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THE KINEMATIC SENSITIVITY OF ROBOTIC MANIPULATORS TO JOINT CLEARANCES

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ABSTRACT

The paper deals with the kinematic sensitivity of robotic manipulators to joint clearances. First, an error prediction model applicable to both serial and parallel manipulators is developed. A clearance model associated with axisymmetrical joints, which are widely used in robotic manipulators, is also proposed. Then, two nonconvex quadratically constrained quadratic programs (QCQPs) are formulated in order to find the maximum reference-point position error and the maximum orientation error of the moving-platform for given joint clearances. Finally, the contributions of the paper are highlighted by means of two illustrative examples.

INTRODUCTION

Accuracy is one of the key features that favor robotic manipulators for many industrial applications. Superior levels of accuracy are achieved by controlling or measuring all possible sources of errors on the pose of the moving platform¹ of a robotic manipulator. Among the most important sources of errors, we find manufacturing errors, assembly errors, compliance in the mechanical architecture, resolution of the servoactuators, backlash in the reducers, and clearances in the joints. As indicated in refs. [1, 2], the errors due to manufacturing, assembly and compliance can be compensated through

calibration and model-based control. Joint clearances, on the contrary, exhibit low repeatability, which generally makes their compensation difficult. For this reason, the focus of this paper is the impact of joint clearances on the pose errors of serial and parallel-mechanism moving platforms.

Joint clearances introduce extraneous degrees of freedom between two connected links. When present, they generally contribute importantly to the degradation of the performance of a mechanism. Besides increasing the point-position and orientation errors of the end effector of a mechanism, they may cause impacts and faster wear of the joints. Because of these undesirable effects, machine designers can eliminate clearances by preloading the pairing elements. However, with preloaded joints, a parallel mechanism is difficult to assemble, as it requires high tolerances and manufacturing costs. When the parallel mechanism is overconstrained, it may even lose degrees of mobility or be impossible to assemble. As a result, joint clearances can be essential for proper functioning of certain mechanisms, but they cannot be neglected when evaluating their accuracy.

We may file the various approaches that have been proposed to compute and quantify the errors due to joint clearances into two categories: stochastic [3] and deterministic [4–9]. In this paper, we focus on deterministic methods, which generally allow to draw firm bounds on the pose errors given possible ranges for

¹In this paper, the end effector of a serial manipulator is referred to as its moving platform.

the clearances.

Wang and Roth [4] essentially proposed a model for clearances in revolute joints embodied as journal bearings. The effect of these clearances under a given wrench applied at the moving platform of a robotic manipulator was then computed by Innocenti [6]. Other researchers attempted to decouple the effect of clearances from the wrench applied at the moving platform by computing all its possible displacements.

This is what was done by Voglewede and Ebert-Uphoff [7], who used workspace generation techniques in order to compute the boundary of the set of moving-platform displacements allowed by clearances at the joints. The solution proposed in ref. [7] is general and very attractive. However, it becomes computationally intensive when high numbers of joints are affected by clearances, and the moving-platform displacements occur in three-dimensional space.

In order to fill this gap, Venanzi and Parenti-Castelli [8] took advantage of the generally small displacements allowed by clearances to treat them as small-displacement screws. The linearized loop-closure equations could then be used to evaluate the effects of clearances. A method for computing the maximum values of the moving-platform screw components due to clearances in a single open kinematic chain was devised. However, as these authors point out, their method does not apply to overconstrained parallel mechanisms, since they rely on the principle of virtual work in order to obtain the relationship between joint and moving-platform displacements.

Recently, Meng et al. [9] modified the formulation proposed in ref. [8] and approximated the clearance model proposed by Wang and Roth [4], which allowed them to formulate the problem of maximizing the components of the moving-platform small-displacement screw for any parallel mechanism. The problems obtained through this reformulation are convex, which guarantees that any local optimization method will converge to the global optimum. The results reported by Meng et al. [9] are very complete, since they provide a fast and efficient method for computing the effect of clearances on the moving-platform pose of a robotic manipulator.

Nevertheless, we believe that it would be interesting to define a minimum number of metrics, in order to draw easily understandable, global pictures of the effect of clearances on the moving platform. In previous reports, the effect of joint clearances is measured sequentially over all the displacement directions of the moving platform. In space, this gives a total of six metrics, over which the designer is to base his design of a robotic manipulator. In this paper, we follow an idea proposed in ref. [10] and elsewhere for measuring the moving-platform sensitivity to actuator displacements. In clear, we define two

indices: one for reference-point displacements of the moving platform, and one for its rotations.

The paper is divided as follows. We first recall the error prediction model proposed by [8]. We then propose a model of clearances in axisymmetrical joints that is slightly different from those reported by [4, 9]. Having formally written all required models, we define the two proposed indices, which take the form of nonconvex optimization problems. We then explain the strategy adopted for solving this optimization problem. We finish with two examples, where we compute the indices for given poses of a spatial serial 3R manipulator and a planar parallel five-bar manipulator.

ERROR PREDICTION MODEL

A methodology is introduced in this section to obtain the variations in the moving platform pose as a function of joint clearances. In this vein, we consider serial and parallel robotic manipulators consisting of a fixed base and a moving-platform, which are mutually connected through m legs. Let us assume that the i^{th} leg is a serial kinematic chain composed of n_i links, the first being connected to the base and the n_i^{th} to the moving-platform. For a serial robotic manipulator m turns to be equal to 1. We mathematically describe these open kinematic chains using the Denavit-Hartenberg parameters.

Parameterization

Let us define frame $\mathcal{F}_{i,j}$, which is attached to the j^{th} link of the i^{th} leg. Moreover, $\mathcal{F}_{i,1}$, $i = 1, \dots, m$, are the reference frames attached to the fixed base and have the same orientation. $\mathcal{P}_i = \mathcal{F}_{i,n_i}$, $i = 1, \dots, m$, are attached to the moving-platform of the robotic manipulator. Each frame is related to the previous one by the screw

$$\mathbf{S}_{i,j} = \begin{bmatrix} \mathbf{R}_{i,j} & \mathbf{t}_{i,j} \\ \mathbf{0}_3^T & 1 \end{bmatrix} \in \text{SE}(3), \quad (1)$$

which takes $\mathcal{F}_{i,j}$ onto $\mathcal{F}_{i,j+1}$, and where $\mathbf{R}_{i,j} \in \text{SO}(3)$ is a 3×3 rotation matrix; $\mathbf{t}_{i,j} \in \mathbb{R}^3$ points from the origin of $\mathcal{F}_{i,j}$ to that of $\mathcal{F}_{i,j+1}$, and $\mathbf{0}_3$ is the three-dimensional zero vector. Moreover, all frames follow the Denavit-Hartenberg convention [11], so that $\mathbf{S}_{i,j}$ may be expressed as

