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► **To cite this version:**

Florence Forbes, Hongliang Lu, Julyan Arbel. Non parametric Bayesian priors for hidden Markov random fields. JSM 2018 - Joint Statistical Meeting, Jul 2018, Vancouver, Canada. pp.1-38. hal-01941679

**HAL Id: hal-01941679**

**<https://hal.science/hal-01941679>**

Submitted on 12 Dec 2018

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# Bayesian Nonparametric Priors for Hidden Markov Random Fields: Application to Image Segmentation

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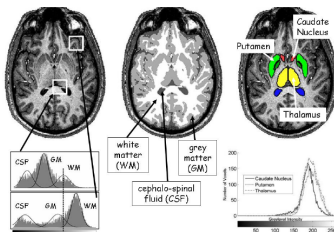
July 2018



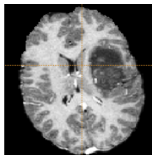
# Unsupervised image segmentation

## Challenges for mixture models (clustering)

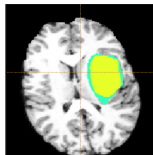
inhomogeneities, noise



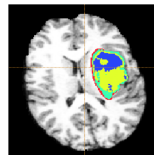
How many segments?



T1 gado



2 classes

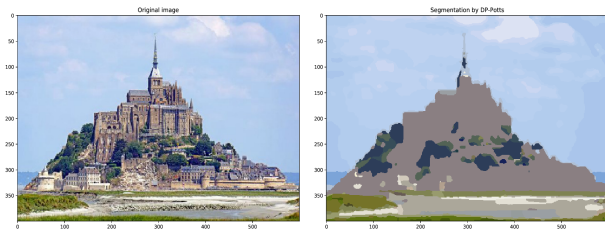


4 classes

## Extensions of Dirichlet Process mixture model with spatial regularization

# Outline of the talk

- 1 Dirichlet process (DP)
- 2 Spatially-constrained mixture model: DP-Potts mixture model
  - Finite mixture model
  - Bayesian finite mixture model
  - DP mixture model
  - DP-Potts mixture model
- 3 Inference using variational approximation
- 4 Some image segmentation results
- 5 Conclusion and future work





# Dirichlet process (DP)

The DP is a central Bayesian nonparametric (BNP) prior<sup>1</sup>.

## Definition (Dirichlet process)

A **Dirichlet process** on the space  $\mathcal{Y}$  is a **random process**  $G$  such that there exist  $\alpha$  (concentration parameter) and  $G_0$  (base distribution) such that for any finite partition  $\{A_1, \dots, A_p\}$  of  $\mathcal{Y}$ , the random vector  $(P(A_1), \dots, P(A_p))$  will be Dirichlet distributed:

$$(P(A_1), \dots, P(A_p)) \sim \text{Dir}(\alpha G_0(A_1), \dots, \alpha G_0(A_p))$$

Notation:  $G \sim \text{DP}(\alpha, G_0)$

The DP is the infinite-dimensional generalization of the Dirichlet distribution.

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<sup>1</sup>Ferguson, T. (1973). A Bayesian analysis of some nonparametric problems. *The Annals of Statistics*, 1(2):209–230.

# Dirichlet process (DP) construction

A DP prior  $G$  can be constructed using three methods:

- The Blackwell-MacQueen urn scheme
- The Chinese Restaurant Process
- The Stick-Breaking construction

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<sup>2</sup>Sethuraman, J. (1994). A constructive definition of Dirichlet priors. *Statistica Sinica*, 4:639-650.

# Dirichlet process (DP) construction

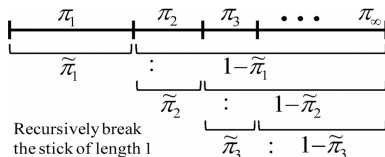
A DP prior  $G$  can be constructed using three methods:

- The Blackwell-MacQueen urn scheme
- The Chinese Restaurant Process
- **The Stick-Breaking construction**

The DP has almost surely **discrete** realizations<sup>2</sup>:

$$G = \sum_{k=1}^{\infty} \pi_k \delta_{\theta_k^*}$$

where  $\theta_k^* \stackrel{\text{iid}}{\sim} G_0$  and  $\pi_k = \tilde{\pi}_k \prod_{l < k} (1 - \tilde{\pi}_l)$  with  $\tilde{\pi}_k \stackrel{\text{iid}}{\sim} \text{Beta}(1, \alpha)$ .



<sup>2</sup>Sethuraman, J. (1994). A constructive definition of Dirichlet priors. *Statistica Sinica*, 4:639-650.

