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# Heuristic strategies in mathematics teacher education

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Here a tutorial is described that focuses on mathematical methods that implicitly appear in the mathematics shown in a Linear Algebra lecture, and makes these methods explicit for the students. Interviews with students at the end of their first semester show some effects.

## Theoretical framework

Mathematics, as all sciences, comprises content (e.g. definitions, theorems, finished proofs, ...), but also mathematical methods.

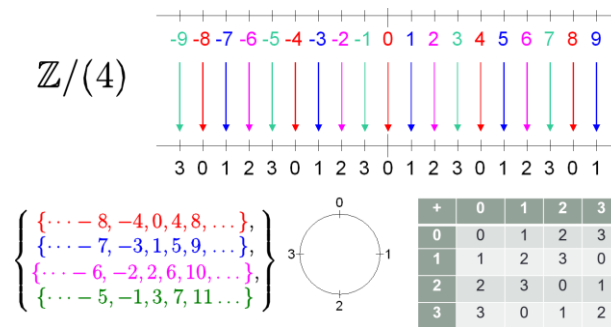
Pólya (1945) underlined the importance of the mathematical methods in the subtitle of “How to solve it -A new aspect of mathematical methods”. We use a broad approach of heuristic strategies as a central part of the mathematical methods shown in the following list:

- *Organise your material / understand the problem:* change the representation of the situation if useful, try out systematically, (Pólya, 1945) use simulations with or without computers, discretize situations,
- *Use the working memory effectively:* combine complex items to supersigns, which represent the concept of ‘chunks’ (Miller, 1956), use symmetry, break down your problem into sub-problems,
- *Think big:* do not think inside dispensable borders, generalise the situation (Pólya, 1945),
- *Use what you know:* use analogies from other problems, trace back new problems to familiar ones, combine partial solutions to get a global solution, use algorithms where possible (Pólya, 1945),
- *Functional aspects:* analyse special cases or borderline cases (Pólya, 1945), in order to optimise you have to vary the input quantity,
- *Organise the work:* work backwards and forwards, keep your approach – change your approach – both at the right moment (Pólya, 1945).

Other mathematical methods are e.g. “proof strategies” which make extensive use of formal language such as mathematical induction, proof by contradiction, proof by exhaustion, the invariance principle or others. Another kind of methods are the use of mathematical language e.g. in depth reading mathematical texts or writing down mathematical proofs correctly.

## Tutorial example

In the tutorial we made mathematical methods implicitly used in the lecture explicit. We also used these methods giving additional explanations and reflected this use afterwards in the tutorial. The concept of quotient spaces in linear Algebra was established on rings. One example given in the lecture was  $\mathbb{Z}/m\mathbb{Z}$ . In the tutorial we reduced this example to the case  $\mathbb{Z}/4\mathbb{Z}$  and gave additional representations of this concept, some of them shown in figure 1.



**Figure 1:  $\mathbb{Z}/4\mathbb{Z}$  in several representations.**

Here the four residue classes are shown in several representations, together with the way of calculating on that structure. The similarity of the calculation on a common clock was stressed as a possible link to school mathematics and further explanations were given. Calculating on the equivalence class that are supersigns from the heuristic point of view was difficult for the students and the similarity to calculating with fractions that are equivalence classes and supersigns as well was discussed. Furthermore, moving between representations was stressed as a method to deepen the understanding of the content or to find a way to solve a problem.

### Students' feedback

At the end of the first semester we interviewed five students who participated in the tutorial over the whole semester. The interviews were transcribed and analyzed using quantum content analysis.

All students remembered well the connection to school mathematics based on mathematical methods that appear in university and school as well. The students also emphasized that visualized explanations with less formal language were very helpful. These explanations using “a change of representation” were remembered well.

The input related to the heuristic strategies was remembered by the students. The concept of “heuristic strategy” was remembered and the students could describe examples given in the tutorial, connecting the examples to the appropriate heuristic strategy.

### Conclusions and looking ahead

Supporting students by stressing mathematical methods, and particularly by stressing heuristic strategies, seems to have a positive impact, but is not realized easily. For this we will continue with this tutorial in the third and fourth semester bringing in heuristic strategies wherever possible.

### References

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