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# Making numbers: Issues in using manipulatives with young children

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*The Making Numbers research Project, funded by the Nuffield Foundation, has developed guidance for teachers of 3 to 9 year olds on the use of manipulatives in the teaching of arithmetic. The project, which consisted of a literature review, survey of teacher use and small-scale teaching investigations, identified key principles and issues for the use of manipulatives. This paper gives a brief overview of project findings, which are reported elsewhere, and discusses issues relating to the early years from the literature review.*

*Keywords: Early childhood mathematics, finger counting, patterns, representations, manipulatives.*

## Background to the project

Making Numbers is a Nuffield Foundation funded project to develop research-informed guidance for teachers of three to nine year olds on the use of manipulatives to teach arithmetic. The project has run for two years and includes a literature review, a survey of current practice and the development of exemplars of good practice through observation and small-scale teaching investigations. The resulting guidance has been published as a fully illustrated book for teachers with accompanying animations for use with children (Griffiths, Back, & Gifford, 2016). The findings of the project are reported in full elsewhere (Griffiths, Back, & Gifford, in press): this paper gives a brief overview, highlights some issues concerning the use of manipulatives with young children and exemplifies key pedagogical principles.

Our definition of manipulatives is “objects that can be handled and moved and are used to develop learners’ understanding of a mathematical situation” (Gifford, Back, & Griffiths, 2015, p.1). This includes the use of everyday and structured materials, characterized by a pedagogical intention, which concurs with recent studies (Swan & Marshall, 2010; Carbonneau, Marley, & Selig, 2013). The project focus was on number sense, prioritizing for this age range counting, cardinality, comparison and composition of numbers. Hence the project title, ‘Making numbers’, emphasises flexibly decomposing and recomposing numbers, which was identified by Boaler (2009) as key to mathematical achievement.

From the survey and interviews with teacher groups, we found that manipulatives were mainly used in the early years and with older low achievers (Gifford et al., 2015). The most common were counters, interlocking cubes and Numicon 10-frame based number plates (Wing, 2001), followed by place value apparatus. Teachers’ choice was influenced by commercial availability and past government policy, but for some it was serendipitous. Most expressed a lack of confidence about how to use manipulatives to teach different aspects of number.

The methodological approach of the literature review was to consider studies and theories from a range of perspectives, in order to gain insights into factors affecting children’s learning. Sources included the history of pedagogy, cognitive and social constructivist theories of learning and quantitative and qualitative empirical studies. We found that experimental studies were contradictory

and inconclusive, identifying only crude factors, such as length of the study or the amount of instructional guidance, as shown by Carbonneau et al.'s (2013) meta-study.

The conclusion was that the effective use of manipulatives depended on some key pedagogical principles (Gifford et al., 2015). These included:

- the careful matching of both manipulatives and activities to the mathematical focus
- the identification and assessment of children's prerequisite understanding
- activities involving comparison, equivalence, analysis and generalisation
- discussion, requiring children to use manipulatives to justify reasoning
- linking manipulatives to abstract symbols
- creating an inclusive mathematics learning community.

Here we identify some issues from the literature review about teachers' use of manipulatives with young children. These concern fingers as manipulatives, discrete and continuous models for number and the learning potential of pattern activities. The pedagogical principles above are exemplified in a small case study from a teaching investigation.

### **Fingers as manipulatives**

The benefits of drawing from a range of perspectives are evident in the consideration of fingers as manipulatives, which are of particular relevance to teachers of young children. Their significance has been underlined by neurological research: brain areas representing fingers and numbers are closely related, according to Wood and Fischer (2008). Young children use mental finger representations for numbers more than adults, suggesting that finger use is significant for the development of number understanding. Gracia-Bafalluy and Noel (2008) found that young children's finger awareness was predictive of their mathematical competence and that training in distinguishing fingers resulted in improvements in subitising, counting and comparing numbers. However, the way that fingers are used for counting varies in different cultures, including counting three to a finger, or counting hands as fives. Bender and Beller (2012) argued that the resulting number concepts also vary according to different languages, some of which support 10s structures more transparently. Jordan (2003) also found that children from low-income families tended not to use fingers to solve problems. This suggests that home practices differentially support children's number understanding and need to be taken into account by teachers.

