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# Math teaching as jazz improvisation: Exploring the ‘highly principled but not determinate’ instructional moves of an expert instructor

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*When engaging students in genuine mathematical problem solving, how can instructors maintain a productive learning environment? In this paper, I examine a series of improvised instructional moves of Alan Schoenfeld, a renowned teacher of mathematical problem solving, and investigate his dilemmas, considerations, and in-the-moment decisions. I use the TRUmath framework to unpack the conflicts that underlie Schoenfeld’s dilemmas, and to propose a tacit teaching heuristic that help explains his hard-to-justify moves. I conclude that Schoenfeld’s in-the-moment decision making is tacitly oriented towards maintaining certain kinds of balances between his pedagogical principles. On the basis of this analysis, I recommend exploring further the use of TRUmath as a framework for analyzing in-the-moment decision making in the context of conflicting pedagogical principles.*

*Keywords: University math teaching, teaching dilemmas, decision making, problem solving.*

## Introduction

I think this kind of teaching is highly principled by not determinate. What I was thinking of is jazz improvisation. It’s anything but random; there are moves that the musician will say would or would not be right; but, there may not be a sound justification for any particular in-the-moment move other than ‘it just felt right’.

In this quote, Prof. Alan Schoenfeld reflects on a series of in-the-moment teaching decisions he made during a problem solving session. Schoenfeld is an expert teacher of mathematical problem solving (MPS hereafter); he has been studying and teaching MPS for more than three decades now. After so many years, Schoenfeld’s instruction seems anything but improvised. For this study, he has reflected on numerous teaching moves that he made during his MPS course, and he was typically able to provide a sound and detailed rationale for his decisions. However, there were certain decisions that Schoenfeld found hard to fully justify, as a key ingredient in their making was a tacit sense of where the class is and how different decisions could work out. In the quote above, Schoenfeld argues that this kind of hard-to-justify decisions makes the instruction of MPS a lot like jazz improvisation, in the sense that both activities are ‘highly principled but not determinate’. In this paper we investigate Schoenfeld’s jazz-like teaching moves through a case study of three hard-to-justify decisions in one MPS session. In this session, Schoenfeld faced a typical dilemma in MPS-oriented lessons: how should a teacher react when a student comes up with a beautiful and original idea that opens the door to a mathematical exploration that seems worthwhile for some of the students, and a step too far for other students? The aim of this paper is to unpack Schoenfeld’s conflicting pedagogical considerations in this case, and to provide insights into his decision making.

The lesson examined in this paper was part of Schoenfeld’s MPS course. Schoenfeld’s teaching in this course has been studied in several papers. For example, Arcavi, Kessel, Meira and Smith (1998) studied Schoenfeld’s teaching in relation to the establishment of classroom norms and MPS heuristics. Schoenfeld’s in-the-moment decision making in this course, which is the focus of this paper, has not been studied so far. This MPS course was given to education graduate students and

prospective teachers and comprised of paper reading, a small scale research project, and engagement in authentic MPS. In terms of goals and pedagogy, the lesson described in this paper is similar to lessons that Schoenfeld has taught in earlier years to undergraduate mathematics students. Therefore, the dilemmas and instructional moves discussed below are not specific to teacher-education courses, and should be viewed in the context of MPS-oriented instruction at university.

There are various approaches for explaining why teachers make the decisions they make as they teach. One approach, which has been gaining much attention in recent years, is to explain teaching decisions in terms of knowledge, goals, and orientations (Schoenfeld, 2010). This approach has been used at the university level in empirical studies (e.g. Pinto, 2013) and also in professional development programs, as an organizing framework for instructors' self-reflections on their teaching (e.g. Schoenfeld, Thomas, & Barton, 2016). However, a notable limitation to the explanatory power of this approach is that instructors' self-reflections are oriented towards what instructors notice in their teaching and have words for. Therefore, there is a need for an organizing framework that would draw attention to various important facets of the work of teaching. One candidate framework is the Teaching for Robust Understanding of Mathematics framework (TRUmath) (Schoenfeld, 2015). In this paper we analyze Schoenfeld's reflections on his teaching from a TRUmath perspective, and examine the use of TRUmath as an organizing structure for instructors' reflections on their dilemmas and decisions that attends to all the major contributors for productive learning environments.

