



HAL
open science

Pole placement problem for linear systems with constrained control. (An Overview)

Hafsa Hamidane, Samira El Faiz, Mohammed Guerbaoui, Abdelali Ed-Dahhak, Abdeslam Lachhab, Benachir Bouchikhi

► **To cite this version:**

Hafsa Hamidane, Samira El Faiz, Mohammed Guerbaoui, Abdelali Ed-Dahhak, Abdeslam Lachhab, et al.. Pole placement problem for linear systems with constrained control. (An Overview). International Meeting on Advanced Technologies in Energy and Electrical Engineering (IMAT3E'18), Nov 2018, Fes, Morocco. hal-01941963

HAL Id: hal-01941963

<https://hal.science/hal-01941963>

Submitted on 2 Dec 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Pole placement problem for linear systems with constrained control. (An Overview)

¹*H. Hamidane, ³S. El Faiz, ¹M. Guerbaoui, ¹A. Ed-dahak, ¹A. Lachhab, ²B. Bouchikhi

¹ hamidanehafs@gmail.com, Modelling, Systems Control and Telecommunications Team, Moulay Ismaïl University, High School of Technology, Meknes, Morocco.

² Sensor Electronic & Instrumentation Team, Moulay Ismaïl University, Faculty of Sciences, Meknes, Morocco.

³ Research Team on Energy and Sustainable Development, Ibn Zohr University Guelmim, Morocco.

An overview of some of the methods used for modern systems and control theory related to the pole placement problems is given. These methods are used in order to develop and synthesis stabilizing controller which studies the stability of the system.

As a particular case; linear time invariant systems in continuous time domain

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

Where $A \in \mathfrak{R}^{n \times n}$ and $B \in \mathfrak{R}^{n \times m}$ are given matrices, x and u represent respectively the state and control vectors of the system; are engaged in this framework to show dynamic systems pole placement development in the state space form.

To investigate the stability of such linear systems, we reconsider the pole placement problems methods used in order to choose a state feedback control law:

$$u = Fx ; (F \in \mathfrak{R}^{m \times n}) \quad (2)$$

such that the closed loop system

$$\dot{x}(t) = (A + BF)x(t) \quad (3)$$

has a specified poles, regarding the constraints notion. These constraints are usually represented as inequalities that can affect the systems performance if they are not taken into consideration when synthesising and developing the controller. For that aim it is preferable to develop the exact mathematical models representing these saturations that can affect the systems state or control law

In the present framework we review the case where the control law u of the system is constrained to evolve in the domain $\Omega \subset R^m$ defined as:

$$\Omega = \{u \in \mathbb{R}^m / -u_{\min} \leq u \leq u_{\max}, u_{\min}, -u_{\max} \geq 0\}$$

Where Ω represents a non symmetric polyhedral set.

In this regard a necessary concept to take into account when involving constraints concept, is the positive invariance approach [1, 2, 4, 8], thus the main objective of such an approach is to obtain a stabilizing regulator F as a unique solution to the nonlinear algebraic equation:

$$FA + FBF = HF \quad (4)$$

Such that the model of equation (3) remains valid.

$H \in \mathbb{R}^{n \times n}$ is a given matrix satisfying some necessary requirements, $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are given matrices.

Based on the cited equation (4) an algorithm developed in [8] will be presented as well to describe the purpose of such an equation in synthesizing the control law of linear constrained systems.

In addition to the mentioned non linear equation, another type of equations that contributes positively in the development of the pole placement solutions is known as Sylvester equation [1, 2, 6, 8], this equation depends mainly on equation (4) to find a control law F for the linear system and it has the form:

$$AX - XH = -B \quad (5)$$

X is the unique solution of equation (5), $H \in \mathbb{R}^{n \times n}$, $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are given matrices. In this order another another algorithm presented in [8], will be considered and clarified.

For both non linear and Sylvester equations, the reduced order technique [3] is also engaged as a method that gives the possibility to solve some classes of constrained linear systems by the projection of the trajectories of the original system onto a subspace associated with the undesirable open-loop eigenvalues which enable to decreases the complexity of some computations.

We also introduce linear matrix inequality (LMI) technique [5, 7] due to its efficiency and simplicity in resolving the pole placement and concomitant problems; in an efficient numerical manner.

Finally and for the same aim some of the numerical examples treated in [8] corroborating and illustrating the usefulness of the proposed synthesis techniques are also suggested. In this order we involve some of the most recently used softwares such Matlab, including latest and various LMI Toolboxes, one to be cite Yalmip Toolbox, to reexecute these examples.

References

1. A. Benzaouia, M. Ait Rami, and S. El Faiz. Stabilization of linear systems with saturation: a Sylvester equation approach, *IMA Journal of Mathematical Control and Information*, 21(3), 247-259. (2004).
2. A. Benzaouia. The resolution of equation $XA + XB = HX$ and the pole assignment problem, *IEEE Transactions on Automatic Control*, 39(10), 2091-2095, (1994).
3. E. B Castelan, J. G. da Silva, & J. E. R. Cury. A reduced-order framework applied to linear systems with constrained controls. *IEEE Transactions on Automatic Control*, 41(2), 249-255, (1996).
4. H. Maarouf. The resolution of the equation $XA + XB = HX$ and the pole assignment problem: a general approach. *Automatica*, 79, 162-166, (2017).
5. M. E Ammar, & G. A Dumont. Automatic tuning of paper machines cross-direction controllers via linear matrix inequalities. *Journal of Electrical Systems and Information Technology*, 2(3), 283-295, (2015).
6. Q. W. Wang, A. Rehman, Z. H. He, and Y. Zhang. Constraint generalized Sylvester matrix equations. *Automatica*, 69, 60-64. (2016).
7. S. Boyd, L. ElGhaoui, E. Feron, and V. Balakrishnan. *Linear matrix inequalities in system and control theory*, (Vol. 15). Siam, (1994).
8. S. ELFaiz. *Commande des systèmes linéaires sous contraintes: stabilisation et placement de poles robuste via LMI*, Doctoral thesis in automatics, Cadi Ayyad University, Faculty of Sciences Semlalia Marrakech, Morocco, (2004).