# Spatially-constrained mixture model: DP-Potts mixture

Clustering/segmentation: Finite mixture models assume data are generated by a finite sum of probability distributions:

$$\mathbf{y} = (\mathbf{y}_1, \dots, \mathbf{y}_N) \text{ with } \mathbf{y}_i = (y_{i1}, \dots, y_{iD}) \in \mathbb{R}^D \text{ i.i.d}$$

$$p(\mathbf{y}_i | \theta^*, \pi) = \sum_{k=1}^K \pi_k F(\mathbf{y}_i | \theta_k^*)$$

where

- $\theta^* = (\theta_1^*, \dots, \theta_K^*)$  and  $\pi = (\pi_1, \dots, \pi_K)$  with  $\theta^*$  class parameters and  $\pi$  mixture weights with  $\sum_{i=1}^K \pi_i = 1$ .
- $\theta^*$  and  $\pi$  can be estimated using EM algorithm.

Equivalently

- $G = \sum_{k=1}^K \pi_k \delta_{\theta_k^*}$  non random
- $\theta_i \sim G$  and  $\mathbf{y}_i | \theta_i \sim F(\mathbf{y}_i | \theta_i)$ .

# Bayesian finite mixture model

In a Bayesian setting, a prior distribution is placed over  $\theta^*$  and  $\pi$ .

Thus, the posterior distribution of parameters given the observations is

$$p(\theta^*, \pi | \mathbf{y}) \propto p(\mathbf{y} | \theta^*, \pi) p(\theta^*, \pi)$$

To generate a data point within a **Bayesian finite mixture model**:

- $\theta_k^* \sim G_0$
- $\pi_1, \dots, \pi_K \sim \text{Dir}(\alpha/K, \dots, \alpha/K)$
- $G = \sum_{k=1}^K \pi_k \delta_{\theta_k^*}$  is a random measure
- $\theta_i | G \sim G$ , which means  $\theta_i = \theta_k^*$  with probability  $\pi_k$
- $\mathbf{y}_i | \theta_i \sim F(\mathbf{y}_i | \theta_i)$

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- $\mathbf{y}_i | \theta_i \sim F(\mathbf{y}_i | \theta_i)$

## Limitation:

Require specifying the number of components  $K$  beforehand.

## Solution:

Assume an infinite number of components using BNP priors.

# DP mixture model

From a Bayesian finite mixture model to a DP mixture model

To establish a DP mixture model, let  $G$  be a DP prior ( $K \rightarrow \infty$ ), namely

$$G \sim \text{DP}(\alpha, G_0)$$

and complement it with a likelihood associated to each  $\theta_i$

To generate a data point within a **DP mixture model**:

- $G \sim \text{DP}(\alpha, G_0)$
- $\theta_i | G \sim G$
- $\mathbf{y}_i | \theta_i \sim F(\mathbf{y}_i | \theta_i)$

# DP mixture model

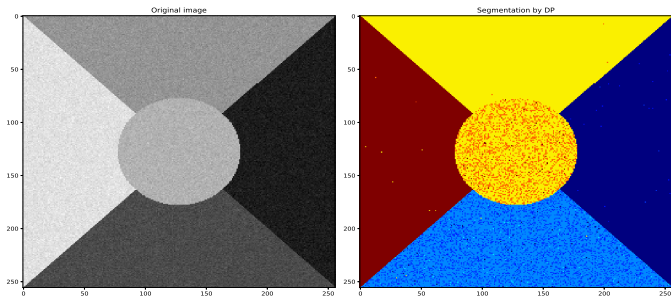
2D point clustering (unsupervised learning) based on the DP mixture model:

Let the data speak for themselves!



# DP mixture model

Application to image segmentation:



## Drawback:

Spatial constraints and dependencies are not considered.

## Solution:

Combine the DP prior with a hidden Markov random field (HMRF).

# DP-Potts mixture model

To solve the issue, we introduce a spatial Potts model component:

$$M(\theta) \propto \exp \left( \beta \sum_{i \sim j} \delta_{z(\theta_i)=z(\theta_j)} \right)$$

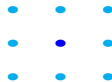
with  $\theta = (\theta_1, \dots, \theta_N)$  and  $\beta$  the interaction parameter.