How should children be taught to use fingers? Sarama and Clements (2009) advised teachers not to discourage children from using fingers until they were confident with mental strategies, in order to prevent reliance on finger counting. Marton and Neuman (1990), using a phenomenological approach, found that older higher attainers, who used recall and derived facts, showed numbers of fingers 'all-at-once', whereas low attainers continued to rely on using fingers to count on and back. 'Finger numbers' encouraged children to analyse numbers, developing subitised images and part-whole number understanding. This model therefore represents key number concepts more effectively. Sinclair and Pimm (2015) reported that using the app 'Touchcounts' rapidly resulted in three year olds showing 'all-at-once' finger numbers. This seems an important skill which young children might also learn in other ways, for instance, when singing number rhymes.

## Discrete and continuous models of number

One current issue relating to young children is about the relative merits of discrete and continuous models of number, for instance using either counters or colour rods which represent numbers as lengths. Usually in England numbers are introduced by counting separate items. However, when children are later introduced to number lines, numbers are represented by intervals on a continuous line and children often count the numerals or marks, rather than the intervals between them. Sarama & Clements (2009, p. 119) recommended caution in using the number line “as a representation for beginning arithmetic”, raising the issue of when and how to introduce it. However, neuroscientific evidence suggests that people intuitively see some kind of mental number line (Wood & Fischer, 2015). This supports renewed interest in teaching young children about number based on measuring lengths, as proposed by Bass (2015) following the approach of Davydov (1975). Bass pointed out that number “arises from measuring one quantity by another, taken to be the ‘unit’” (2015, p. 100). He argued that introducing numbers as chosen units for measuring quantities provides a more coherent model of numbers which can also include fractions. It supports multiplicative and proportional reasoning and early understanding of algebraic principles such as inverse and commutativity. This argument also supports the use of colour rods.

Fuson (2009) argued that number paths or tracks are more comprehensible for young children, as they present adjacent squares which are more obviously countable than intervals on a line. Laski & Siegler (2014) found that children who regularly played number track games, reading the numbers aloud as they counted moves, improved their awareness of number magnitude and arithmetical achievement. It has subsequently been argued that children’s engagement with ordinal number has been overlooked as a means of developing understanding. Sarama & Clements (2009, p. 93) suggested that children were linking the distance model with counting moves and numbers spoken:

connections between the numerical magnitudes and all the visual-spatial, kinaesthetic, auditory and temporal cues in the games (i.e., all the magnitudes increase together: numerals, distance moved, number of moves, number of counting words etc.) may provide a rich mental model for a mental number line.

It therefore seems that we may underestimate children’s capabilities to combine the various models to construct complex networks of numerical understanding. According to Clements and Sarama (2009), four year olds may be in the process of developing a mental number line connecting different ‘quantification schemes’, including discrete number ideas based on subitising and counting with continuous ideas about duration and length.

Some manipulatives have attempted to integrate discrete and continuous models of number. For instance, bead strings present countable beads, usually coloured in groups of five or 10, along a line: Beishuizen (2010) reported that teaching children to find a number on a bead string and then on a number line helped them to develop mental calculation. Some older apparatus, such as Montessori’s numeric rods and the Stern counting board, present rods arranged as ‘staircases’ of discrete cardinal numbers, with numerals alongside. These afford a clear image of the value of numerals in sequence increasing by one, but are not common in schools, having been replaced by rods made of interlocking cubes. An approach we have developed is to use centimetre cubes alongside a ruler: this has the advantage of providing countable objects, while demonstrating that the individual items counted are

in the intervals between the numerals. This not only shows the numbers increasing by one and but also that each successive number includes all previous ones, the idea of hierarchical inclusion or ‘nested numbers’ (Clements & Sarama, 2009, p. 20). Combining both models also provides opportunities for children to discuss what is the same and what is different, as advocated by Harries, Barmby and Suggate (2008). This is an area where teachers need to be aware of the complexities involved in the differences between continuous and discrete models and further investigation is needed of how children reconcile these.

