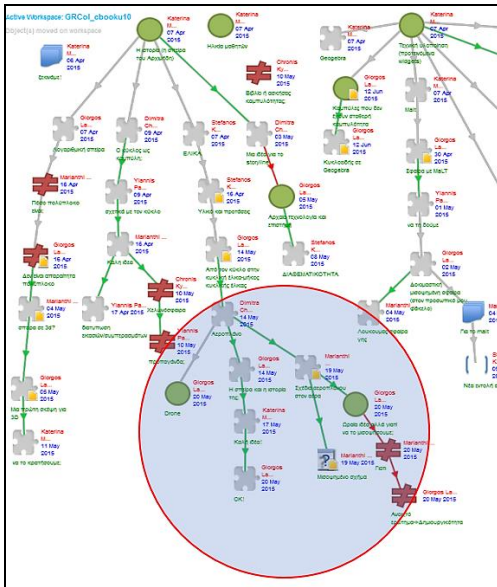


Figure 1: Excerpt from the CoICode workspace depicting critical episode 1

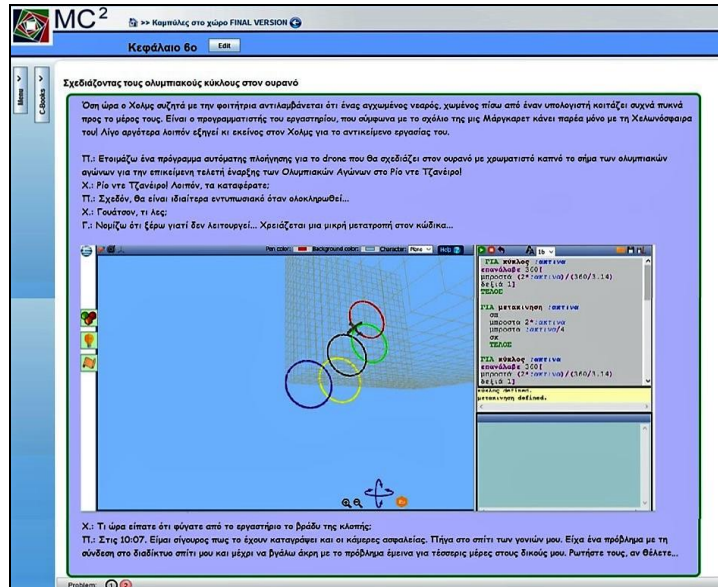


Figure 2: A 'Curves in Space' c-book page asking students to fix the code for designing the Olympic rings

## Results

### Critical episode: The design of a widget instance

The episode selected (see Figure 1) started one month after the outset of the design process, it lasted 8 days (14/5-20/5/15) and the participants in it were three CoI members: George, Mathematics teacher and graduate student in Mathematics education, Dimitra, Literature teacher and graduate student in ICT in Education, specialised in creative writing and Marianthi, MA ICT in Education graduate and developer. At that time an exchange of resources was taking place on the mathematical idea of Helix-Spiral between a senior mathematician (Stefanos) and George. The discussion is initiated by Dimitra (14/5) who, inspired by the airplane functionality in MaLT+, suggests that students calculate the distance covered by airplanes performing spiral movement during air shows. George (14/5) responds enthusiastically, elaborates on Dimitra's idea and provides a Wikipedia link on jets streams. Marianthi then puts forth a suggestion on a *half-baked* widget instance (i.e., a 'buggy' procedure where students are asked to experiment, figure out what is wrong or superfluous in the code and correct it):

Marianthi (19/5): [...] in MaLT+ I created a procedure where the airplane movement forms the Olympic rings. I am sending you the complete code so that we can half-bake it, e.g. it can turn by a 45-degree angle in the last two turns so that the rings do not come out straight (attaches 'Olympic\_correct.txt')

Marianthi (19/5): I am sending you the one I wrote with the wrong angles (attaches 'Olympic\_wrong.txt')

George (20/5): I like it a lot! I suggest not to half-bake it, but ask students to create it by themselves by looking at an image of the Olympic rings [...]