The DP mixture model is thus extended:

- $G \sim \text{DP}(\alpha, G_0)$
- $\theta | M, G \sim M(\theta) \times \prod_i G(\theta_i)$
- $\mathbf{y}_i | \theta_i \sim F(\mathbf{y}_i | \theta_i)$



4-neighbours



8-neighbours

# DP-Potts mixture model

Other spatially-constrained BNP mixture models + inference algorithms:

- DP or PYP-Potts partition model + MCMC<sup>3</sup>
- Hemodynamic brain parcellation (DP-Potts) + PARTIAL VB<sup>4</sup>
- DP or PYP-Potts + Iterated Conditional Mode (ICM)<sup>5</sup>

## Markov chain Monte Carlo (MCMC):

- Advantage: asymptotically exact
- Drawback: computationally expensive

## Variational Bayes (VB):

- Advantage: much faster
- Drawback: less accurate, no theoretical guarantee

We propose a **DP-Potts mixture model** based on a **general stick-breaking construction** that allows a **natural Full VB algorithm** enabling scalable inference for large datasets and straightforward generalization to other priors.

<sup>3</sup>Orbanz & Buhmann (2008); Xu, Caron & Doucet (2016); Sodjo, Giremus, Dobigeon & Giovannelli (2017)

<sup>4</sup>Albughdadi, Chaari, Tournet, Forbes, Ciuciu (2017)

<sup>5</sup>Chatzis & Tsechpenakis (2010); Chatzis (2013)

# DP-Potts: Stick breaking construction

Stick breaking construction of DP:  $G \sim DP(\alpha, G_0)$

- $\theta_k^* | G_0 \sim G_0$
- $\tau_k | \alpha \sim \mathcal{B}(1, \alpha), k = 1, 2, \dots$
- $\pi_k(\tau) = \tau_k \prod_{l=1}^{k-1} (1 - \tau_l), k = 1, 2, \dots$
- $G = \sum_{k=1}^{\infty} \pi_k(\tau) \delta_{\theta_k^*}$

+

- $\theta_i | G \sim G$
- $\mathbf{y}_i | \theta_i \sim F(\mathbf{y}_i | \theta_i)$

= Dirichlet Process Mixture Model (DPMM)

# DP-Potts: Stick breaking construction

## Stick breaking construction of DPMM

- $\theta_k^* | G_0 \sim G_0$
- $\tau_k | \alpha \sim \mathcal{B}(1, \alpha), k = 1, 2, \dots$
- $\pi_k(\tau) = \tau_k \prod_{l=1}^{k-1} (1 - \tau_l), k = 1, \dots$
- $G = \sum_{k=1}^{\infty} \pi_k(\tau) \delta_{\theta_k^*} \implies$
- $\theta_i | G \sim G$
- $\mathbf{y}_i | \theta_i \sim F(\mathbf{y}_i | \theta_i)$

## Stick breaking construction of DPMM

- $\theta_k^* | G_0 \sim G_0$
- $\tau_k | \alpha \sim \mathcal{B}(1, \alpha), k = 1, 2, \dots$
- $\pi_k(\tau) = \tau_k \prod_{l=1}^{k-1} (1 - \tau_l), k = 1, \dots$
- $\theta_i = \theta_k^*$  with probability  $\pi_k(\tau)$
- $\mathbf{y}_i | \theta_i \sim F(\mathbf{y}_i | \theta_i)$

# DP-Potts: Stick breaking construction

Using assignment variables  $z_i$

## DPMM view

- $\theta_k^* | G_0 \sim G_0$
- $\tau_k | \alpha \sim \mathcal{B}(1, \alpha), k = 1, 2, \dots$
- $\pi_k(\tau) = \tau_k \prod_{l=1}^{k-1} (1 - \tau_l), k = 1, \dots$
- $\theta_i = \theta_k^*$  with probability  $\pi_k(\tau)$
- $\mathbf{y}_i | \theta_i \sim F(\mathbf{y}_i | \theta_i)$   $\implies$

## Mixture/Clustering view

- $\theta_k^* | G_0 \sim G_0$
- $\tau_k | \alpha \sim \mathcal{B}(1, \alpha), k = 1, 2, \dots$
- $\pi_k(\tau) = \tau_k \prod_{l=1}^{k-1} (1 - \tau_l), k = 1, \dots$
- $p(z_i = k | \tau) = \pi_k(\tau)$
- with  $z_i = z(\theta_i) = k$  when  $\theta_i = \theta_k^*$
- $\mathbf{y}_i | z_i, \theta^* \sim F(\mathbf{y}_i | \theta_{z_i}^*)$