$$\mathbf{S}_{i,j} = \mathbf{S}_{i,j,\theta} \mathbf{S}_{i,j,b} \mathbf{S}_{i,j,a} \mathbf{S}_{i,j,\alpha}, \quad (2)$$

where

$$\mathbf{S}_{i,j,\alpha} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \alpha_{i,j} & \sin \alpha_{i,j} & 0 \\ 0 & -\sin \alpha_{i,j} & \cos \alpha_{i,j} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (3)$$

$$\mathbf{S}_{i,j,a} \equiv \begin{bmatrix} 1 & 0 & 0 & a_{i,j} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (4)$$

$$\mathbf{S}_{i,j,b} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & b_{i,j} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (5)$$

$$\mathbf{S}_{i,j,\theta} \equiv \begin{bmatrix} \cos \theta_{i,j} & \sin \theta_{i,j} & 0 & 0 \\ -\sin \theta_{i,j} & \cos \theta_{i,j} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (6)$$

and where $\alpha_{i,j}$, $a_{i,j}$, $b_{i,j}$ and $\theta_{i,j}$ represent the link twist, the link length, the link offset, and the joint angle, respectively. In the case of a revolute joint, $\theta_{i,j}$ is a variable, whereas in the case of a prismatic joint, $b_{i,j}$ is a variable.

The Moving-Platform Pose

Thence, the pose of the sixth link with respect to the fixed frame $\mathcal{F}_{i,1}$ may be expressed as

$$\mathbf{P}_i = \prod_{j=1}^{n_i} \mathbf{S}_{i,j}, \quad (7)$$

for a given leg i . Provided that the joints are perfectly rigid in all directions but one, that the links are perfectly rigid and that the geometry of the robotic manipulator is known exactly, we have

$$\mathbf{P} = \mathbf{P}_1 = \mathbf{P}_2 = \cdots = \mathbf{P}_m. \quad (8)$$

However, if we consider small clearances in all the joints, we must include small errors in Eq. (7) of \mathbf{P}_i for Eq. (8) to hold true.

Joint-Clearance Errors

Taking into account clearances in the joints, the frame $\mathcal{F}_{i,j}$ associated with link j of leg i is shifted to $\mathcal{F}'_{i,j}$. Provided it is small, this error on the pose of joint $j+1$ with respect to joint j may be represented by the small-displacement screw

$$\delta \mathbf{s}_{i,j} \equiv \begin{bmatrix} \delta \mathbf{r}_{i,j} \\ \delta \mathbf{t}_{i,j} \end{bmatrix} \in \mathbb{R}^6, \quad (9)$$

where $\delta \mathbf{r}_{i,j} \in \mathbb{R}^3$ represents the small rotation taking frame $\mathcal{F}_{i,j}$ onto $\mathcal{F}'_{i,j}$, while $\delta \mathbf{t}_{i,j} \in \mathbb{R}^3$ points from the origin of $\mathcal{F}_{i,j}$ to that of $\mathcal{F}'_{i,j}$. $\delta \mathbf{s}_{i,j}$ can also be represented as the 4×4 matrix

$$\delta \mathbf{S}_{i,j} = \begin{bmatrix} \delta \mathbf{R}_{i,j} & \delta \mathbf{t}_{i,j} \\ \mathbf{0}_3^T & 0 \end{bmatrix} \in \text{se}(3), \quad (10)$$

where $\delta \mathbf{R}_{i,j} \equiv \partial(\delta \mathbf{r}_{i,j} \times \mathbf{x}) / \partial \mathbf{x}$ is the cross-product matrix of $\delta \mathbf{r}_{i,j}$.

Error on the Moving-Platform Pose

Because of joint clearances, the frame \mathcal{P}_i is shifted to \mathcal{P}'_i . In $\text{SE}(3)$, the displacement taking frame $\mathcal{F}_{i,j}$ onto $\mathcal{F}'_{i,j}$ is given by the matrix exponential of $\delta \mathbf{S}_{i,j}$, $e^{\delta \mathbf{S}_{i,j}}$. As a result, the screw that represents the pose of the shifted moving-platform may be computed through the i^{th} leg as

$$\mathbf{P}'_i = \prod_{j=1}^{n_i} e^{\delta \mathbf{S}_{i,j}} \mathbf{S}_{i,j}, \quad (11)$$

where screw \mathbf{P}'_i takes frame $\mathcal{F}_{i,1}$ onto \mathcal{P}'_i when taking errors into account.

In order to obtain the moving-platform pose error, the screw $\Delta \mathbf{P}_{i|\mathcal{P}_i}$ that takes the nominal moving-platform pose \mathcal{P}_i onto the shifted one \mathcal{P}'_i through the i^{th} leg is expressed in frame \mathcal{P}_i as

$$\Delta \mathbf{P}_{i|\mathcal{P}_i} = \mathbf{P}_i^{-1} \mathbf{P}'_i, \quad (12)$$

$$= \prod_{j=n_i}^1 \mathbf{S}_{i,j}^{-1} \prod_{j=1}^{n_i} \left(e^{\delta \mathbf{S}_{i,j}} \mathbf{S}_{i,j} \right), \quad (13)$$

$$= \mathbf{S}_{i,n_i}^{-1} \cdots \mathbf{S}_{i,1}^{-1} e^{\delta \mathbf{S}_{i,1}} \mathbf{S}_{i,1} e^{\delta \mathbf{S}_{i,2}} \mathbf{S}_{i,2} \cdots e^{\delta \mathbf{S}_{i,n_i}} \mathbf{S}_{i,n_i},$$

$$\approx \prod_{j=n_i}^1 \mathbf{S}_{i,j}^{-1} \prod_{j=1}^{n_i} \left((\mathbf{1}_{4 \times 4} + \delta \mathbf{S}_{i,j}) \mathbf{S}_{i,j} \right), \quad (14)$$

$$= \prod_{j=n_i}^1 \mathbf{S}_{i,j}^{-1} \prod_{j=1}^{n_i} \left(\mathbf{S}_{i,j} + \delta \mathbf{S}_{i,j} \mathbf{S}_{i,j} \right), \quad (15)$$

$$\approx \prod_{j=n_i}^1 \mathbf{S}_{i,j}^{-1} \left(\prod_{j=1}^{n_i} \mathbf{S}_{i,j} + \delta \mathbf{S}_{i,1} \mathbf{S}_{i,1} \mathbf{S}_{i,2} \cdots \mathbf{S}_{i,n_i} \right. \\ \left. + \mathbf{S}_{i,1} \delta \mathbf{S}_{i,2} \mathbf{S}_{i,2} \cdots \mathbf{S}_{i,n_i} + \mathbf{S}_{i,1} \mathbf{S}_{i,2} \delta \mathbf{S}_{i,3} \mathbf{S}_{i,3} \cdots \mathbf{S}_{i,n_i} \right. \\ \left. + \cdots + \mathbf{S}_{i,1} \mathbf{S}_{i,2} \cdots \mathbf{S}_{i,n_i-1} \delta \mathbf{S}_{i,n_i} \mathbf{S}_{i,n_i} \right), \quad (16)$$

