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## [6.6]CHIRALANE: A REMARKABLY SYMMETRIC CHIRAL MOLECULE

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**Abstract:** *We present [6.6]chiralane, a rotationally symmetric, rigid, extremal chiral molecule with some remarkable properties.*

### 1. INTRODUCTION

[6.6]Chiralane is a point group **T** highly condensed polycycloalkane C<sub>27</sub>H<sub>28</sub> (Figure 1) containing 68 single bonds. There are five quaternary carbons, 16 tertiary carbons and six secondary carbons (methylenes). The connection table of the full molecular graph with atomic coordinates is given in the appendix.

[6.6]Chiralane (central atom), several [m.n] homologs ( $m, n = 5$  or greater), and [m]chiralanes (no central atom) were designed by Schwartz to explore maximally chiral nanostructures (Schwartz, 2004).

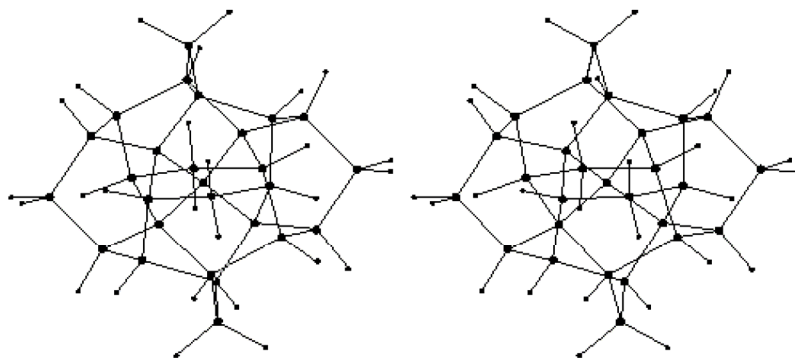


Figure 1: Stereogram of Hartree Fock/6-31G(d)-modeled [6.6]chiralane.

## 2. GRAPH SYMMETRIES AND NOMENCLATURE

The molecular graph of the chiralane has 55 nodes, 68 edges, and 1 component. Thus, the Euler formula requires the associated molecular graph to have  $68+1-55=14$  independent cycles. The difference between dependent cycles and independent cycles may be illustrated by naphthalene: it has three cycles (containing respectively 6, 6, and 10 carbons), but only two of them are independent because the third one (a bicyclic ring) is deduced from the other two. The chiralane has only one cyclic system, containing all carbon atoms. An example compound with two cyclic systems is diphenylmethane, in which the two cyclic systems are joined by bonds not included in a cycle. The smallest chiralane rings are homochiral twist-boat conformation six-membered rings. All larger rings are polycyclic, and are composed from six-membered rings. The structural formula is usefully described in terms of concentric layers around a central atom called the focus (Petitjean 1992). The first layer of atoms contains the neighbours of the focus (i.e. the atoms bonded to the focus), the second layer contains the next neighbours, and so on. The focus is usually set at the center of the graph, the center being a node bearing the largest number of concentric layers. For clarity, we further consider the hydrogen-suppressed graph.

[6.6]Chiralane possesses a unique center with three concentric layers around it. The carbons of each layer are noted according to their layer number and to their number of hydrogens, as indicated in the first column of Table 1.

Carbon type	Layer	Hydrogens	Total carbons per layer	Atom numbers range
Focus	0	0	1	1
C(1)	1	0	4	2-5
CH(2)	2	1	12	6-17
CH(3)	3	1	4	18-21
CH <sub>2</sub> (3)	3	2	6	22-27

**Table 1:** Carbon types classified by their distance from the focus.

The full molecular graph of [6.6]chiralane has 768 automorphisms, but its hydrogen-suppressed graph has only twelve automorphisms. The difference arises from the six methylene groups that multiply the number of automorphisms by  $2^6$ . The twelve automorphisms define five classes of equivalent atoms. Remarkably, these classes correspond exactly to the 5 carbon types defined in Table 1. According to Petitjean (2007), a graph having more than one automorphism is symmetric.