# DP-Potts: Stick breaking construction

Using assignment variables  $z_i$

## Stick breaking of DPMM

- $\theta_k^* | G_0 \sim G_0$
- $\tau_k | \alpha \sim \mathcal{B}(1, \alpha), k = 1, 2, \dots$
- $\pi_k(\tau) = \tau_k \prod_{l=1}^{k-1} (1 - \tau_l)$
- $p(z_i = k | \tau) = \pi_k(\tau)$
- $\mathbf{y}_i | z_i, \theta^* \sim F(\mathbf{y}_i | \theta_{z_i}^*)$

## Stick breaking of DP-Potts

- $\theta_k^* | G_0 \sim G_0$
- $\tau_k | \alpha \sim \mathcal{B}(1, \alpha), k = 1, 2, \dots$
- $\pi_k(\tau) = \tau_k \prod_{l=1}^{k-1} (1 - \tau_l)$
- $p(\mathbf{z} | \tau, \beta) \propto \prod_i \pi_{z_i}(\tau) \exp(\beta \sum_{i \sim j} \delta_{z_i = z_j})$
- $\mathbf{z} = \{z_1, \dots, z_N\}$
- $\mathbf{y}_i | z_i, \theta^* \sim F(\mathbf{y}_i | \theta_{z_i}^*)$

**NB:** Well defined for every stick breaking construction ( $\sum_{k=1}^{\infty} \pi_k = 1$ ) :

e.g. Pitman-Yor ( $\tau_k | \alpha, \sigma \sim \mathcal{B}(1 - \sigma, \alpha + k\sigma)$ )

# Inference using variational approximation

Clustering/ segmentation task:

- Estimating  $\mathbf{Z}$
- while parameters  $\Theta$  unknown , eg.  $\Theta = \{\tau, \alpha, \theta^*\}$

## Bayesian setting

Access the intractable  $p(\mathbf{Z}, \Theta | \mathbf{y}, \Phi)$  approximate as  $q(\mathbf{z}, \Theta) = q_z(\mathbf{z})q_\theta(\Theta)$

## Variational Expectation-Maximization

Alternate maximization in  $q_z$  and  $q_\theta$  ( $\phi$  are hyperparameters) of the Free Energy:

$$\begin{aligned} \mathcal{F}(q_z, q_\theta, \phi) &= E_{q_z q_\theta} \left[ \log \frac{p(\mathbf{y}, \mathbf{Z}, \Theta | \phi)}{q_z(\mathbf{z})q_\theta(\Theta)} \right] \\ &= \log p(\mathbf{y} | \phi) - KL(q_z q_\theta, p(\mathbf{Z}, \Theta | \mathbf{y}, \phi)) \end{aligned}$$



# DP-Potts Variational EM procedure

Joint DP-Potts (Gaussian) Mixture distribution

$$p(\mathbf{y}, \mathbf{z}, \boldsymbol{\tau}, \alpha, \boldsymbol{\theta}^* | \phi) = \prod_{j=1}^N p(y_j | z_j, \boldsymbol{\theta}^*) p(\mathbf{z} | \boldsymbol{\tau}, \beta) \prod_{k=1}^{\infty} p(\tau_k | \alpha) \prod_{k=1}^{\infty} p(\boldsymbol{\theta}_k^* | \rho_k) p(\alpha | s_1, s_2)$$

- $p(y_j | z_j, \boldsymbol{\theta}^*) = \mathcal{N}(y_j | \mu_{z_j}, \Sigma_{z_j})$  is Gaussian
- $p(\mathbf{z} | \boldsymbol{\tau}, \beta)$  is a **DP-Potts model**
- $p(\tau_k | \alpha)$  is Beta  $\mathcal{B}(1, \alpha)$
- $p(\boldsymbol{\theta}_k^* | \rho_k) = \mathcal{NIW}(\mu_k, \Sigma_k | m_k, \lambda_k, \Psi_k, \nu_k)$  is Normal-inverse-Wishart
- $p(\alpha | s_1, s_2) = \mathcal{G}(\alpha | s_1, s_2)$  is Gamma