## **The potential of pattern**

There is a range of possible factors affecting young children’s understanding of manipulatives used to represent number relations. From Piagetian theory young children are seen as needing sensory experiences in order to learn, and from Vygotskian theory as being able to use symbols, such as fingers, from an early age. Recent theories suggest differences in young children’s spontaneous and intuitive appreciation of number and pattern. Hannula and Lehtinen (2005) found that some very young children displayed a tendency to spontaneously focus on numerical features of a situation (SFON), while others did not, and this predicted later achievement. Mulligan and Mitchelmore (2009) found that children varied considerably in their awareness of mathematical pattern and structure (AMPS) and that this was also linked to mathematical achievement. They identified different stages of AMPS: some children focused on non-mathematical surface features of patterns, some children noticed one or two mathematical elements in a pattern and others could reproduce and continue patterns, by identifying the components and relationships. Mulligan and Mitchelmore also found that pattern awareness could be taught to young children, suggesting that this facility is learned from experience: Papic, Mulligan and Mitchelmore (2011) reported a successful intervention with four year olds, which improved awareness of pattern and mathematical structures. This presents a potentially powerful approach, both in building on young children’s strengths with visuo-spatial memory and in developing pre-algebraic understanding.

## **Staircase patterns**

One of our teaching investigations, into the learning of numbers to 20 with a class of 30 six year old children, exemplified the importance of pattern awareness as well as some of the key principles identified in the literature review. In a series of small group teaching sessions, we aimed to teach children about the numbers from 10 to 20, sometimes referred to as the ‘teen numbers’ (Gifford & Thouless, 2016). We used a ‘staircase’ pattern of ‘teen numbers’ made of rods of interlocking cubes arranged in number order. The rods for one to 10 were of different colours, with the rod for ten consisting of a stick of green cubes; the rods for 11 to 19 then had 10 green cubes with the rods for numbers 1 to 9 attached, repeating the same colour sequence. Numerals were presented alongside the rods 1 to 9. We invited the children to find a single rod, such as 18. We found that children used a variety of strategies to do this: most picked a rod at random and counted the cubes from one. Magda counted in ones along the 20 rod and then along the 18 rod and explained “I knew one less would be 19 and one less would be 18”. Only one child identified rods by counting on from ten. Some children made errors in counting, either in the word sequence, or in matching words to cubes when pointing.

This activity demonstrated the pedagogical principle of carefully selecting manipulatives to match mathematical concepts and showed how this affected the accessibility of those concepts. We wanted

the children to recognize the sticks as representing the teen numbers as composed of ten and another number, but it was not easy to see there were ten green cubes and recognize the ‘ten’, unlike with Numicon plates which children had previously identified as ‘ten’. However, the staircase arrangement enabled some children to recognize the ‘one more than’ pattern, which would not have been so evident with Numicon. When trying to find the ‘thirteen’ rod, six year Lucasz, who was not yet fluent in English, said, ‘because after 2, 3. Because it’s like ten, 1, 2, 3.’ In doing this he made a sweeping gesture across the 11, 12 and 13 rods, then across to rods 1, 2 and 3. Later, Magda, whose home language was Czech and who was more fluent in English, said, ‘It’s a bit like counting up stairs. Like counting 1, 2, 3 but 11, 12, 13.’ The children thereby implicitly identified two types of pattern, a repeating units pattern and the stair pattern of equal intervals of one, which is the most basic arithmetic sequence. This was evidence of early algebraic thinking, in that they noticed mathematical features, identified the relationship between elements and observed regularities (Blanton et al., 2015, cited by Kieran, Pang, Schifter, & Ng, 2016).