$$= \mathbf{1}_{4 \times 4} + \mathbf{S}_{i,n_i}^{-1} \cdots \mathbf{S}_{i,1}^{-1} \delta \mathbf{S}_{i,1} \mathbf{S}_{i,1} \mathbf{S}_{i,2} \cdots \mathbf{S}_{i,n_i} \\ + \mathbf{S}_{i,n_i}^{-1} \cdots \mathbf{S}_{i,2}^{-1} \delta \mathbf{S}_{i,2} \mathbf{S}_{i,2} \mathbf{S}_{i,3} \cdots \mathbf{S}_{i,n_i} \\ + \cdots + \mathbf{S}_{i,n_i}^{-1} \delta \mathbf{S}_{i,n_i} \mathbf{S}_{i,n_i}, \quad (17)$$

$$= \mathbf{1}_{4 \times 4} + \sum_{j=1}^{n_i} \left(\prod_{k=n_i}^j \mathbf{S}_{i,k}^{-1} \delta \mathbf{S}_{i,j} \prod_{l=j}^{n_i} \mathbf{S}_{i,l} \right). \quad (18)$$

From Eq. (17), we see that end-effector displacement $\Delta \mathbf{P}_{i|\mathcal{P}_i}$ is small, since it is composed of the identity matrix plus a finite sum of small-displacement screws. Therefore, $\Delta \mathbf{P}_{i|\mathcal{P}_i}$ may be as well represented with the small displacement screw

$$\delta \mathbf{p}_{i|\mathcal{P}_i} = \sum_{j=1}^{n_i} \left(\prod_{k=n_i}^j \mathbf{S}_{i,k}^{-1} \delta \mathbf{S}_{i,j} \prod_{l=j}^{n_i} \mathbf{S}_{i,l} \right). \quad (19)$$

Alternatively, the small-displacement screw taking frame \mathcal{P}_i onto frame \mathcal{P}'_i may be computed as a vector in \mathbb{R}^6 , namely, $\delta \mathbf{p}_{i|\mathcal{P}_i}$. To this end, let us recall that the adjoint map of screw $\mathbf{S}_{i,j}$ is

$$\text{adj}(\mathbf{S}_{i,j}) \equiv \begin{bmatrix} \mathbf{R}_{i,j} & \mathbf{O}_{3 \times 3} \\ \mathbf{T}_{i,j} \mathbf{R}_{i,j} & \mathbf{R}_{i,j} \end{bmatrix}, \quad (20)$$

where $\mathbf{T}_{i,j} \equiv \partial(\mathbf{t}_{i,j} \times \mathbf{x}) / \partial \mathbf{x}$ is the cross-product matrix of $\mathbf{t}_{i,j}$. Then, we may express $\delta \mathbf{s}_{i,j}$ in frame $\mathcal{F}_{i,j+1}$ by simply computing $\text{adj}(\mathbf{S}_{i,j}^{-1}) \delta \mathbf{s}_{i,j}$. As a result, $\delta \mathbf{s}_{i,j}$ may be expressed in frame \mathcal{P}_i through the product

$$\left(\prod_{k=n_i}^j \text{adj}(\mathbf{S}_{i,k}^{-1}) \right) \delta \mathbf{s}_{i,j}, \quad (21)$$

and the small-displacement screw taking \mathcal{P}_i onto \mathcal{P}'_i simply becomes

$$\delta \mathbf{p}_{i|\mathcal{P}_i} = \sum_{j=1}^{n_i} \prod_{k=n_i}^j \text{adj}(\mathbf{S}_{i,k}^{-1}) \delta \mathbf{s}_{i,j} \quad (22)$$

$$\equiv \sum_{j=1}^{n_i} \prod_{k=n_i}^j (\text{adj}(\mathbf{S}_{i,k}))^{-1} \delta \mathbf{s}_{i,j} \quad (23)$$

It is noteworthy that $\delta \mathbf{p}_{i|\mathcal{P}_i}$ is expressed in the frame attached to the moving platform, i.e., \mathcal{P}_i . For the evaluation of the pose errors on the moving-platform, the small-displacement screw taking \mathcal{P}_i onto \mathcal{P}'_i has to be expressed in the reference attached to the fixed base, i.e., $\mathcal{F}_{i,1}$. Let $\delta \mathbf{p}_{i|\mathcal{F}_{i,1}}$ be this small-displacement screw expressed in $\mathcal{F}_{i,1}$:

$$\delta \mathbf{p}_{i|\mathcal{F}_{i,1}} = \prod_{j=1}^{n_i} (\mathbf{N}_{i,j}) \delta \mathbf{p}_{i|\mathcal{P}_i}, \quad (24)$$

where

$$\mathbf{N}_{i,j} \equiv \begin{bmatrix} \mathbf{R}_{i,j} & \mathbf{O}_{3 \times 3} \\ \mathbf{O}_{3 \times 3} & \mathbf{R}_{i,j} \end{bmatrix}. \quad (25)$$

As a result,

$$\delta \mathbf{p}_{i|\mathcal{F}_{i,1}} = \prod_{j=1}^{n_i} \mathbf{N}_{i,j} \sum_{j=1}^{n_i} \prod_{k=n_i}^j (\text{adj}(\mathbf{S}_{i,k}))^{-1} \delta \mathbf{s}_{i,j} \quad (26)$$

$$= \sum_{j=1}^{n_i} \left(\prod_{l=1}^{n_i} \mathbf{N}_{i,l} \prod_{k=n_i}^j (\text{adj}(\mathbf{S}_{i,k}))^{-1} \right) \delta \mathbf{s}_{i,j} \quad (27)$$

The following compact form may be used:

$$\delta \mathbf{p}_{i|\mathcal{F}_{i,1}} = \mathbf{M}_i \delta \mathbf{s}_i, \quad (28)$$

where

$$\mathbf{M}_i \equiv [\mathbf{M}_{i,1} \ \mathbf{M}_{i,2} \ \cdots \ \mathbf{M}_{i,n_i}], \quad (29)$$

$$\mathbf{M}_{i,j} \equiv \prod_{l=1}^{n_i} (\mathbf{N}_{i,l}) \prod_{k=n_i}^j (\text{adj}(\mathbf{S}_{i,k}))^{-1}, \quad (30)$$

$$\delta \mathbf{s}_i \equiv [\delta \mathbf{s}_{i,1}^T \ \delta \mathbf{s}_{i,2}^T \ \cdots \ \delta \mathbf{s}_{i,n_i}^T]^T. \quad (31)$$

MODELING THE CLEARANCES IN AN AXISYMMETRICAL JOINT

Intuitively, clearances in a joint are best modeled by bounding its associated errors below and above. Assuming that the lower and upper bounds are the same, this generally yields six parameters that bound the error screw $\delta \mathbf{s}_{i,j}$. In the case of a revolute joint, however, we can take advantage of the axisymmetry to reduce the number of parameters. In the case of a prismatic joint, we assume that the prismatic joints are also axisymmetrical, which encompasses most hydraulic and pneumatic cylinders, as well as ball screws.