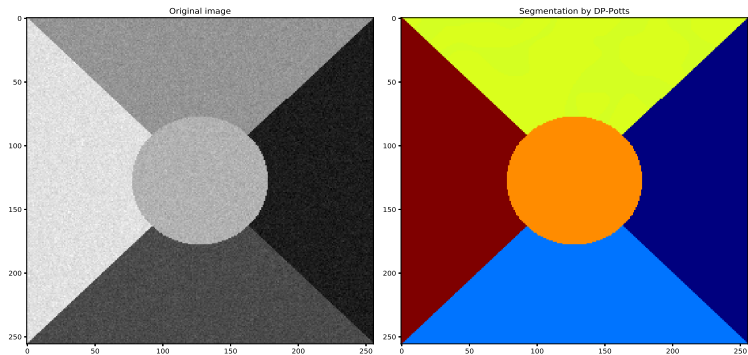
Usual **truncated** variational posterior,  $q_{\tau_k} = \delta_1$  for  $k \geq K$  (eg.  $K = 40$ )

$$q(\mathbf{z}, \boldsymbol{\Theta}) = \prod_{j=1}^N q_{z_j}(z_j) q_{\alpha}(\alpha) \prod_{k=1}^{K-1} q_{\tau_k}(\tau_k) \prod_{k=1}^K q_{\boldsymbol{\theta}_k^*}(\mu_k, \Sigma_k)$$

- E-steps: VE-Z, VE- $\alpha$ , VE- $\boldsymbol{\tau}$  and VE- $\boldsymbol{\theta}^*$
- M-step:  $\phi$  updating straightforward except for  $\beta$

# Some image segmentation results

Model validation and verification:



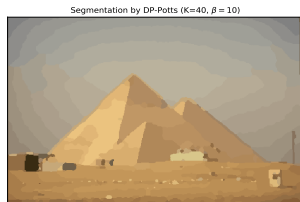
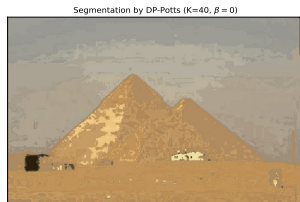
Segmented image using DP-Potts model with  $\beta = 2.5$ .

# Some image segmentation results

Convergence of the VB algorithm initialized by the k-means++ clustering:

# Some image segmentation results

Segmentation results for Berkeley Segmentation Dataset:

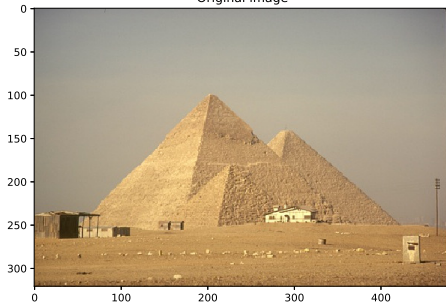


The segmentation results obtained by DP-Potts model with  $\beta = 0, 1, 5$ .

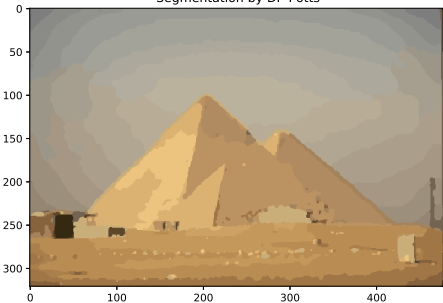
# Some image segmentation results

Segmentation with estimated  $\beta = 1.66$

Original image



Segmentation by DP-Potts



# Quantitative evaluation of the segmentations

**Probabilistic Rand Index** on 154 color (RGB) images with ground truth (several) from Berkeley dataset (1000 superpixels). But Manual ground truth segmentations are subjective !

PRI results with DP-Potts model

	Mean	Median	St.D.
K=10	71.48	72.54	0.1040
K=20	73.64	73.42	0.0935
K=40	75.33	76.47	0.0853
K=50	75.81	76.31	0.0873
K=60	76.55	77.12	0.0848
K=80	<b>77.06</b>	<b>78.30</b>	<b>0.0835</b>

PRI results from Chatzis 2013

PRI (%)	DPM	iHMRF	MRF-PYP	GC
Mean	74.15	75.50	76.49	76.10
Median	75.49	76.89	78.08	77.59
St.D.	0.084	0.082	0.079	0.083

**Computation time** : Berkeley 321x481 image reduced to 1000 superpixels takes **10-30 s** on a PC with CPU Intel(R) Core(TM) i7-5500U CPU 2.40GHz and 8GB RAM

# Conclusion and future work

- A general DP-Potts model and the associated VB algorithm were built.
- The DP-Potts model was applied to image segmentation and tested on different types of datasets.
- Impact of the interaction parameter  $\beta$  on the final results is significant.
- An estimation procedure was proposed for  $\beta$

# Conclusion and future work

- A general DP-Potts model and the associated VB algorithm were built.
  - The DP-Potts model was applied to image segmentation and tested on different types of datasets.
  - Impact of the interaction parameter  $\beta$  on the final results is significant.
  - An estimation procedure was proposed for  $\beta$
- 
- How does  $\beta$  influence the number of components?
  - Extend the model with other priors (Pitman-Yor process, Gibbs-type priors, etc.).
  - Try other variational approximations (truncation-free)
  - Investigate theoretical properties of BNP priors under structural constraints (time, spatial) ....
  - ... for other applications, such as discovery probabilities, etc.