All monocycles found in [6.6]chiralane are cyclohexane rings, and all bicyclic systems based upon two cyclohexane rings exist in [6.6]chiralane: bicyclo[2.2.2]octane, bicyclo[3.3.1]nonane, bicyclo[4.4.0]decane (decalin), and spiro[5.5]undecane. There are 30 cyclohexane rings. The full list is given in the appendix. These 30 rings are partitioned into three classes:

Class 1: 12 rings Focus-C(1)-CH(2)-CH(3)-CH(2)-C(1)-Focus

Class 2: 6 rings Focus-C(1)-CH(2)-CH<sub>2</sub>(3)-CH(2)-C(1)-Focus

Class 3: 12 rings C(1)-CH(2)-CH(3)-CH(2)-CH<sub>2</sub>(3)-CH(2)-C(1)

Kuratowski's theorem (Berge 1973) requires the [6.6]chiralane molecular graph to be non-planar. It contains as minor the complete graph  $K_5$ , the five nodes being the focus and its four neighbours C(1).

Given its highly condensed cyclic but non-planar graph structure, [6.6]chiralane resisted assignment of a systematic name following nomenclature rules such as the IUPAC Blue Book, Section A (1979), and its electronic version (ACD/labs, 1997). Fullerenes' planar graphs readily yield to nomenclature rules (Fowler et al. 2007).

Nomenclator software (ChemInnovation Software 2006) assigned (3*S*,5*S*,6*S*,7*S*,9*S*,10*S*,11*S*,13*S*,14*S*,15*S*,17*S*,19*S*,21*S*,22*S*,23*S*,25*S*,2*R*,18*R*,26*R*,27*R*)tetracyclo[13.11.1.0<sup>1,18</sup>.0<sup>2,7</sup>.0<sup>2,11</sup>.0<sup>3,14</sup>.0<sup>5,27</sup>.0<sup>6,25</sup>.0<sup>9,26</sup>.0<sup>10,19</sup>.0<sup>13,18</sup>.0<sup>17,22</sup>.0<sup>21,26</sup>.0<sup>23,27</sup>]heptacosane.

All carbon atoms are members of homochiral twist-boat cyclohexane rings. The assignment of *R*-configuration atoms is then curious. HyperChem detected twelve *S*-

configuration atoms only. The central atom is an undistorted tetrahedral  $sp^3$ -hybridized carbon bearing four rigorously identical homochiral substituents. It possesses no improper (rotation-reflection, alternating) axes of symmetry and is therefore chiral - but appears not to be labeled by existing nomenclature rules.

### 3. GEOMETRIC DATA

All bond lengths and valence angles have been computed. The bond lengths depend only on the bond type. Their values are reported in Table 2. They are unremarkable compared to *trans*-decalin values. The longer twelve surface bonds between tertiary carbons are typical of their class. The molecule as a whole is in slight compression compared to diamond with 1.54456 Å C-C bonds (Hom et al. 1975).

Bond type	Bond length	Total bonds
Focus-C(1)	1.536	4
C(1)-CH(2)	1.540	12
CH(2)-CH(3)	1.564	12
CH(2)-CH <sub>2</sub> (3)	1.539	12
C-H bond	1.086	28

Table 2: Bonds lengths, Å.

Except around the focus, bond angles are progressively twisted from the unstrained  $sp^3$ -hybridized carbon value expected to be  $\arccos(-1/3)$  or 109.5°. The values are reported in Table 3.

Carbon type	types of neighbours	Valence angle	Total bonds
Focus	C(1), C(1)	109.5	6
C(1)	Focus, CH(2)	103.5	12
C(1)	CH(2), CH(2)	114.8	12
CH(2)	C(1), CH(3)	104.2	12
CH(2)	C(1), CH(3)	106.2	12
CH(2)	C(1), H	112.3	12
CH(2)	CH(3), CH(3)	111.7	12
CH(2)	CH(3), H	111.2	12
CH(2)	CH(3), H	110.9	12
CH(3)	CH(2), CH(2)	107.7	12
CH(3)	CH(2), H	111.1	12
CH(3)	CH(2), CH(2)	105.9	6
CH(3)	CH(2), H	111.1	12
CH(3)	CH(2), H	111.2	12
CH(3)	H, H	106.4	6

Table 3: Valence angles, degrees.

