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***PÉPITE* Online automated assessment and student learning: The domain of equations in the 8th grade**

Stéphane Sirejacob¹, Françoise Chenevotot-Quentin² and Brigitte Grugeon-Allys³

¹Laboratoire de Didactique André Revuz (LDAR), Université Paris Diderot, Paris, France; stephane.sirejacob@etu.univ-paris-diderot.fr

²Laboratoire de Didactique André Revuz (LDAR), Université d'Artois, Universités Cergy Pontoise, Paris Diderot, Paris Est Créteil, Rouen, France; chenevotot.francoise@neuf.fr

³Laboratoire de Didactique André Revuz (LDAR), Université Paris Est Créteil, Universités d'Artois, Cergy Pontoise, Paris Diderot, Rouen, France; brigitte.grugeon-allys@u-pec.fr

“Pépité” is an online automated assessment tool for elementary algebra for students in secondary education (12-16 years-old) in France. Pépité was initially developed for students at the end of compulsory schooling in France (16 years-old). At CERME9, we presented its transfer at different school levels and illustrated it with the design of Pépité test for grade 8th students. Information provided by Pépité allows identifying students’ consistent reasoning and calculation in order to organize teaching corresponding to students’ learning needs. In this paper, we focus on the use of Pépité test for grade 8th students to learn the domain of equations. We defined an epistemological reference of the algebraic domain that allows us not only to build the tasks selected for the test and to analyze students’ responses but also to propose suitable courses adapted to students’ learning needs.

Keywords: diagnostic assessment, Information and Communication Technology (ICT), elementary algebra; equations, student’s profile, teaching suggestions.

Context of the study

This paper deals with the issue “Digital assessment of and for learning” of TWG16 “Students Learning Mathematics with Technology and Other Resources”. Since the 1990s, our team has developed several multidisciplinary projects (Delozanne & al., 2010; Grugeon-Allys & al., 2012) concerning the design, development and use of online tools for diagnostic assessment and student learning. One of these tools, named “*Pépité*”, is relevant for learning elementary algebra for students of secondary education (12-16 years) in France. We have disseminated *Pépité* online tool on platforms¹ largely used by teachers and students.

In this paper, we deal with the use of *Pépité* online assessment for learning the domain of equations for grade 8th students. First, we revisit the theoretical foundations of *Pépité* online assessment. Then, we illustrate it with *Pépité* assessment for grade 8th students in France (13-14 years). We specify both the didactical model and the computer model that automatically generates generic tasks, analyses students’ work and provides descriptions of students’ profiles. Finally, we discuss the potentialities

¹ Pépité tools are available on LaboMep platform (developed by Sésamath, a French maths’ teachers association): <http://www.labomep.net/> and on WIMS environment (an educational online learning platform spanning learning from primary school to the university in many disciplines).

of *Pépité* online assessment to propose suitable courses adapted to students' learning needs for the domain of equations for grade 8th.

The theoretical and methodological framework

In the educational system, assessment is a complex issue. Usually, assessment results are generated from standardized and psychometric models. Studies highlight the strengths and limitations of such approaches to make instructional decisions (Kettelin-Geller & Yovanoff, 2009). To identify the features of appropriate online assessment for learning, we have chosen both a cognitive and epistemological approach and also an anthropological approach, the potentialities of which are described in Grugeon-Allys & al. (2012).

Epistemological and cognitive approach

Designing a test requires the selection of a set of tasks that enables the assessment to be realized. We agree with Vergnaud who stated, "Studying learning of an isolated concept, or an isolated technique, has no sense" (Vergnaud, 1986, p. 28). Furthermore, Vergnaud introduced a strong assumption: dialectics between genesis of a student's knowledge and mathematical knowledge structure. Beyond a quantitative analysis of responses, we have to define a qualitative didactical analysis (based on a collection of students' responses to the tasks) to identify the type of procedures and knowledge used by students in solving the tasks. To provide descriptions of a student's consistent reasoning, it is necessary to define a reference for modelling the mathematical competence, in a mathematical domain, at a particular school grade.

Anthropological approach

The epistemological approach is not sufficient so as to take into account the impact of the institutional context on students' learning. According to the anthropological approach, mathematical knowledge is strongly connected to the institutions where it has to live, to be learnt and to be taught; it is strongly connected to mathematical practices (curricula, etc.). Chevallard (1999) analyses knowledge in terms of praxeology, that is to say in terms of type of tasks, techniques used to solve these tasks, technological discourses developed in order to produce, explain and justify techniques, and last, theory that justifies technological discourses.