# References

- 1 M. Albughdadi, L. Chaari, J.-Y. Tourneret, F. Forbes, and P. Ciuciu. A Bayesian non-parametric hidden Markov random model for hemodynamic brain parcellation. *Signal Processing*, 135:132-146, 2017.
- 2 S. P. Chatzis. A Markov random field-regulated Pitman-Yor process prior for spatially constrained data clustering. *Pattern Recognition*, 46(6): 1595-1603, 2013.
- 3 S. P. Chatzis and G. Tsechpenakis. The infinite hidden Markov random field model. *IEEE Trans. Neural Networks*, 21(6):1004-1014, 2010.
- 4 F. Forbes and N. Peyrard. Hidden Markov Random Field Selection Criteria based on Mean Field-like approximations. *IEEE PAMI*, 25(9):1089-1101, 2003
- 5 P. Orbanz and J. M. Buhmann. Nonparametric Bayesian image segmentation. *International Journal of Computer Vision*, 77(1-3):25-45, 2008.
- 6 J. Sodjo, A. Giremus, N. Dobigeon, J.F. Giovannelli, A generalized Swendsen-Wang algorithm for Bayesian nonparametric joint segmentation of multiple images, *ICASSP*, 2017.
- 7 R. Xu, F. Caron, and A. Doucet. Bayesian nonparametric image segmentation using a generalized Swendsen-Wang algorithm. *ArXiv e-prints*, February 2016.

*Thank you for your attention!*

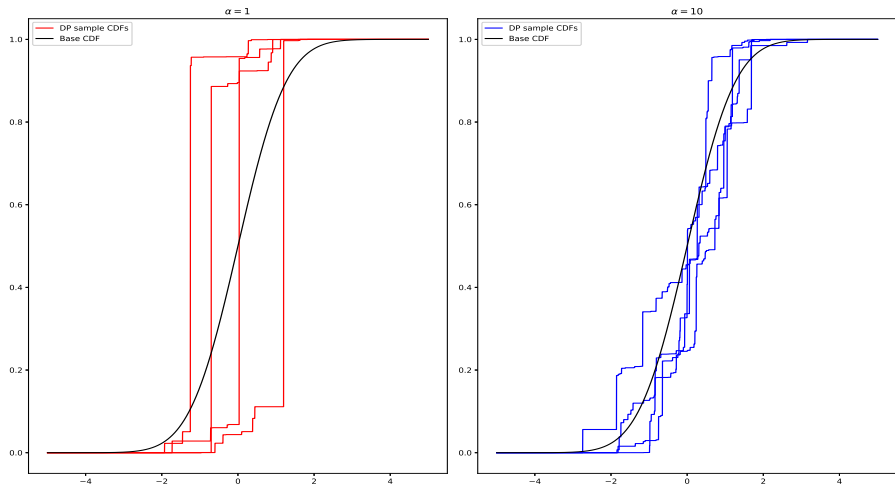
contact: [florence.forbes@inria.fr](mailto:florence.forbes@inria.fr)



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- Contact: [florence.forbes@inria.fr](mailto:florence.forbes@inria.fr)

# Stick breaking construction



DP simulations with  $G_0$  being a standard normal distribution  $\mathcal{N}(0, 1)$  and  $\alpha = 1, 10$  using the Stick-Breaking representation.

# Variational EM

## General formulation, at iteration ( $r$ )

$$\text{E-Z } q_z^{(r)}(\mathbf{z}) \propto \exp \left( E_{q_\theta^{(r-1)}} [\log p(\mathbf{y}, \mathbf{z}, \Theta | \phi^{(r-1)})] \right)$$

$$\text{E-}\Theta \ q_\theta^{(r)}(\Theta) \propto \exp \left( E_{q_z^{(r)}} [\log p(\mathbf{y}, \mathbf{Z}, \Theta | \phi^{(r-1)})] \right)$$

$$\text{M-}\phi \ \phi^{(r)} = \arg \max_{\phi} E_{q_z^{(r)} q_\theta^{(r)}} [\log p(\mathbf{y}, \mathbf{Z}, \Theta | \phi)]$$