## **Setting and methods**

This paper examines a lesson taught by Alan Schoenfeld in a "Mathematical Thinking and Problem Solving" course at the Graduate School of Education at UC Berkeley. The lesson took place during the 11<sup>th</sup> week of the semester. The class comprised of 21 students – graduate students in the school of education and students from teacher preparation programs. The class met once a week for a 3-hour lesson and every lesson included an MPS part where students worked alone or in small groups on a list of problems and then reconvened to share ideas and solutions. The author videotaped the lessons and took notes. After each lesson, Schoenfeld wrote down some reflections on his dilemmas, his instructional moves and decisions, and their impact on the lesson. In addition, the author conducted three 1-hour interviews with Schoenfeld at different stages of the semester that focused on where the class is with respect the learning trajectories for the course.

The analysis in this study is based in part on the TRUmath framework, which seeks to characterize the main contributors for productive learning environments. This framework was derived through a comprehensive literature review by distilling the factors that shape learning in classrooms into a small number of "equivalence classes". These classes are represented through five dimensions: (1) the richness of the mathematics, (2) cognitive demand and opportunities for "productive struggle", (3) equitable access to content for all students, (4) students' opportunities to develop agency, ownership, and positive mathematical identities; and (5) formative assessment. According to the TRUmath framework, these five dimensions are both necessary and sufficient for studying learning environments in the sense that instruction needs to do well along these dimensions in order to produce mathematically proficient students. Figure 1 provides a brief account of each dimension.

| The Mathematics                                                                                                                                                                                                                                                                                                                                    | Cognitive Demand                                                                                                                                                                                                                                                                                                            | Access to Mathematical Content                                                                                                                                                                                                                                                                                                                     | Agency, Authority, and Identity                                                                                                                                                                                                                                                                                                                                                    | Formative Assessment                                                                                                                                                                                                                                                                                   |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.</i> | <i>The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that students are lost at sea.</i> | <i>The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of students get most of the "air time" is not equitable.</i> | <i>The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.</i> | <i>The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to move forward.</i> |

**Figure 1 – The five dimensions of the TRUmath framework (Schoenfeld, 2015)**

In this paper, we explore TRUmath's explanatory power on Schoenfeld's own instruction, and use the five dimensions as an organizing structure for the discussion of his dilemmas and considerations. Schoenfeld is one of the leading developers of the TRUmath framework, and therefore it is particularly suitable for exploring his decision making. We examine a sequence of three hard-to-justify decisions, first from an outside observer perspective based on the videos of the lessons; then from Schoenfeld's inner perspective based on his post-lesson written reflections and the interviews; and finally, from a TRUmath perspective, where the dimensions are used to organize and compare Schoenfeld's various considerations, to unpack the pedagogical conflicts that underlie his dilemmas, and to investigate what kinds of balances he achieved in his in-the-moment decisions.

### The Square-ness task

The discussion analyzed in the paper revolves around the Square-ness task, given in the figure below, which was designed by Judah Schwartz in cooperation with members of the Balanced Assessment Group at the Harvard Graduate School of Education. In the lesson, Schoenfeld framed this task as "an introduction to the game mathematicians play", and directed students to start with their intuition as to what it means for a rectangle to be more 'squarish', and then mathematize this intuition by "coming up with a mathematical characterization that would enable anyone to perform some sort of operations on a rectangle [...] and obtain a number that would tell, in some sense or other, how close to being a square that rectangle is."

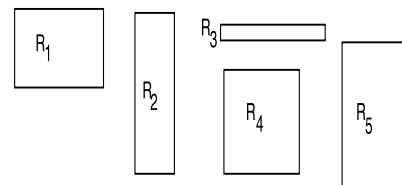
Below you will find a collection of rectangles.

(a) Define a mathematical measure that allows you to tell which rectangle is the "most square" and which rectangle is the "least square".

(b) Define a different measure that achieves the same result.

(c) Is one measure "mathematically superior" to the other?

Argue why, and be prepared to defend your choice to the class



## Analysis

It's not the note you play that's the wrong note – it's the note you play afterwards that makes it right or wrong. (Miles Davis)

Taking a cue from a renowned jazz improviser, I maintain that improvised teaching decisions should not be examined in isolation but rather as part of the flow of instructional moves that teachers make during a lesson. Accordingly, this investigation of Schoenfeld's improvised teaching moves examines three hard-to-justify decisions in the context of Schoenfeld's instruction throughout the MPS session. The analysis is presented as a narrated description of the whole session that comprises of three threads: an outside-observer description of Schoenfeld's moves; a synthesis of Schoenfeld's reflections on his moves, as explicated in the interviews; and a TRUmath perspective on three challenging teaching dilemmas and their hard-to-justify resolutions.