Recall that frame $\mathcal{F}_{i,j}$ is attached to joint j of leg i according to the Denavit-Hartenberg convention. As a result, its Z -axis is aligned with the revolute-joint axis. Moreover, the origin of $\mathcal{F}_{i,j}$ may be chosen to lie at the centroid of the revolute joint as shown in Fig. 1. In this case, the Z components of $\delta \mathbf{r}_{i,j}$ and $\delta \mathbf{t}_{i,j}$ —both defined in Eq. (9)—are axial components, while the X and Y components are radial. Accordingly, the error bounds are written as

$$\delta r_{i,j,X}^2 + \delta r_{i,j,Y}^2 \leq \Delta \beta_{i,j,XY}^2, \quad (32)$$

$$\delta r_{i,j,Z}^2 \leq \Delta \beta_{i,j,Z}^2, \quad (33)$$

$$\delta t_{i,j,X}^2 + \delta t_{i,j,Y}^2 \leq \Delta b_{i,j,XY}^2, \quad (34)$$

$$\delta t_{i,j,Z}^2 \leq \Delta b_{i,j,Z}^2, \quad (35)$$

where $\delta \mathbf{r}_{i,j} \equiv [\delta r_{i,j,X} \ \delta r_{i,j,Y} \ \delta r_{i,j,Z}]^T$ and $\delta \mathbf{t}_{i,j} \equiv [\delta t_{i,j,X} \ \delta t_{i,j,Y} \ \delta t_{i,j,Z}]^T$. Notice that previous works [4, 8, 9] have proposed geometric models of the embodiments and clearances of several types of revolute and prismatic joints based on bushings. Since many robotic mechanical systems rely on ball bearings, and not bushings, to allow motion of their joints, these geometrical analyzes are not always applicable. Therefore, we prefer using the generic mathematical inequalities (32–35), which are suitable for any axisymmetrical joint.

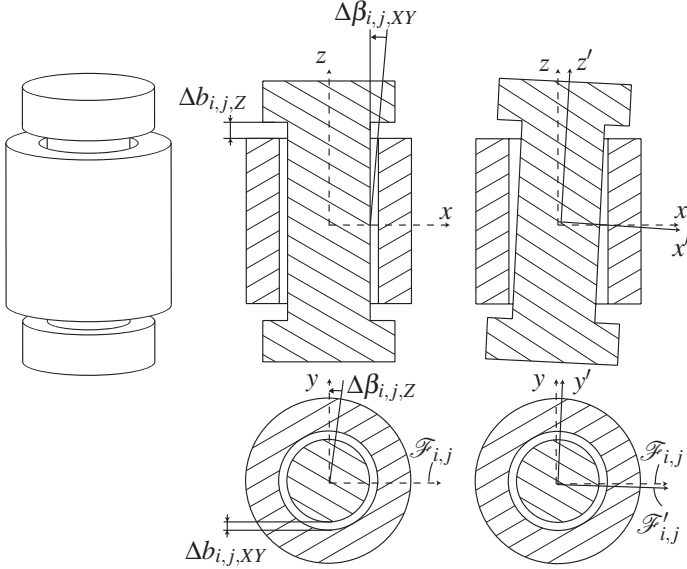


FIGURE 1. CLEARANCE-AFFECTED REVOLUTE JOINT

THE MAXIMUM MOVING-PLATFORM POSE ERRORS

This section aims at finding the maximum reference-point position error and the maximum orientation error of the moving-platform for given joint clearances and to identify the joints that need the tightest tolerances. Let us begin with the general case, where one seeks to identify the maximum errors due to joint clearances in the complete mechanism. For that matter, it would be interesting to compute the maximum rotation and maximum reference-point displacement due to joint clearances.

Formally, the maximum moving-platform rotation r_{\max} due

to joint clearances is obtained by solving the problem

$$\begin{aligned}
 -r_{\max}^2 \equiv \text{minimize} \quad & - \sum_{k=1,2,3} (\mathbf{e}_{6,k}^T \delta \mathbf{p})^2, \\
 \text{over} \quad & \delta \mathbf{p}, \delta \mathbf{s}_{i,j}, j = 1, \dots, n_i, i = 1, \dots, m, \\
 \text{subject to} \quad & (\mathbf{e}_{6,1}^T \delta \mathbf{s}_{i,j})^2 + (\mathbf{e}_{6,2}^T \delta \mathbf{s}_{i,j})^2 - \delta \beta_{XY,i,j}^2 \leq 0, \\
 & (\mathbf{e}_{6,3}^T \delta \mathbf{s}_{i,j})^2 - \delta \beta_{Z,i,j}^2 \leq 0, \\
 & (\mathbf{e}_{6,4}^T \delta \mathbf{s}_{i,j})^2 + (\mathbf{e}_{6,5}^T \delta \mathbf{s}_{i,j})^2 - \delta b_{XY,i,j}^2 \leq 0, \\
 & (\mathbf{e}_{6,6}^T \delta \mathbf{s}_{i,j})^2 - \delta b_{Z,i,j}^2 \leq 0, \\
 & \delta \mathbf{p} = \mathbf{M}_i \delta \mathbf{s}_i, \\
 & j = 1, \dots, n_i, i = 1, \dots, m,
 \end{aligned} \tag{36}$$

where $\mathbf{e}_{j,k} \in \mathbb{R}^6$ is defined such that $\mathbf{1}_{j \times j} \equiv [\mathbf{e}_{j,1} \ \mathbf{e}_{j,2} \ \dots \ \mathbf{e}_{j,j}]$.