Marianthi (20/5): [...] I think if it's half-baked it will be more challenging for students to correct it than create it from scratch. Also, we can focus on specific mathematical topics like the turn angle or the distance of cycles.

George (20/5): [...] Since the unit addresses senior students it would more creative to allow them work without such restrictions. If we half-bake it though, wouldn't it be better to use variables for the angles?

George (20/5) refers to drones as a more innovative alternative to airplanes and designs two alternative versions of the widget instance in which he adopts Marianthi's proposal. Finally, one of his versions was incorporated in the c-book without further negotiation including his suggestion of imprinting the traces of a drone instead of a plane (see Figure 2). This episode shows how the collective resource system is enriched through the sharing, reflection and transformation of individual resources to boundary resources. What is more, boundary crossing interactions between CoI members allowed the cross-fertilization of diverse perspectives: mathematics, digital tools development and creative writing. Dimitra, having studied existing resources is inspired to articulate the airplane idea stating in what ways it deviates from what has been heard before. Marianthi turns Dimitra's idea into a 'tangible' object, i.e., a widget instance, while George expresses considerations initiating an interesting exchange on the pedagogical affordances of different types of activities. He brings CMT to the fore and poses the challenge to other members to directly argue on specific pedagogical and technical affordances of the proposed activities. The final version of the instance appears in the c-book as a result of the *coordination* of George's and Marianthi's ideas. Social creativity is thus enhanced by exchanging, discussing on and modifying half-baked curve designs acting as boundary objects, allowing the communication and coordination of diverse perspectives. Mathematical resources thus take a mediational role between diverse perspectives undergoing several transformations and revisions until they are reified as widget instances in the c-book. As teachers negotiate over an emergent mathematical construction, they are challenged to reflect on and reconsider their beliefs and practices as well as their meanings of mathematical objects and relationships, thus expand their learning.

### **The evolutionary path of the narrative**

The path presented below is related to the evolution of the narrative of the c-book. The respective path includes 52 contributions and stretches along the entire workspace. Early on in the design process, CoI members were concerned with devising a narrative that, together with appropriate widget instances, would provide opportunities for mathematization and meaning making around curvature. The mathematical affordances of various digital tools also became an early topic of discussion so that tools, narrative and mathematical concepts were interrelated in the design of the c-book. Below we provide decisive contributions from individual CoI members and stress the social nature of the processes involved in the development of the scenario from its first appearance to its incorporation in the c-book. At that time a number of widget instances designed to afford creativity and meaning making in curvature took the role of boundary objects by evolving through multiple cycles. However, a cohesive narrative that would incorporate and join together these elements was pending, despite the fact that some interesting ideas had been already suggested. The path sheds light into how the CoI members'

conceptions about their productions in terms of didactical design (widget instances and corresponding learning activities) intertwined with their ideas about the narrative of the c-book. Stefanos (22/6) presents a -rather loose- synthesis of his own and other members' ideas on the c-book narrative integrated in a new version of the c-book: the history of curves, two detectives working to solve a crime, a 3d printer laboratory, and solving riddles related to spirals. George (24/6) reacts enthusiastically and attaches an elaborated version of Stefanos' story incorporating Sylvie's comments on enhancing the story: two renown detectives (Hercule Poirot and Sherlock Holmes) try to solve a mysterious robbery in a laboratory, which is connected to constructions related to curvature. Sylvie, a teacher and creative writing specialist who joins the discussion at that time, presents a totally different idea on the structure of the scenario relying on contemporary characters, which fuels an intense debate. Stefanos (25/6) objects to Sylvie's suggestion on the grounds that the storyline should blend with the widget instances so that students follow a learning trajectory working with tools of gradual increase in complexity. He also posts a document in which he justifies his rationale for building his own version in which mathematical concepts are presented in a coherent and meaningful way. George and Katerina (computer mediated communication specialist) react to Stefanos' post:

George (26/6): Very insightful comments, especially in relation to the way the current narrative supports the smooth integration of the learning sequence on curvature. [...] Wouldn't it be better to make some corrections without discarding what we've done until now? [...] (He attaches 'What a strange morning in the laboratory.doc' where he expands his previous version to include logarithmic spirals).