A reference epistemological praxeology for algebraic knowledge

For a given mathematical domain, we defined a reference epistemological praxeology (Garcia, Gascon, Higuera & Bosch, 2006) that makes it possible to create an *a priori* design that describes features of an appropriate assessment. For algebraic knowledge, such reference is based on results from didactics of algebra (Chevallard, 1989; Artigue & al., 2001; Kieran, 2007). In its *tool* dimension (Douady, 1985), there are tasks for *generalizing*, *modelling*, *substituting*, *proving*. In its *object* dimension, there are tasks focused on calculus with algebraic expressions (*calculating*, *substituting* a number for a letter, *developing*, *factorizing*) or equations (*solving*). This reference makes it possible to define appropriate technology for an intelligent and controlled algebraic calculus, based on equivalence of algebraic expressions and the dialectic between numeric and algebraic treatment modes.

The domain of equations for grade 8th students

The three following types of tasks are specifically related to equations (we will give some precise examples later about our experimentations):

- Modelling and putting a problem into equation (tool dimension). These tasks motivate the production of an equation in order to solve modelling problems and require semiotic conversions (Duval, 1993).
- Solving an equation using an algebraic technique; proving that two equations are equivalent (object dimension). These tasks use the concept of equivalence and require transformational activity (Kieran, 2007).
- Testing if a number is a solution of an equation; identifying the degree of an equation (object dimension). These tasks are based on substitution and polynomial properties.

Features of *Pépîte* online assessment

The *Pépîte* online diagnostic assessment is based on a reference epistemological praxeology of the algebraic domain, both for designing tasks and for analyzing responses to the test. We will base this on the *Pépîte* test for grade 8th students.

The didactical model

Pépîte test

The test (targeting 13-14 years old students) is composed of 10 diagnostic tasks and 22 individual items covering the types of tasks defined below (Table 1). The tasks may be multiple-choice or open-ended items (Figure 1).

Types of tasks	Number of items	Test items
Calculus	4 / 22	7.1 / 7.2 / 8.1 / 8.2
Producing numerical expressions	1 / 22	5
Producing algebraic expressions	2 / 22	3.1 / 6
Translation or recognition	14 / 22	1.1 / 1.2 / 1.3 / 2.1 / 2.2 / 2.3 / 3.2 / 4.1 / 4.2 / 9.1 / 9.2 / 9.3 / 9.4 / 10
Problem solving in different mathematics frameworks	1 / 22	6

Table 1: Organization of the 8th grade level test in terms of types of tasks

Exercise 6: Proof and calculation program

A student says to another student: “You will always find the same result if you take a number, you add 6 to that number, you multiply the result by 3, you subtract triple the initial number”.

Is this statement true for any number? Justify your answer.

Justification

Result

The statement is true for any given number: true or false?

Figure 1: Example of generalization task

Responses analysis: the multidimensional model of algebraic assessment

At the local assessment level (for each task), students' responses are not only evaluated as correct/incorrect, but also according to their technological discourse, that justifies the techniques. The analysis concerns validity of response (V) and seven dimensions: meaning of the equal sign (E), algebraic writings produced during symbolic transformations (EA), numerical writings produced during symbolic transformations (EN), use of letters as variables (L), algebraic rationality (J), connections between a semiotic register to another (T) and skills with negative and decimal numbers (N) (Table 2) (Grugeon-Allys, 2015). We code the responses with assessment criteria, which depend on knowledge and reasoning involved in the techniques².

Assessment dimensions	Assessment criteria
Validity of response	V0: No answer V1: Valid and optimal answer V2: Valid but non optimal answer V3: Invalid answer Vx: Unidentified answer
Algebraic writings produced during symbolic transformations	EA41: Incorrect rules make linear expressions $a^2 \rightarrow 2a$ EA42: Incorrect rules gather terms ...
Connections between a semiotic register to another	T1: Correct translation T2: Correct but non optimal translation T3: Incorrect translation taking into account the relationships T4: Incorrect translation without taking into account the relationships Tx: No interpretation

Table 2: The multidimensional model of algebraic assessment (partial view)

We illustrate the multidimensional model of algebraic assessment on the task “Proof and calculation program” (Figure 1). In order to solve this task, two *a priori* strategies are possible: an arithmetic strategy using a number or an algebraic strategy mobilizing a variable. Several incorrect techniques can illustrate an arithmetic strategy (Table 3) according to the rules used to translate or transform numeric expressions. Algebraic strategy may be incorrect (J3) if the conversion rules (T3 or T4) or algebraic transformation rules (EA3 or EA4) are inadequate (Table 4).