The discussion of the Square-ness task began with a short introduction by Schoenfeld, after which he invited students to present their candidates measures. One of the students, Sophie, approached the board and suggested that the square-ness of a rectangle with side-lengths  $a$  and  $b$  will be defined by: "The ratio  $a/b$  ought to be close to 1". Two students objected to this definition, arguing that it is not well defined and that it should specify that  $a$  is the shortest side length. Sophie disagreed at first, claiming that "it does not matter", but was eventually persuaded. She added "where  $a \leq b$ " to the written definition, and walked back to her seat. At this point, Schoenfeld intervened:

Ok, I love it when the class takes over and raises mathematical objections. The question is, if you're characterizing the square-ness of this figure and you're getting a number, shouldn't we get the same number if we happen and bring it down this way instead? (Draws a figure of a rectangle rotated by 90 degrees) It's the same rectangle, so whatever measure you have should bring you the same number. If you have say, a 1 by 3 rectangle, then you get 1 over 3 which is not the same distance from 1 as 3 over 1; so, it begins to be problematic unless you lay it down so that 'a' is the smaller one of the two [side lengths], and then take 'a' over 'b'.

After making this remark, Schoenfeld leaned quietly against the wall and waited for the students to react. In his reflections, Schoenfeld noted that up to this point the discussion took off just as he intended, as the students were engaged in defining, comparing and criticizing measures, and by doing so, expressing their implicit expectations from a measure. Schoenfeld noted that it is quite typical that the first candidate measure is based either on the ratio or on the difference between adjacent side lengths of the rectangle. The measure Sophie suggested has the nice property that it can be defined in a way that makes it invariant under rotations and scaling. The students' debate on whether Sophie's measure was well defined did not address explicitly the properties of the measure. The students seemed more occupied with figuring out the exact routine for computing the measure. Nevertheless, Schoenfeld explained in the interview that this debate provided him with an opportunity to acknowledge, respond to and build on students' ideas, while rephrasing these ideas in a way that fit his goals – to engage students in discussing the desired properties of measures.

Emmy was the first to comment on Sophie's example: "I think that I can probably find a quadrilateral that is not a square, but would have that, hmm... would be a square under that measure of square-ness, but it is not a rectangle, is that OK?"

Schoenfeld's immediate response was to the entire class: "Do you want to take a vote? Is that OK?" The class seemed divided with some students wondering whether considering quadrilaterals other than rectangles is allowed, while others expressing interest in seeing Emmy's example. Schoenfeld agreed with the students that the problem is stated just for rectangles, but as several students responded in disappointment, he paused to make a quick evaluation and to decide how to proceed.

In his reflections on this moment, Schoenfeld noted that Emmy's example came as a surprise, and too soon with respect to where he felt the class was. He explained that he was more expecting students to propose another candidate measure, which he could compare to Sophie's measure; or to point out that Sophie's measure is invariant under scaling, which would have provided another opportunity to discuss properties of measures. Schoenfeld reflected on his dilemma: on the one hand, the class seemed eager to see Emmy's example; Emmy's comment was well aligned with his own agenda of discussing the desired properties of measures, as it pointed out the fact that Sophie's example might not be generalizable to parallelograms. He also considered this comment as an authentic and beautiful example of 'doing mathematics' and he wanted to acknowledge this; and, coming from a student, this generalization felt natural and organic rather than an artificial teaching move, making it even more appealing. On the other hand, Schoenfeld noted that he was worried that Emmy's example might steer the discussion towards arbitrary quadrilaterals, and he was not sure that the class was ready for this level of abstraction. He wanted more students to participate in the discussion and considered putting Emmy's example on hold so other students could present their candidate measures for rectangles. He also noted that he had examined some of the student work on this problem and that there were a few important insights that he wanted to draw from that work.

In the classroom, Schoenfeld responded almost instantly:

Now, before I throw [Emmy] out of class (laughing), let's examine what [she] said. One of the properties one might like for, hmm... any definition is to ask the question: what class of objects this definition applies to? [...] So, this measure (points at the Sophie's definition on the board) works for rectangles, but being my psychic self, I think the figure Emmy had in mind was a family of rhombuses (draws two rhombuses on the board), all of which have a measure 1 according to the definition, if we think in terms of side lengths; but they don't look like a square!