## VE-Z, VE- $\alpha$ , VE- $\tau$ , and VE- $\theta^*$

e.g. VE-Z step divides into  $N$  VE- $Z_j$  steps ( $q_{z_j}(z_j) = 0$  for  $z_j > K$ )

$$q_{z_j}(z_j) \propto \exp \left( E_{q_{\theta_{z_j}^*}} [\log p(y_j | \theta_{z_j}^*)] + E_{q_\tau} [\log \pi_{z_j}(\tau)] + \beta \sum_{i \sim j} q_{z_i}(z_j) \right)$$

# Estimation of $\beta$

M- $\beta$  step: involves  $p(\mathbf{z}|\boldsymbol{\tau}, \beta) = \mathcal{K}(\beta, \boldsymbol{\tau})^{-1} \exp(V(\mathbf{z}; \boldsymbol{\tau}, \beta))$

with  $V(\mathbf{z}; \boldsymbol{\tau}, \beta) = \sum_i \log \pi_{z_i}(\boldsymbol{\tau}) + \beta \sum_{i \sim j} \delta_{(z_i = z_j)}$

$$\begin{aligned}\hat{\beta} &= \arg \max_{\beta} E_{q_{\mathbf{z}} q_{\boldsymbol{\tau}}} [\log p(\mathbf{z}|\boldsymbol{\tau}; \beta)] \\ &= \arg \max_{\beta} E_{q_{\mathbf{z}} q_{\boldsymbol{\tau}}} [V(\mathbf{z}; \boldsymbol{\tau}, \beta)] - E_{q_{\boldsymbol{\tau}}} [\log \mathcal{K}(\beta, \boldsymbol{\tau})]\end{aligned}$$

## Two difficulties

- (1)  $p(\mathbf{z}|\boldsymbol{\tau}, \beta)$  is intractable (normalizing constant  $\mathcal{K}(\beta, \boldsymbol{\tau})$ , typical of MRF)
- (2) it depends on  $\boldsymbol{\tau}$  (typical of DP)

## Two approximations

- (1) "standard" Mean Field like approximation<sup>a</sup>
- (2) Replace the random  $\boldsymbol{\tau}$  by a fixed  $\tilde{\boldsymbol{\tau}} = E_{q_{\boldsymbol{\tau}}}[\boldsymbol{\tau}]$

---

<sup>a</sup>Forbes & Peyrard 2003

# Approximation of $p(\mathbf{z}|\boldsymbol{\tau}; \beta)$

$$p(\mathbf{z}|\boldsymbol{\tau}, \beta) \approx \tilde{q}_z(\mathbf{z}|\beta) = \prod_{j=1}^N \tilde{q}_{z_j}(z_j|\beta)$$

$$\tilde{q}_{z_j}(z_j = k|\beta) = \frac{\exp(\log \pi_k(\tilde{\boldsymbol{\tau}}) + \beta \sum_{i \in N(j)} q_{z_i}(k))}{\sum_{l=1}^{\infty} \exp(\log \pi_l(\tilde{\boldsymbol{\tau}}) + \beta \sum_{i \in N(j)} q_{z_i}(l))} \quad \text{and} \quad \tilde{\boldsymbol{\tau}} = E_{q_{\boldsymbol{\tau}}}[\boldsymbol{\tau}]$$

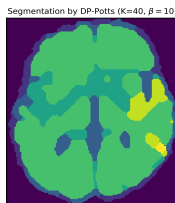
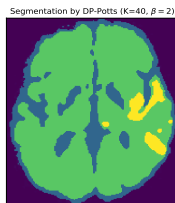
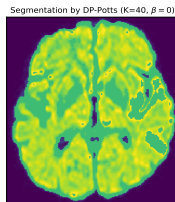
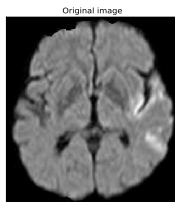
$\beta$  is estimated at each iteration by setting the approximate gradient to 0

$$E_{q_z q_{\boldsymbol{\tau}}} [\nabla_{\beta} V(\mathbf{z}; \boldsymbol{\tau}, \beta)] = \sum_{k=1}^K \sum_{i \sim j} q_{z_j}(k) q_{z_i}(k)$$

$$\nabla_{\beta} E_{q_{\boldsymbol{\tau}}} [\log \mathcal{K}(\beta, \boldsymbol{\tau})] = E_{p(\mathbf{z}|\boldsymbol{\tau}, \beta) q_{\boldsymbol{\tau}}} [\nabla_{\beta} V(\mathbf{z}; \boldsymbol{\tau}, \beta)] \approx \sum_{k=1}^K \sum_{i \sim j} \tilde{q}_{z_j}(k|\beta) \tilde{q}_{z_i}(k|\beta)$$