Notice that r_{\max} is independent of $\delta \mathbf{t}_{i,j}$, $j = 1, \dots, n_i$, $i = 1, \dots, m$, i.e., the errors on the positions of the joint centroids. This may be verified by noticing that the point-displacement component of the errors on the joints in the i^{th} leg, $\delta \mathbf{s}_i$, always lies in the nullspace of the first three rows of \mathbf{M}_i . Accordingly, their contribution to the objective (through the equality of Eq. (36)) is null. Furthermore, for the proposed error model, the rotation and point-displacement errors, $\delta \mathbf{r}_{i,j}$ and $\delta \mathbf{t}_{i,j}$, are completely decoupled in the inequality constraints. In conclusion, since $\delta \mathbf{t}_{i,j}$ does not affect the objective and is decoupled from the optimization variables that affect the objective, its associated constraints may be removed from Eq. (36). This leaves us with the simpler equivalent form

$$\begin{aligned}
 -r_{\max}^2 \equiv \text{minimize} \quad & - \sum_{k=1,2,3} (\mathbf{e}_{6,k}^T \delta \mathbf{p})^2, \\
 \text{over} \quad & \delta \mathbf{p}, \delta \mathbf{s}_{i,j}, j = 1, \dots, n_i, i = 1, \dots, m, \\
 \text{subject to} \quad & (\mathbf{e}_{6,1}^T \delta \mathbf{s}_{i,j})^2 + (\mathbf{e}_{6,2}^T \delta \mathbf{s}_{i,j})^2 - \delta \beta_{XY,i,j}^2 \leq 0, \\
 & (\mathbf{e}_{6,3}^T \delta \mathbf{s}_{i,j})^2 - \delta \beta_{Z,i,j}^2 \leq 0, \\
 & \delta \mathbf{p} = \mathbf{M}_i \delta \mathbf{s}_i, \\
 & j = 1, \dots, n_i, i = 1, \dots, m.
 \end{aligned} \tag{37}$$

The expression of the maximum point-displacement p_{\max} due to joint clearances is the same as that of r_{\max} , except for

the objective function, namely,

$$\begin{aligned}
-p_{\max}^2 \equiv \text{minimize} \quad & - \sum_{k=4,5,6} (\mathbf{e}_{6,k}^T \delta \mathbf{p})^2, \quad (38) \\
\text{over} \quad & \delta \mathbf{p}, \delta \mathbf{s}_{i,j}, j = 1, \dots, n_i, i = 1, \dots, m, \\
\text{subject to} \quad & (\mathbf{e}_{6,1}^T \delta \mathbf{s}_{i,j})^2 + (\mathbf{e}_{6,2}^T \delta \mathbf{s}_{i,j})^2 - \delta \beta_{XY,i,j}^2 \leq 0, \\
& (\mathbf{e}_{6,3}^T \delta \mathbf{s}_{i,j})^2 - \delta \beta_{Z,i,j}^2 \leq 0, \\
& (\mathbf{e}_{6,4}^T \delta \mathbf{s}_{i,j})^2 + (\mathbf{e}_{6,5}^T \delta \mathbf{s}_{i,j})^2 - \delta b_{XY,i,j}^2 \leq 0, \\
& (\mathbf{e}_{6,6}^T \delta \mathbf{s}_{i,j})^2 - \delta b_{Z,i,j}^2 \leq 0, \\
& \delta \mathbf{p} = \mathbf{M}_i \delta \mathbf{s}_i, \\
& j = 1, \dots, n_i, i = 1, \dots, m,
\end{aligned}$$

In the case of p_{\max} , the objective depends on both the rotation and the point-displacement errors. Therefore, Eq. (38) cannot be simplified as was Eq. (36).

Computing the Maximum Moving-Platform Pose Errors

The problems of Eqs. (37) and (38) are nonconvex quadratically constrained quadratic programs (QCQPs). Although their feasible sets are convex—all the constraints of both problems are convex—their objectives are concave, making the computation of their global minima $-r_{\max}^2$ and $-p_{\max}^2$ non-trivial, at least in general. Notice that some specific instances of nonconvex QCQPs were proven to have equivalent convex formulations [12–14], and hence, allow for the computation of solutions in polynomial time. However, the problems of Eqs. (37) and (38) do not correspond to any of these classes of QCQPs. This calls for an additional effort in order to reliably and efficiently compute the global optima r_{\max} and p_{\max} .

We begin by rewriting Eq. (38) in a form that is similar to the original problem in ref. [15]. This requires the elimination of the equality constraints, which may be achieved through a change of variables. To this end, let us resort to QR factorizations of \mathbf{M}_i^T ,

$$\begin{aligned}
\mathbf{M}_i^T &= \mathbf{V}_i \mathbf{U}_i, \quad (39) \\
&= [\mathbf{V}_{i,1} \ \mathbf{V}_{i,2}] \begin{bmatrix} \mathbf{U}_{i,1} \\ \mathbf{0}_{(6n-6) \times 6} \end{bmatrix}, \\
&= \mathbf{V}_{i,1} \mathbf{U}_{i,1}, \quad (40)
\end{aligned}$$

$i = 1, \dots, m$, where $\mathbf{V}_i \in \mathbb{R}^{6n_i \times 6n_i}$ is orthogonal, $\mathbf{U}_i \in \mathbb{R}^{6n_i \times 6n_i}$ is upper-triangular, $\mathbf{V}_{i,1} \in \mathbb{R}^{6 \times 6}$, $\mathbf{V}_{i,2} \in \mathbb{R}^{6 \times (6n_i-6)}$, and $\mathbf{U}_{i,1} \in \mathbb{R}^{6 \times 6}$ is an upper-triangular matrix. This QR factorization may be computed using the Householder method. Notice that, from its structure, \mathbf{M}_i is bound to bear its full row rank of six. As a result, $\mathbf{U}_{i,1}$

is also of full rank, *i.e.*, it is invertible. Moreover, we define

$$\delta \mathbf{q}_i \equiv \begin{bmatrix} \delta \mathbf{q}_{i,1} \\ \delta \mathbf{q}_{i,2} \end{bmatrix} = \mathbf{V}_i^T \delta \mathbf{s}_i, \quad (41)$$

where $\delta \mathbf{q}_{i,1} \in \mathbb{R}^6$ and $\delta \mathbf{q}_{i,2} \in \mathbb{R}^{6n_i-6}$, so that

$$\delta \mathbf{s}_i = \mathbf{V}_{i,1} \delta \mathbf{q}_{i,1} + \mathbf{V}_{i,2} \delta \mathbf{q}_{i,2}. \quad (42)$$

This allows us to rewrite the equality constraints as

$$\delta \mathbf{p} = \mathbf{M}_i \delta \mathbf{s}_i = \mathbf{U}_{i,1}^T \mathbf{V}_{i,1}^T \delta \mathbf{s}_i = \mathbf{U}_{i,1}^T \delta \mathbf{q}_{i,1}, \quad (43)$$

or,

$$\delta \mathbf{q}_{i,1} = \mathbf{U}_{i,1}^{-T} \delta \mathbf{p}, \quad (44)$$

where $\mathbf{U}_{i,1}^{-T}$ is the inverse of the transpose of $\mathbf{U}_{i,1}$. Upon substituting Eq. (44) into Eq. (42), we obtain

$$\delta \mathbf{s}_i = \mathbf{V}_{i,1} \mathbf{U}_{i,1}^{-T} \delta \mathbf{p} + \mathbf{V}_{i,2} \delta \mathbf{q}_{i,2}. \quad (45)$$