Note that Schoenfeld's response opened the door for Emmy's example, but that he presented the example himself, as he understood it, rather than letting Emmy present it in her own words. Looking back at his response, Schoenfeld noted that it is hard to justify, claiming that on one hand, it is highly principled in the sense that it is consistent with his goals and orientations, as explicated in his reflections on his dilemma. On the other hand, this decision was not determined by principles, as it was based in part on a tacit sense of where the class was, and how well things could work out.

We now turn to analyze Schoenfeld's hard-to-justify decision from a TRUmath perspective. One option Schoenfeld had was to invite Emmy to present her example in her own words. This option is well aligned with the Formative Assessment and Agency dimensions. Moreover, Schoenfeld considered Emmy's idea to be "a beautiful example of doing mathematics", and since his goals were to discuss properties of measures, he considered this option to also be well aligned with the Mathematics dimension. However, Schoenfeld's reflections suggest that he found this option less appealing from the perspectives of the Access and Cognitive Demand dimensions. He explained that

it is essential that students understand and relate to the goals of the exploration. Emmy, who might still be struggling to formulate her idea, could end up leading a discussion that the rest of the students could not engage with productively. Another option Schoenfeld considered was flatly rejecting Emmy's example, or putting it on a back burner. This option would have given Schoenfeld more control over the lesson, which has merits in terms of the Mathematics, Cognitive Demand and Access dimensions. However, Schoenfeld considered this route potentially harmful in terms of Authority, Agency and Identity, and Formative Assessment. Schoenfeld's response represents an alternative to these two options. He acknowledged and built on Emmy's idea (Formative Assessment), lowering the risk of being perceived as rejecting her thinking (Authority, Agency and Identity). However, he did so by proposing two visual examples of rhombuses, making the discussion more concrete and accessible (Cognitive Demand and Access). Moreover, Schoenfeld provided a crisp outlining of the topic of the discussion: "what class of objects this definition applies to" (Access), orienting the discussion towards the properties of measures (Mathematics). To summarize, Schoenfeld considered the first two options to be potentially beneficial as well as potentially harmful; his reaction chose a middle ground that he still considered beneficial, and safe.

The lesson continued with Sophie, Emmy and a few other students discussing how to modify Sophie's measure to make it 'more square'. This discussion led to a new candidate measure for square-ness: the product of the ratio between adjacent side lengths and the ratio between adjacent angles. However, Emmy criticized this measure, claiming that while this measure has the nice property that squares are separated from other shapes, she can no longer see what kind of ordering this measure induces on parallelograms, and whether this ordering has anything to do with her initial intuition as to what square-ness should mean. Several students endorsed this criticism, and the class abandoned this measure. One student suggested that it might not be possible to find a measure that works for both rectangles and rhombuses, and the whole classroom discussion started to break up into several concurrent discussions. At this point, Schoenfeld intervened:

I'll point out that what we're doing right now is exactly the business mathematicians are engaged in. [...] We start with rectangles and see candidate measures for rectangles; then the question is, what about parallelograms? Trapezoids? Arbitrary quadrilaterals? Is it possible to find a measure that could characterize square-ness for all of those? We only got one definition of square-ness of rectangles so far, and I want to see a few more. It is possible that if we are just looking at rectangles any of the candidate measures will do, although some might be easier to calculate, some might correspond more to your intuition in terms of how square something is. And then, as we move on, only some of those definitions work for more objects. That's the game mathematicians play. So, we have two directions to go. We have this definition (wipes the board clean and writes 'a/b closest to 1 where  $a \leq b$ '). We can ask, are there any other characterizations, or reasons to like them more or less; and we can ask do they generalize and how much, which can also get us to a discussion about just what properties of definitions in general do we want, and what properties do we want in this particular case. The floor is open.

Reflecting on this intervention, Schoenfeld noted that this was a point where he sensed the class was indeed not ready for the exploration they initiated, as he anticipated might happen; in his message to the students, he was trying to steer the discussion back to rectangles, while making sure he is still giving due credit to the exploration the students were engaged with, framing it as the 'game

mathematicians play'. Schoenfeld considered this decision as essentially based on a tacit evaluation of where the class is. In the interview, he used TRUmath terms to make this evaluation somewhat more explicit: He explained that he was reading from the students' facial expressions that some students were getting disconnected, signaling Access was becoming an issue; he also noted that the fact students starting to question whether the problem could be solved at all was for him a signal that the Cognitive Demand of the task might be too high. Schoenfeld concluded that in his intervention he was implicitly trying to attend to the Access and Cognitive Demand dimensions.

