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# Polynomial approximation: From history to teaching

Rahim Kouki<sup>1</sup> and Yassine Hachaïchi<sup>2</sup>

<sup>1</sup>University of Tunis El Manar, LAMSIN-ENIT; Tunisia; [rahim.kouki@gmail.com](mailto:rahim.kouki@gmail.com)

<sup>2</sup>University of Tunis El Manar, LAMSIN ENIT; Tunisia; [hachaichi.ens@gmail.com](mailto:hachaichi.ens@gmail.com)

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## Introduction and research problem

Our motivation for engaging in this research project relies on recurrent students' difficulties in the appropriation of the concept of Taylor approximation. Despite the richness of this concept as a tool in many fields of application (e.g calculation of limits, local study of a function and the study of physical phenomena), for what we know, there is little research on this topic in mathematics education. Beyond the lack of comprehension of the notion of convergence for Taylor series (Martin, 2013), we hypothesize that the articulation between syntax and semantics is not clear for students, creating a fuzzy area that prevents them from being able to apply Taylor approximation as a tool in applied mathematics. In this paper, we summarize the main results of our epistemological investigation and the first element of a didactic study of teaching material on this topic, in order to identify paths for improving its teaching and learning.

## Theoretical framework

Throughout our epistemological and didactic investigations, we refer to the dialectics between semantics (graphic interpretation, dynamic interpretation and numerical approximation) and syntax (the different formulations of the Taylor approximation), including their articulation with the *tool-object dialectic* (Douady, 1991) and the *registers of semiotic representations* (Duval, 2006). (this approach is developed and used in Kouki, to appear).

## Epistemological investigation

We outline the evolution of the mathematical object known as “Taylor approximation” through a historical and epistemological investigation. Our study emphasizes the co-existence of different types of techniques mainly with geometric, algebraic and dynamic origins that contributed to the emergence and development of this object (Kouki, Belhaj Amor, & Hachaïchi, 2016).

Our research identifies key moments in the development of infinitesimal calculus and local approximations which led to the calculus as it is taught nowadays. In particular, the works of Roberval, Euler, Fermat, Leibniz, Newton, until Taylor and Weierstrass are significant in the development of Taylor approximations.

## Didactic investigation

Based on our epistemological investigation, we have conducted a didactic study of the Tunisian curriculum, textbooks and course handouts on Taylor approximation. The Tunisian curriculum, three handwritten courses and four handbooks were analyzed. Through this study, we confront the various dimensions of this concept revealed by the epistemological investigation with those that are present in teaching materials.

This study supports the hypothesis that there are gaps in the teaching of Taylor approximation. In particular, some of them are due to insufficiencies of graphical representations and numerical approximation, corresponding to a deficit of semantic work, the emphasis being put on the syntax.

## Conclusion

The epistemological and didactical investigations summarized in this paper are part of a larger project aiming at developing propositions for enhancing the teaching and learning of Taylor approximation. This enhancement can help to foster an appropriation of the concept allowing to use it in the various applications undergraduate students could meet in further studies, in particular in applied mathematics.

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