Let us regroup all remaining optimization variables into the array

$$\delta \mathbf{u} \equiv [\delta \mathbf{p}^T \ \delta \mathbf{q}_{1,2}^T \ \delta \mathbf{q}_{2,2}^T \ \dots \ \delta \mathbf{q}_{m,2}^T]^T \in \mathbb{R}^v,$$

where $v = 6 + 6 \sum_{k=1}^m (n_k - 1)$, so that $\delta \mathbf{s}_i$ may be expressed as

$$\delta \mathbf{s}_i \equiv \Upsilon_i \delta \mathbf{u}, \quad (46)$$

where

$$\Upsilon_i \equiv \begin{bmatrix} \mathbf{V}_{i,1} \mathbf{U}_{i,1}^{-T} & \mathbf{V}_{i,2} & \mathbf{0}_{6n_i \times 6 \sum_{k=i+1}^m (n_k-1)} \end{bmatrix} \in \mathbb{R}^{6n_i \times v}, \quad (47)$$

for $i = 1$

$$\Upsilon_i \equiv \begin{bmatrix} \mathbf{V}_{i,1} \mathbf{U}_{i,1}^{-T} & \mathbf{0}_{6n_i \times 6 \sum_{k=1}^{i-1} (n_k-1)} & \mathbf{V}_{i,2} \\ \mathbf{0}_{6n_i \times 6 \sum_{k=i+1}^m (n_k-1)} \end{bmatrix} \in \mathbb{R}^{6n_i \times v}, \quad (48)$$

for $i = 2, \dots, m-1$ and

$$\Upsilon_i \equiv \begin{bmatrix} \mathbf{V}_{i,1} \mathbf{U}_{i,1}^{-T} & \mathbf{0}_{6n_i \times 6 \sum_{k=1}^{i-1} (n_k-1)} & \mathbf{V}_{i,2} \end{bmatrix} \in \mathbb{R}^{6n_i \times v} \quad (49)$$

for $i = m$.

Finally, we substitute Eq. (46) into Eq. (38) in order to come up with an optimization problem that contains only inequality constraints, namely,

$$\begin{aligned} -p_{\max}^2 &\equiv \text{minimize} & f_0 &\equiv -\delta \mathbf{u}^T \mathbf{F}_0 \delta \mathbf{u}, & (50) \\ &\text{over} & \delta \mathbf{u}, & \\ &\text{subject to} & f_{i,j,k}(\delta \mathbf{u}) &\equiv \delta \mathbf{u}^T \mathbf{F}_{i,j,k} \delta \mathbf{u} - 1 \leq 0, \\ & & k &= 1, \dots, 4, j = 1, \dots, n_i, \\ & & i &= 1, \dots, m, \end{aligned}$$

where

$$\mathbf{F}_0 \equiv \sum_{l=4,5,6} \mathbf{e}_{v,l} \mathbf{e}_{v,l}^T, \quad (51)$$

$$\mathbf{F}_{i,j,1} \equiv (1/\delta \beta_{XY,i,j}^2) \Upsilon_i^T (\mathbf{e}_{6n_i,6j-5} \mathbf{e}_{6n_i,6j-5}^T + \mathbf{e}_{6n_i,6j-4} \mathbf{e}_{6n_i,6j-4}^T) \Upsilon_i, \quad (52)$$

$$\mathbf{F}_{i,j,2} \equiv (1/\delta \beta_{Z,i,j}^2) \Upsilon_i^T \mathbf{e}_{6n_i,6j-3} \mathbf{e}_{6n_i,6j-3}^T \Upsilon_i, \quad (53)$$

$$\mathbf{F}_{i,j,3} \equiv (1/\delta b_{XY,i,j}^2) \Upsilon_i^T (\mathbf{e}_{6n_i,6j-2} \mathbf{e}_{6n_i,6j-2}^T + \mathbf{e}_{6n_i,6j-1} \mathbf{e}_{6n_i,6j-1}^T) \Upsilon_i, \quad (54)$$

$$\mathbf{F}_{i,j,4} \equiv (1/\delta b_{Z,i,j}^2) \Upsilon_i^T \mathbf{e}_{6n_i,6j} \mathbf{e}_{6n_i,6j}^T \Upsilon_i. \quad (55)$$

In the scope of this paper, optimization problem (50) is solved with ModeFrontier software [16] in order to find the maximum reference-point position error and the maximum orientation error of the moving-platform for given joint clearances and for any robot configuration. This optimization problem could be also solved by means of Interval Analysis [17] or with the algorithm proposed in [15].

ILLUSTRATIVE EXAMPLES

The kinematic sensitivities of a 3R serial manipulator and a five-bar parallel manipulator to joint clearances are analyzed in this section in order to highlight the contributions of the paper. The lengths are expressed in meter, while the angles are given in radian in these examples.

3R Serial Manipulator

Figure 2 illustrates the 3R serial manipulator under study. It is composed of one leg, i.e., $m = 1$, that contains three revolute

joints, i.e., $n_1 = 3$. The first and the second joints axes are orthogonal, while the second and the third ones are parallel. As a consequence, the Denavit-Hartenberg parameters of the manipulator take the form:

$$\{\alpha_{1,1}, a_{1,1}, b_{1,1}, \theta_{1,1}\} = -\pi/2, 0, 10, \theta_{1,1} \quad (56)$$

$$\{\alpha_{1,2}, a_{1,2}, b_{1,2}, \theta_{1,2}\} = 0, 5, 0, \theta_{1,2} \quad (57)$$

$$\{\alpha_{1,3}, a_{1,3}, b_{1,3}, \theta_{1,3}\} = 0, 5, 0, \theta_{1,3} \quad (58)$$

$\theta_{1,j}$, $j = 1, \dots, 3$ being the revolute joint angles. The er-

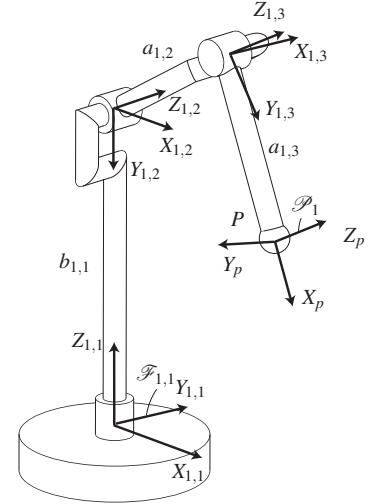


FIGURE 2. 3R SERIAL MANIPULATOR

ror bounds characterizing the joint clearances and defined in Eqs.(32–35) are assumed to be:

$$\Delta \beta_{1,j,XY} = 0.01 \quad (59)$$

$$\Delta \beta_{1,j,Z} = 0.01 \quad (60)$$

$$\Delta b_{1,j,XY} = 0.01 \quad (61)$$

$$\Delta b_{1,j,Z} = 0.01 \quad (62)$$

Figure 3 shows the end-effector reference-point position error due to the joint clearances, the nominal Cartesian coordinates of the end-effector P in base frame $\mathcal{F}_{1,1}$ being equal to

$$\mathbf{p}_{|\mathcal{F}_{1,1}} \equiv [p_x \ p_y \ p_z]^T = [5 \ 0 \ 6]^T \quad (63)$$