# Some image segmentation results

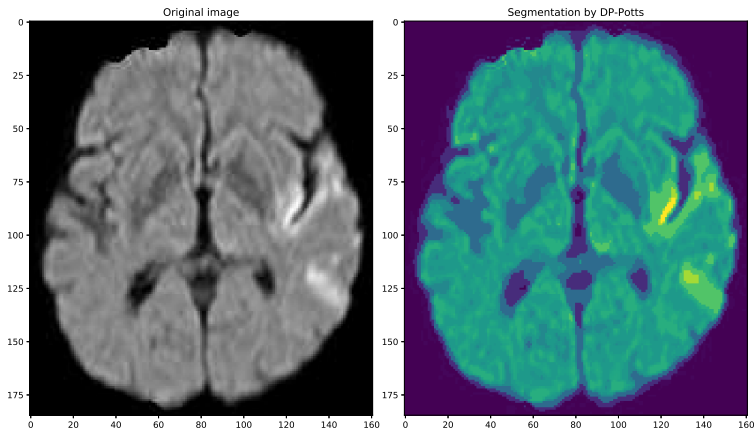
Segmentation results for medical images: all hyperparameters fixed



The segmentation results obtained by DP-Potts model with  $\beta = 0, 1, 5$ .

# Some image segmentation results

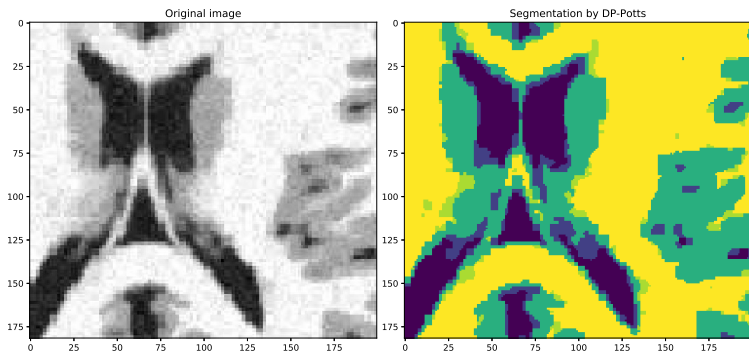
Segmentation with estimated hyperparameters ( $\beta = 0.75$ )





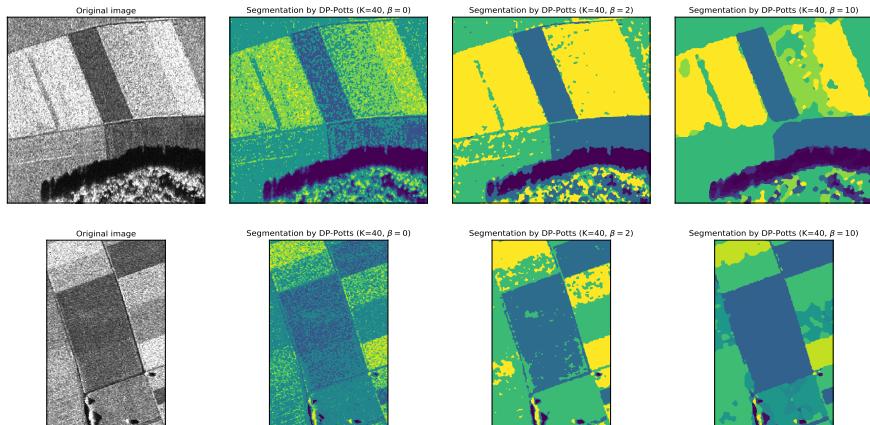
# Some image segmentation results

Segmentation with estimated  $\beta = 0.96$  (pixels with partial volume)



# Some image segmentation results

Segmentation results for SAR images:

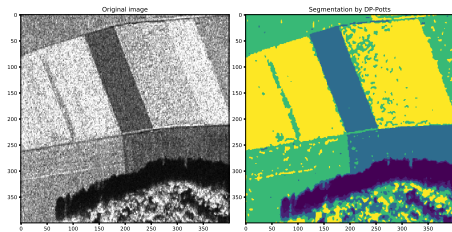


The segmentation results obtained by DP-Potts model with  $\beta = 0, 1, 5$ .

# Some image segmentation results

Segmentation results with estimated  $\beta$

$\beta = 1.11$



$\beta = 1.02$