All 30 spatial cyclohexane skeletons have been pairwise compared with the CSR freeware (Petitjean 1998), which optimally superposes two molecules and returns their

largest common spatial motif together with their RMS (Root Mean Square) distance, i.e. the quadratic mean distance of the atom-pairs defined by the common motif in each molecule. Here, the two molecules are always two cyclohexane rings. Among the 435 pairs of cyclohexane rings, the computed common spatial motif is always the full six-membered ring. All RMS distances within any of the three classes of cyclohexane rings mentioned in section 2 are null: a privileged conformation is associated with each class of ring. The RMS distances values between rings of each three classes are: 0.050 Å (class 1 - class 2), 0.069 Å (class 1 - class 3), and 0.079 Å (class 2 - class 3). The respective largest distances among all carbon-pairs are 0.078 Å, 0.105 Å, and 0.093 Å. So, the three privileged conformations are very similar. All rings are chiral. The rings involving a secondary carbon (class 2) offer a  $C_2$  rotational symmetry and the other ones offer a slightly distorted  $C_2$  rotational symmetry. The three classes of rings have a twist-boat conformation. The deviation from perfect direct rotational symmetry is measured with the direct symmetry index (DSI) and the deviation from achirality is measured with the chiral index (CHI), these two indices being computed with the QCM freeware (Petitjean 1999). The definition of the chiral index is based on the normalized minimized root mean square distance between the set and its rotated and translated mirror image, and take a more general form in the case of continuous and/or infinite sets, which are beyond the scope of this paper (Petitjean 2002). The DSI and CHI values are reported in Table 4. A null value indicates a perfect symmetry or achirality. The maximal theoretical value for DSI and CHI is equal to 1.

Ring class	DSI	CHI
1	0.00014	0.07280
2	0.00000	0.07258
3	0.00019	0.05087

**Table 4:** The direct symmetry index and the chiral index for each class of rings.

#### 4. SYMMETRY AND CHIRALITY

[6.6]Chiralane has the following rotational symmetries: three orthogonal axes of order 2, each one being the bisector of two of the six C(1)-focus-C(1) valence angles; and four axes of order 3, each of them passing through one of the four focus-C(1) bond. [6.6]Chiralane has all the direct rotational symmetries of the regular tetrahedron, but none of its mirror symmetries or higher order improper symmetries!

The focus is the center of the following regular polyhedra:

Atome type	Number of vertices	Geometric figure
C(1)	4	regular tetrahedron
CH(3)	4	regular tetrahedron
CH(3)	6	regular octahedron
H bonded to CH(3)	4	regular tetrahedron

**Table 5:** Some geometrical figures defined by the [6.6]chiralane atoms.

Given [6.6]chiralane as a set of 55 points in the 3D space together with the equivalence classes induced by the associated graph automorphisms, the chiral index takes the value  $CHI=0.9824$ . Similarly, the chiral index of its skeleton takes the value  $CHI=1.0000$ . The chiral index being a continuous function of the coordinates and of the weights, attributing weights proportional to the atomic masses of the 55 atoms would not substantially change the value of the chiral index.

$CHI=1$  is the largest value permitted for the chiral index. This maximal value can be reached only when the variance matrix is proportional to the identity matrix (Petitjean 2002). Both [6.6]chiralane and its skeleton satisfy to this condition. For the skeleton, the corresponding standard deviation is 1.768 Å on each axis. For the full set of the 55 points, the standard deviation is 1.399 Å on each axis, the calculation being done on a geometric basis, i.e. all 55 punctual atoms have the same weight.