As a matter of fact, Fig. 3 illustrates the range of end-effector reference-point position errors in frame \mathcal{P}_1 , the latter being attached to the end-effector of the manipulator in its nominal con-

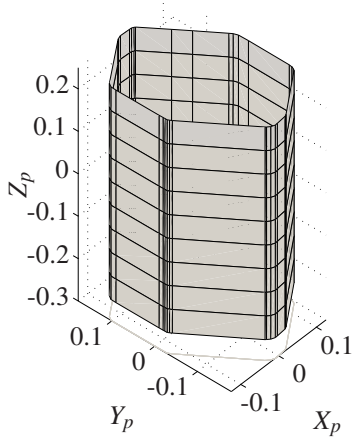


FIGURE 3. THE RANGE OF END-EFFECTOR REFERENCE-POINT POSITION ERRORS IN FRAME \mathcal{P}_1 FOR THE NOMINAL END-EFFECTOR POSITION DEFINED WITH EQ. (63)

figuration. It turns out that the maximum reference-point position errors of P along X_p , Y_p and Z_p are equal to 0.12, 0.19 and 0.22, respectively.

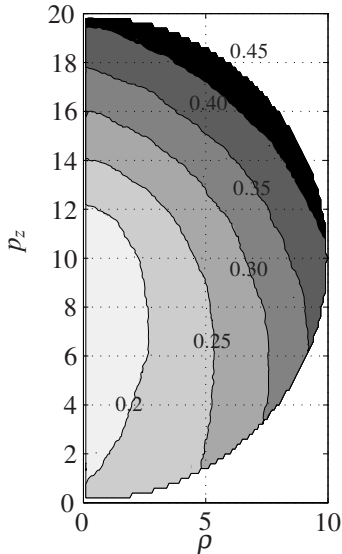


FIGURE 4. ISOCONTOURS OF p_{max} THROUGHOUT W_s

Figure 4 shows the isocontours of the maximum point-displacement p_{max} throughout the manipulator workspace W_s , which is defined with respect to ρ and p_z , $\rho \equiv \sqrt{p_x^2 + p_y^2}$. p_{max} is evaluated at each grid-point by means of optimization problem (50). It appears that p_{max} varies between 0.2 and 0.54 throughout W_s .

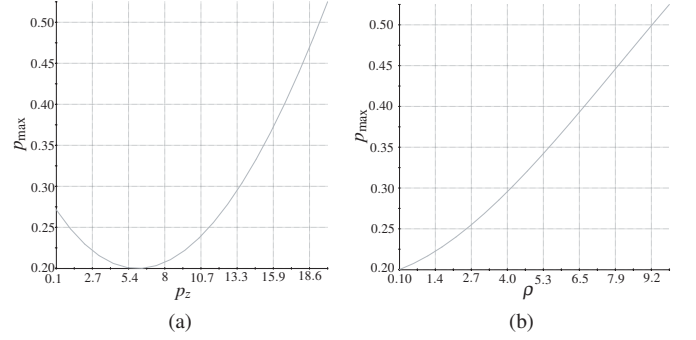


FIGURE 5. THE MAXIMUM POINT-DISPLACEMENT p_{max} OF THE END-EFFECTOR AS A FUNCTION OF: (a) p_z ; (b) ρ

Notice that the maximum orientation error on the end-effector is constant throughout W_s and is equal to 0.043 rad.

Figures 5(a)-(b) show the trend in the maximum point-displacement p_{max} of the end-effector as a function of the Z-coordinate of the end-effector in frame $\mathcal{F}_{1,1}$ and ρ , respectively. From Fig. 5(a), p_{max} is a minimum when p_z is equal to 5.8. From Fig. 5(b), the higher ρ , the higher p_{max} .

Five-bar Parallel Manipulator

Figure 6 illustrates the five-bar parallel manipulator under study. It is composed of two legs, i.e., $m = 2$, that contain two revolute joints, i.e., $n_1 = n_2 = 2$. All revolute joints axes are parallel. As a consequence, the Denavit-Hartenberg parameters of the manipulator take the form:

$$\{\alpha_{1,1}, a_{1,1}, b_{1,1}, \theta_{1,1}\} = 0, 5, 0, \theta_{1,1} \quad (64)$$

$$\{\alpha_{1,2}, a_{1,2}, b_{1,2}, \theta_{1,2}\} = 0, 10, 0, \theta_{1,2} \quad (65)$$

$$\{\alpha_{2,1}, a_{2,1}, b_{2,1}, \theta_{2,1}\} = 0, 5, 0, \theta_{2,1} \quad (66)$$

$$\{\alpha_{2,2}, a_{2,2}, b_{2,2}, \theta_{2,2}\} = 0, 10, 0, \theta_{2,2} \quad (67)$$

$\theta_{i,j}$, $i = 1, \dots, 2$, $j = 1, \dots, 2$, is the angle of the j th revolute joint of the i th leg. The distance c between the axes of the revolute joints attached to the base is equal to 5. The error bounds characterizing the joint clearances and defined in Eqs.(32–35) are assumed to be:

$$\Delta\beta_{i,j,XY} = 0.01 \quad (68)$$

$$\Delta\beta_{i,j,Z} = 0.01 \quad (69)$$

$$\Delta b_{i,j,XY} = 0.1 \quad (70)$$

$$\Delta b_{i,j,Z} = 0.1 \quad (71)$$

Notice that the revolute joint connecting the two legs is supposed to be perfect, i.e., without any clearance.