Solutions	Reasoning and technological discourse	Coding
For number 5 $(5 + 6) \times 3 - 3 \times 5 = 18$	Correct arithmetic strategy with global expression that uses parenthesis	V3, L5, EA1, J2, T1
For number 5 $5 + 6 = 11$; $11 \times 3 = 33$; $3 \times 5 = 15$; $33 - 15 = 18$	Correct arithmetic strategy with partial expressions	V3, L5, EA1, J2, T2
For number 5 $5 + 6 \times 3 - 3 \times 5 = 8$	Erroneous arithmetic strategy with global expression that uses no parenthesis	V3, L5, EA3, J2, T3
For number 5 $5 + 6 = 11 \times 3 = 33 - 3 = 30 \times 5 = 150$	Erroneous arithmetic strategy with calculus by step (procedural aspect)	V3, L5, EA3, J2, T4

Table 3: A priori analysis for arithmetic strategies

² Contrary to usual practices in assessment, we do not attribute a code by technique for each task. This would lead to a multiplicity of codes on various tasks and would be unusable for a cross analysis on all the tasks of the test.

Solutions	Reasoning and technological discourse	Coding
$(x + 6) \times 3 - 3 \times x$ $= 3x + 18 - 3x$ $= 18$	Correct algebraic strategy with global expression that uses parenthesis	V1, L1, EA1, J1, T1
$(x+6) \times 3 = 3x + 18;$ $(3x + 18) - 3x = 18;$	Correct algebraic strategy with calculus by step (procedural aspect)	V2, L1, EA1, J1, T2
$x + 6 \times 3 - 3 x$ $= x + 18 - 3 x$ $= - 2x + 18$	Erroneous algebraic strategy with global expression that uses no parenthesis	V3, L3, EA32, J3, T3
$(x + 6) \times 3 = 3x + 18 = 21x ;$ $21x - 3x = 18x;$	Erroneous algebraic strategy with calculus by step (procedural aspect)	V3, L3, EA42, J3, T4

Table 4: A priori analysis for algebraic strategies

Student's profile, groups and differentiated strategies

The *Pépité* diagnostic assessment proposes both individual and collective assessment. The individual assessment, at the global assessment level (on a set of tasks), builds the student's profile which aims to identify features of algebraic knowledge and skills for the seven dimensions. The collective assessment locates a student on a scale with four components: skill in Algebraic Calculus (CA), skill in Numerical Calculations (coded CN), Use of Algebra for solving tasks (UA) and flexibility in Translating a semiotic register to another (TA). For each of those four components, different technological levels and appropriate benchmarks have been identified (Grugeon-Allys & al., 2012). Regarding to a class, students are divided into three groups according to their skill in Algebraic Calculus: CA1 (group A) - reasoned and controlled calculation preserving the equivalence of expressions -, CA2 (group B) - calculation based on syntactic rules often in blind, not always preserving the equivalence of expressions - and CA3 (group C) - meaningless and non-operative calculation. Therefore, for a given learning objective, it is possible to assign tasks to each group, depending to didactical variables related to the associated technological levels (Delozanne & al., 2010, Pilet & al., 2013).

The computer model

An iterative process between educational researchers, computer scientists and teachers was used to design and test different *Pépité* prototypes in order to improve the didactical model. We defined the conceptual IT model of classes of tasks, which allows the characterizing of equivalent tasks (Delozanne & al, 2008). The software *PépiGen* (Delozanne & al., 2008) automatically generates the tasks and their analyses, at different grade levels. It uses *Pépinère*, a computer algebra system, to generate anticipated student correct or incorrect answers (according to the *a priori* analysis). *Pépité* automatically calculates a student's profile as well as profiles for groups of students. According to a learning objective defined by the teacher, *Pépité* generates tasks adapted to the related technological levels (Grugeon-Allys & al., 2012).

Results and discussion

The information provided by *Pépité* diagnostic assessment allows the teacher to identify students with close profiles in algebra. Then, *Pépité* automatically generates differentiated routes corresponding to these algebraic profiles. As mentioned above, these routes were designed on the basis of a reference epistemological praxeology.