[6.6]Chiralane is a quantitatively maximally chiral molecule. There are other chirality measures (Petitjean 2003) that may be interesting to apply. Thus, a set which maximizes some chirality measure is not, in general, of maximal chirality for the other chirality measures. Rassat and Fowler (2004) have shown that any chiral tetrahedron is the most chiral for some legitimate choice of degree of chirality. However, a CIP-like scheme based on edge length allows classification of chiral tetrahedra, free of labels and considered as geometric objects, into left- and right-handed forms, despite the chiral connection between enantiomers (Fowler and Rassat 2006). An isolated formula unit does not predict the quantitative chirality of its aggregated bulk. Crystallographic space groups such as enantiomorphic  $P3_121$  and  $P3_221$  can be predictive. Optical rotation arises from electronic effects. It does not measure mass distribution chirality. 2-Norbornanone with  $[\alpha]_D = 29.8^\circ$  and 2-norbornone with  $[\alpha]_D = 1146^\circ$  (both in homogeneous solution) are nearly superposable chiral mass distributions (Wiberg et al. 2006, Crawford et al. 2007). Resolved chiral alkanes with no additional chromophores typically exhibit small optical rotations,  $[\alpha]_D$  of a few degrees. Resolved chiral globular cycloalkanes can do much better. Twistane,  $C_{10}H_{16}$  as fused twist-boat cyclohexanes,

has  $[\alpha]_{\text{D}} = 446^\circ$  or a molar rotation of  $60,800^\circ$  (Nakazaki et al. 1982). [6.6]Chiralane has calculated  $[\alpha]_{\text{D}} = 692^\circ$  or a molar rotation of  $244,000^\circ$  (Autschbach et al. 2002).

Each of the twelve CH(2) tertiary carbons is asymmetric, because its four neighbours are always a quaternary carbon C(1), a tertiary carbon CH(3), a secondary carbon CH<sub>2</sub>(3), and a hydrogen. Owing to the data in appendix, the twelve CH(2) configurations are *S*. They would be *R* for the other [6.6]chiralane enantiomer. Then, none of the other carbons is asymmetric, due to the molecular graph automorphisms structure of [6.6]chiralane: the four neighbours of the focus are equivalent, three of the four neighbours of the C(1) carbons are equivalent, the three non hydrogen neighbours of the CH(3) are equivalent, and the CH<sub>2</sub>(3) methylene groups are of course non-asymmetric. Even the subrule of preference *R*>*S* cannot help to order the equivalent carbons. Attributing a flag to the enantiomers such as left/right, *d/l*, *R/S*, *P/M*,  $\Delta/\Lambda$ , etc. is a challenging problem given its graph automorphisms structure. We propose to conventionally assign [6.6]chiralane CH(2) atom configuration.

## 5. CONCLUSION

[6.6]Chiralane is a calculated structure. Though no synthesis is evident, it could be shelved in an air-filled bottle. Consider a central quaternized boron anion or nitrogen cation. Lord Kelvin's frugal definition of chirality (Thomson 1904) goes back to 1893 (Bentley 2009): "I call any geometrical figure, or group of points, chiral, and say that it has chirality if its image in a plane mirror, ideally realized, cannot be brought to coincide with itself ". There are no equations, symmetry arguments, or mentions of composition (color). If a set and its mirror image cannot be superposed (by translation and rotation), they are chiral - e.g., a scalene triangle in 2D. [6.6]Chiralane exemplifies this definition while possessing a center carbon atom whose chiral configuration remains outside nomenclature. Any student would be heartened to learn of its audacity.

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## APPENDIX

The connection table of the [6.6]chiralane is given below. It is split into two parts: the first part contains the carbons (Table 6), from which is derived the list of the six-membered rings (Table 7), and the second part contains the hydrogens (Table 8). The internal atomic numbering has been set in order to have a clear view of the spatial arrangement of the atoms. The cartesian coordinates are issued from a Hartree-Fock/6-31G(d) ab initio computation using Gaussian 98. It is left to the reader to see whether or not some other internal numbering would help in understanding both the graph structure and the spatial structure of the [6.6]chiralane.