Katerina (27/7): [...] I don't think that the new version rejects previous constructs and ideas [...] but rather promotes them by organically binding them with a fresh, creative story.

Up to this point there are two opposing views on the scenario; a mathematics oriented strict, structured and robust learning scenario mainly supported by a senior member who is an experienced Mathematician, teacher and researcher (Stefanos) and a more innovative one which embodies a set of characters and situations of contemporary culture. This tension is released when George (28/7) replying to Katerina, posts a new synthetic version of the scenario. In the next two versions of the scenario Sylvie, Katerina and George collaborate so that Sylvie presents a more robust synthetic version that organically integrates the designed widget instances. The coordination of the two prevalent perspectives in the design of the c-book, i.e., the mathematics and the creative writing perspective, made possible the infusion of creative elements in the narrative, while not losing sight of its mathematical focus on curvature. It is noteworthy that these two perspectives are not only gradually reconciled after the catalytic intervention of George, they also enrich each other; the senior mathematician later proposes two additional ideas on the scenario much more innovative than his initial ones, while the creative writing specialist, after closely collaborating with mathematicians, comes to adjust her story in a synthetic version. These reflective processes are essential for the interweavement of widget instances with the narrative into a concise whole. Social creativity is thus facilitated by the meshing of the Sherlock Holmes narrative with curvature, which can only have emerged because of the diversity in the CoI. Furthermore, the process of story versioning boosts social creativity as it allows for the generation of new ideas which capitalize on, object to and finally synthesize previous ones. It is an ongoing process where ideas are adjusted, adapted and combined to produce new documents.

## Conclusions

The analysis of social creativity in the design process of a c-book on Curvature focused on the boundary crossing interactions between the CoI members and the role of the narrative and the widget instances as key resources for the development of social creativity. Two important boundary crossing processes, *coordination* and *reflection*, have enhanced social creativity establishing communication between different communities of practice: Mathematics, Literacy/creative writing and Digital tools development. Reflection, on the other hand, is the process which gave ground to the fertile synthesis of different views. Moreover, the story and its versions as a key resource was paramount to social creativity within the CoI. The story versioning process allowed for warm debate and idea exchange to take place: it created common ground for all CoI members to unfold their expertise, as well as the meshing of narrative with constructionist artefacts-widgets on curvature. As a result, a collective document, that is the c-book, was developed, associating various shared resources (activities, widget instances, text, and CMT representations) and a scheme for interweaving all these elements in a coherent whole. The issues that emerged during the construction of successive c-book versions challenged teachers' perceptions with respect to the teaching and learning of curvature resulting in innovative approaches fostering creativity and meaning-making. Embedding the comparison of constant to incremental turn and torsion changes to generate spirals in space within a Sherlock Holmes 'who dun it' story involved stepping out of curriculum structures for curvature and making a new conceptual field available to students connecting curvature with functions and 3D geometry.

The use of different theoretical perspectives, i.e., Social Creativity, Documentational Genesis and Boundary Crossing, has helped us gain a deeper insight in the phenomena in question. In our framework of analysis, the Social Creativity perspective provides the lens through which the social dimension of teachers' documentational genesis process can be approached. Moreover, resources take the role of boundary objects allowing the coordination of diverse perspectives leading to the generation of creative products (the c-book as an end-product). Thus, by coordinating these theoretical approaches, we seek to develop a networked understanding (Prediger, Bikner-Ahsbals, & Arzarello, 2008) of the collective design of a c-book as a novel digital medium to foster students' creative mathematical thinking; a single theory would not suffice for understanding such a complex process. Even though drawing connections between theories is not a trivial task, such networks are potentially powerful and useful for the further development of mathematics education as a scientific field. (Prediger et al, 2008).

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