Differentiated routes for learning equations

Three differentiated routes were created concerning equations. The first route “Motivating the production of an equation and solving it with an equation solver” motivates the production of an equation. It includes tasks like “*equalizing two calculation programs*” (see the example below). Students have to solve them using an equation solver. The second route “Algebraic resolution of an equation” requires technologies for solving equations by algebraic methods (by using the concept of equivalence of equations). In the last route “Algebraic resolution of problems that lead to an equation”, tasks that require a problem to be expressed as an equation and then solved, such as, “*equalizing two perimeters of dynamic figures*”, are proposed.

We give now two examples of tasks for the first route. The first one aims to introduce equations and to highlight the inadequacy of arithmetic techniques to solve problems of first degree. As we can see, thanks to a thoughtful choice of the didactic variables, this task prevents arithmetic strategies – because of the presence of the unknown in both calculation programs – or “trial and errors” methods – because the solution of this problem which is $\frac{7}{3}$ cannot be easily obtained by successive trials. Algebraic techniques are necessary.

For groups A, B and C		
Program A	Program B	
Choose a start number Multiply it by 3 Add 5 to the result	Choose a start number Multiply it by 6 Subtract 2 to the result	Alex and Brenda choose the same start number. Alex tests the calculation program A and Brenda tests the program B. Then, Alex and Brenda find the same final result. Which start number did they choose?

Table 5: Task for motivating the production of and equation and solving it with an equation solver

The second task is differentiated (Table 6) to take into account students' algebraic activity and makes the students work on semiotic conversions (from the representation register of algebraic writing to the representation register of calculation program). Differentiation relies on didactical variables: the left member of the equation for group A is a product and solving the equation needs to use the distributive property, while the equation for groups B and C do not require it to be solved. Moreover, the multiplication sign is used for groups B and C to suggest that one or more multiplications are expected in the expression.

For group A	For groups B and C
Write a problem with two calculation programs that correspond to the equation $2(x + 7) = 5 - 3x$.	Write a problem with two calculation programs that correspond to the equation $2 \times x + 7 = 5 - 3 \times x$.

Table 6: Task for working on semiotic conversions solver

Experimentation in a grade 8th class

We now present the results of research carried out in 2016 with a mathematics teacher we will call M2. M2 has been working in a REP establishment (high-priority education network) for three years. We chose him because he is not an expert. After an observation phase (6 hours) of his teaching practices, we proposed a whole teaching sequence to him on equations that takes into account the main epistemological aspects of the reference epistemological praxeology. M2 was free to adapt this sequence to his practices; however, both teacher and researcher have worked together to plan the implementation in the class.

M2 is required to introduce equations in his grade 8th class. First, his 20 students (14 years old) completed the *Pépîte* test. Then, they were been divided into three groups A, B and C. Only one student belonged to group A (reasoned and controlled calculation preserving the equivalence of expressions). The others students belonged to groups B (15 students who can calculate correctly expressions but without using semantic rules) and C (4 students who do not understand the calculus on algebraic expressions). M2 proposed to his students the three routes mentioned above, in the same order. Due to the fact that most of his students were in group B (15/20), M2 chose to give the same tasks to the whole class. After working on the three routes, the 20 students completed a written test on equations. We chose to focus on two tasks from this test to present our results. The first task was about solving three first-degree algebraic equations. Depending on the equation they solved, 7 to 11 students among the 20 students found the correct solutions. We particularly studied how many students used an algebraic technique. We observed that 17 out of 20 students solved the equations using the equivalence of equations. Even if they did not find the right solution, they had a strategy and transformed the equations in order to “eliminate” the unknown; they respected the concept of equivalence to do so. For the second task, equalizing two calculation programs (as presented above in table 5), 11 out of 20 students succeed for putting the problem into an equation.

Discussion

The *Pépîte* assessment tool, based on an epistemological reference of the algebraic domain, allows the teacher to identify students’ consistent reasoning and calculation in order to plan differentiated courses adapted to grade 8th students’ learning needs for the domain of equations. The mathematics routes tested in our experimentation seemed to have effects on the students’ technological level: most of them used algebraic techniques to put a problem into equation. But this experimentation only concerns one teacher. So, in the ERASMUS + project “Advise me” which has just started in September 2016, we aim to carry out a larger scale research study.

We intend to validate these results for the field of arithmetic of integers for grade 3-4 pupils. Grapin (2015) carried out a multidimensional model of assessment for this new domain in elementary school. She defined an epistemological reference of arithmetic of integers to design an assessment tool in order to define pupils’ profiles and to highlight the epistemological aspects of arithmetic to work according to pupils’ learning needs. She organized an experimentation to study the evolution of pupils’ profiles according to differentiated routes adapted to students’ learning needs for the domain of arithmetic of integers. Data analysis is underway.

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