Carbon	Layer	x	y	z	neighbours	H atoms
1	0	0.0000	0.0000	0.0000	2 3 4 5	
2	1	0.8868	0.8868	0.8868	1 7 11 15	
3	1	0.8868	-0.8868	-0.8868	1 6 12 17	
4	1	-0.8868	0.8868	-0.8868	1 9 10 16	
5	1	-0.8868	-0.8868	0.8868	1 8 13 14	
6	2	2.0795	-1.2281	0.0261	3 20 22	28
7	2	2.0795	1.2281	-0.0261	2 21 22	29
8	2	-2.0795	-1.2281	-0.0261	5 18 23	30
9	2	-2.0795	1.2281	0.0261	4 19 23	31
10	2	0.0261	2.0795	-1.2281	4 21 24	32
11	2	-0.0261	2.0795	1.2281	2 19 24	33
12	2	-0.0261	-2.0795	-1.2281	3 18 25	34
13	2	0.0261	-2.0795	1.2281	5 20 25	35
14	2	-1.2281	0.0261	2.0795	5 19 26	36
15	2	1.2281	-0.0261	2.0795	2 20 26	37
16	2	-1.2281	-0.0261	-2.0795	4 18 27	38
17	2	1.2281	0.0261	-2.0795	3 21 27	39
18	3	-1.4370	-1.4370	-1.4370	8 12 16	40
19	3	-1.4370	1.4370	1.4370	9 11 14	41
20	3	1.4370	-1.4370	1.4370	6 13 15	42
21	3	1.4370	1.4370	-1.4370	7 10 17	43
22	3	3.0063	0.0000	0.0000	6 7	44 45
23	3	-3.0063	0.0000	0.0000	8 9	46 47
24	3	0.0000	3.0063	0.0000	10 11	48 49
25	3	0.0000	-3.0063	0.0000	12 13	50 51
26	3	0.0000	0.0000	3.0063	14 15	52 53
27	3	0.0000	0.0000	-3.0063	16 17	54 55

**Table 6:** Connection table and cartesian coordinates (Å) of the [6.6]chiralane skeleton.

Class 1	Class 2	Class 3
1-2-7-21-10-4-1	1-2-7-22-6-3-1	2-7-21-10-24-11-2
1-2-7-21-17-3-1	1-2-11-24-10-4-1	2-7-22-6-20-15-2
1-2-11-19-14-5-1	1-2-15-26-14-5-1	2-11-19-14-26-15-2
1-2-11-19-9-4-1	1-3-12-25-13-5-1	3-6-20-13-25-12-3
1-2-15-20-13-5-1	1-3-17-27-16-4-1	3-6-22-7-21-17-3
1-2-15-20-6-3-1	1-4-9-23-8-5-1	3-12-18-16-27-17-3
1-3-6-20-13-5-1		4-9-19-11-24-10-4
1-3-12-18-16-4-1		4-9-23-8-18-16-4
1-3-12-18-8-5-1		4-10-21-17-27-16-4
1-3-17-21-10-4-1		5-8-18-12-25-13-5
1-4-9-19-14-5-1		5-8-23-9-19-14-5
1-4-16-18-8-5-1		5-13-20-15-26-14-5

**Table 7:** The 30 six-membered rings of the [6.6]chiralane (each first member appears twice).

H atom	x	y	z	neighbour
28	2.6076	-2.1192	-0.3001	6
29	2.6076	2.1192	0.3001	7
30	-2.6076	-2.1192	0.3001	8
31	-2.6076	2.1192	-0.3001	9
32	-0.3001	2.6076	-2.1192	10
33	0.3001	2.6076	2.1192	11
34	0.3001	-2.6076	-2.1192	12
35	-0.3001	-2.6076	2.1192	13
36	-2.1192	-0.3001	2.6076	14
37	2.1192	0.3001	2.6076	15
38	-2.1192	0.3001	-2.6076	16
39	2.1192	-0.3001	-2.6076	17
40	-2.0643	-2.0643	-2.0643	18
41	-2.0643	2.0643	2.0643	19
42	2.0643	-2.0643	2.0643	20
43	2.0643	2.0643	-2.0643	21
44	3.6565	-0.0192	-0.8693	22
45	3.6565	0.0192	0.8693	22
46	-3.6565	-0.0192	0.8693	23
47	-3.6565	0.0192	-0.8693	23
48	0.8693	3.6565	0.0192	24
49	-0.8693	3.6565	-0.0192	24
50	-0.8693	-3.6565	0.0192	25
51	0.8693	-3.6565	-0.0192	25
52	-0.0192	-0.8693	3.6565	26
53	0.0192	0.8693	3.6565	26
54	-0.0192	0.8693	-3.6565	27
55	0.0192	-0.8693	-3.6565	27

**Table 8:** Cartesian coordinates (Å) of the hydrogens atoms of the [6.6]chiralane.

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