At this point, Emmy suggested: "I have an idea, but I don't know how to turn it into a measure [...] I have a measure that would split out squares, but I don't know how to make it order everything else. Should I share it? (Schoenfeld nods) OK, my theory is that if you have a given a perimeter for a quadrilateral, the square will have the maximum area. So, I want something that takes perimeters and determine whether or not, hmm... determines whether or not that's the maximum area for that circumstance and then order everything else according to how not maximum it is, or something."

Emmy's suggestion led to a rapid exchange between her and Sophie, while the rest of the students remained quiet. In the interview, Schoenfeld referred to this moment as another challenging dilemma that led to a hard-to-justify decision. While he considered Emmy's comment to be mathematically inspiring, it also impeded his attempts to lead the discussion back to rectangles. Schoenfeld explained that he guessed Emmy's idea is intuitively based on the isoperimetric theorem, and he estimated that forming a measure for arbitrary quadrilaterals on the basis of this intuition might prove too difficult for most students, potentially reducing their confidence and sense of efficacy even further. Thus, even though following up on Emmy's comment was very appealing from the Mathematics and Formative Assessment perspectives, this option seemed very risky from the perspectives of Access, Cognitive Demand and Identity. However, Schoenfeld found that while his principles directed him to object to exploring Emmy's idea, his sense of the class suggested otherwise: he sensed that the students were quiet but not passive, that they were actively listening to Emmy and Sophie. Consequently, Schoenfeld explained he decided to try and provide the class with just enough scaffolding to enable more students to engage productively in the new exploration:

Ok, let's take what we do know and see if we can turn this to a measure. Hmmm... you may have heard [...] of this thing known as the isoperimetric theorem ... the general theorem is that if you take any figure whatsoever for a fixed perimeter, the circle is the figure with the largest area. If you limit yourself to quadrilaterals, to rectangles, it turns out that for any given perimeter the square is the figure with the largest possible area. So, the question is whether we can turn that into a measure we can use, and then think about abstracting this into some of these other figures.

In TRUmath terms, Schoenfeld's decision can be expressed as an attempt to amend the level of Cognitive Demand so to increase Access. The intervention paid off. Four more students joined Sophie and Emmy and participated in the exploration. It took just a few minutes of discussion for Sophie to come up with a measure that works: "*The perimeter over four, squared, over the area of the shape*". The class enthusiastically picked up on this suggestion, and eventually endorsed it.

## Discussion

In this paper, we examined a sequence of three in-the-moment decisions. Schoenfeld's first decision was to open the door to Emmy's original idea, but present it in his own words; when the exploration



of Emmy's idea seemed too challenging for the class, Schoenfeld's second decision was to try and steer the discussion back to the original problem; and finally, a quick evaluation of where the class was led Schoenfeld to reverse his second decision and allow an even more challenging exploration. Schoenfeld considered his decision making to be highly principled in the sense that his decisions were well aligned with his explicit orientations and goals; however, in his reflections, he also observed that some of these decisions were hard to fully justify since they were strongly influenced by a tacit sense of where the class is and how things could work out. This sense of the class is a resource Schoenfeld developed over years of teaching the course; his reflections suggest that this resource has a crucial role in his decision making when faced with challenging dilemmas: it helps resolve pedagogical conflicts that rise from tensions between competing goals and orientations. The TRUmath framework proved to be useful for unpacking these tensions by providing an organizing structure for the different considerations and the conflicts they present. For example, in the context of Emmy's original idea, when examining three alternative options, we found that Schoenfeld considered two of the options to very well aligned with some of the dimensions, but also potentially harmful from the perspective of the other dimensions. The TRUmath analysis suggested that Schoenfeld chose a path that he considered more moderate across all five dimensions in terms of potential gains and risks. This analysis led Schoenfeld to suggest a teaching heuristic that may have tacitly guided him: keep the lesson productive from the perspective of each dimension, and avoid the temptation to excel in just one or two dimensions at the expense of the other dimensions.

This paper illustrates the theoretical potential of TRUmath as a framework for explaining decisions made in light of conflicting goals and orientations, and the practical potential of TRUmath as an organizing structure for teacher reflection that highlights the gains and risks entailed in different instructional moves. As Schoenfeld is both the subject of this study and one of the developers of TRUmath, further research is required to assess TRUmath's explanatory power for other instructors.

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