This activity thereby also demonstrated the principle of prompting analysis, by simply asking children to say how they identified a number. In our teaching investigations we have been impressed by the way patterns can fascinate children. On one occasion, when children were rushing by our table outside a classroom on their way to lunch, two six year old boys halted in their tracks in order to gaze at the staircase pattern and one child sat down to join us. We found that patterns like these engaged a range of children, who had differing expertise with numbers, in solving problems and noticing relationships. While some children may have had high levels of AMPS (Mulligan and Mitchelmore, 2009), the striking pattern made with manipulatives helped all children to focus on number relationships. Presenting number patterns in this way contrasts with other approaches which advocate that early years mathematics pedagogy should be based on realistic “context situations” (van den Heuvel-Panhuizen, 2008, p. 20).

The activity also prompted discussion, stimulating both Lucasz and Magda to express themselves creatively using language and gesture, as shown above, making connections and elaborating ideas. Lucasz may have been an example of a child using gesture to express emergent mathematics learning, as described by Garber, Alibali and Goldin-Meadow (1998) or he may have already been familiar with this pattern and been able to articulate it in Polish: we do not know. However, this showed that a lack of fluency did not prevent a child in the early stages of learning a language from trying to express a mathematical relationship, and also the importance of gesture in supporting mathematical discussion between children who are not using their home languages. The manipulatives pattern thereby supported the linking of different modes of representation, not only verbal and visual, but also kinaesthetic and, through the use of numeral cards, abstract symbols.

This activity also exemplified the need to identify and assess prerequisite understanding. Some children could not reliably count objects to 20, despite having been assessed as doing this the previous year. Most of the children counted from one instead of counting on from 10, even when it was pointed out to them that there were always 10 green cubes. This may have been because they lacked skills of counting on or were not yet able to unitise ten as an item. As Cobb (1995) pointed out, if children do not have a concept of ‘a ten’ they will not be able to ‘see’ it even when demonstrated. Similarly, some children could add 10 and 3 and instantly say “13”, but seemed not to understand the inverse relationship sufficiently to apply this to decomposing 13 into 10 and 3 and did not use this knowledge

to identify 13 as 10 and 3. Therefore some children may have lacked the prerequisite understanding and skills to access what the manipulatives were intended to represent.

The final principle shown by this example, is that of an inclusive learning community. The children were not in ‘ability’ groups, as is common in English early years classrooms, in which case Lucasz and Magda would not have been in the same group. We do not know if Magda would have articulated the pattern in this way, if she had not first listened to Lucasz, who was able to hear his idea expressed differently. Latoya, in her second session, identified the 15 rod by counting on from 10, saying, ‘10, 11, 12, 13, 14, 15’; this was a skill she had not used previously but had observed another child using in the previous session. Both the lack of grouping and the open activity, allowing for a range of solution strategies, facilitated children in learning from others.

In our study we also observed successful practice, particularly with colour rods, which similarly showed children engaged by patterns and stimulated to discuss mathematical relationships. These experiences imply that developing children’s focus on patterns is a promising avenue for mathematical pedagogy in the early years.

## Implications

The issues discussed here suggest some potentially profitable avenues for future research with mathematical manipulatives and young children and some implications for practice. Firstly, we might build on home practices in finger counting and develop young children’s use of ‘all at once’ finger numbers. We also might investigate how children reconcile differences between discrete and continuous models of number, through comparing and discussing representations. These issues highlight the need to consider carefully exactly how manipulatives might foster learning of particular aspects of mathematics. There are promising avenues for early years mathematics pedagogy in using manipulatives to build on children’s interest in patterns and to develop children’s own expressions of mathematical relationships. However, an important prerequisite for all of these is a focus on children’s understanding of mathematical relationships rather than on performing calculations, as suggested by Ma (2015), which also has implications for early years mathematics curricula.

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