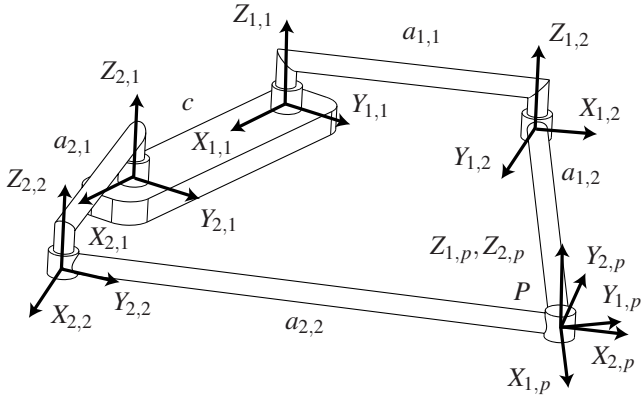


FIGURE 6. FIVE-BAR PARALLEL MANIPULATOR

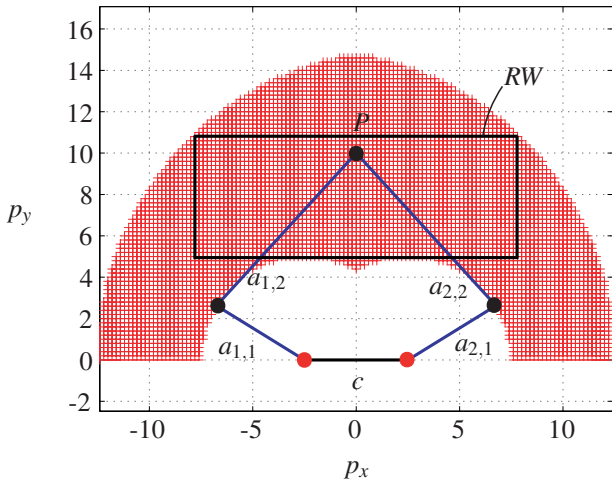


FIGURE 7. THE WORKSPACE AND REGULAR WORKSPACE RW OF THE FIVE-BAR PARALLEL MANIPULATOR UNDER STUDY

Figure 7 illustrates the workspace of the five-bar parallel manipulator under study and its regular workspace RW . The regular workspace of a manipulator is a regular-shaped part of its workspace. The regular workspace shape is up to the designer. It may be a cube, a parallelepiped, a cylinder or another regular shape [18]. A reasonable choice is a shape that “fits well” the one of the singular surfaces. It turns out that a rectangle suits well for the five-bar parallel manipulator. The regular workspace RW of the manipulator under study is obtained by means of a Tabu search Hooke and Jeeves algorithm [19].

Figure 8 shows the isocontours of the maximum point-displacement, p_{\max} , of point P throughout RW . Likewise, Fig. 9 shows the isocontours of the maximum orientation error, r_{\max} , of the moving-platform throughout RW . Notice that p_{\max} and r_{\max} are evaluated at each grid-point of RW by means of optimization problem (50). It is apparent that p_{\max} varies between

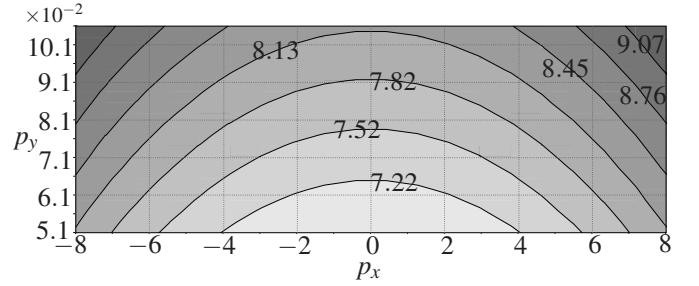


FIGURE 8. ISOCONTOURS OF p_{\max} THROUGHOUT RW

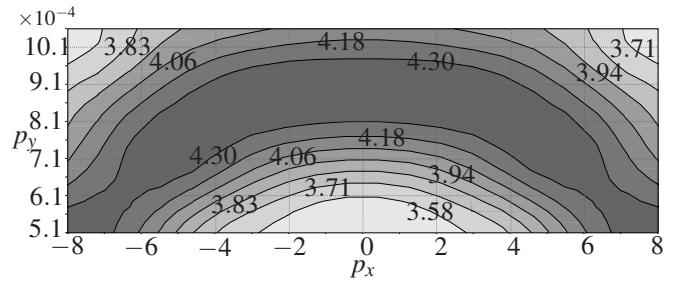


FIGURE 9. ISOCONTOURS OF r_{\max} THROUGHOUT RW

7.15×10^{-2} and 9.12×10^{-2} , while r_{\max} varies 3.49×10^{-4} rad and 4.41×10^{-4} rad throughout RW .

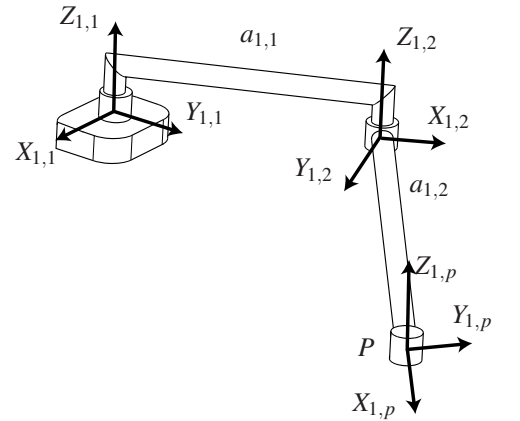


FIGURE 10. 2R SERIAL MANIPULATOR

Figure 10 depicts one leg of the five-bar parallel manipulator under study, namely, a 2R serial manipulator. Figure 11 shows the isocontours of the maximum point-displacement, p_{\max} , of its end-effector P throughout RW . It appears that p_{\max} varies between 3.58×10^{-1} and 5.19×10^{-1} . Moreover, the maximum orientation error of its moving-platform is constant throughout RW and is equal 2.28×10^{-2} rad.

It is noteworthy the maximum orientation error of the end-effector of any serial manipulator due to joint clearances is constant throughout its workspace.

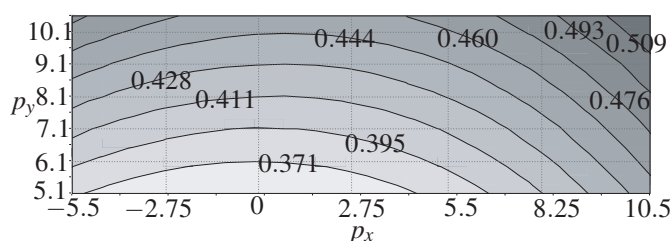


FIGURE 11. ISOCONTOURS OF p_{\max} THROUGHOUT RW

As a result, the maximum point-displacement of the moving-platform of the serial manipulator is about five times larger than the one of its parallel counterpart. Similarly, the maximum orientation error of the moving-platform of the serial manipulator is about fifty times larger than the one of the parallel manipulator. As a matter of fact, the parallel manipulator turns to be quite more accurate than its serial counterpart as the revolute joint that connects the legs is assumed to be perfect and the link lengths are relatively large in comparison with joint clearances.

CONCLUSIONS

The paper dealt with the kinematic sensitivity of robotic manipulators to joint clearances. First, an error prediction model applicable to both serial and parallel manipulators was developed. A clearance model associated with axisymmetrical joints, which are widely used in robotic manipulators, was also proposed. Then, two nonconvex quadratically constrained quadratic programs (QCQPs) were formulated in order to find the maximum reference-point position error and the maximum orientation error of the moving-platform for given joint clearances. Finally, the contributions of the paper were highlighted by means of serial and parallel manipulators. It turns out that 2R serial manipulators are more sensitive to joint clearances than five-bar parallel manipulators. This result makes sense as joint clearances are compensated in parallel manipulators.

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