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# Young children's aesthetic development in the context of mathematical explanation

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*Mathematicians routinely report that beauty is both a reward and a motivation for the work they do. However, how and to what extent children can appreciate mathematical beauty is an open question. This exploratory study looks at young children (ages 6-12, with a focus on the younger years) as they evaluate different explanations of claims about even numbers and triangular numbers. While our results are fairly speculative, we provide case studies which illustrate possible kinds of aesthetic reactions, and some of the factors which might impact on those reactions.*

*Keywords: Aesthetics, explanation, even numbers, triangular numbers, affect.*

## Introduction

There is little doubt that mathematicians have rich, aesthetic lives (Sinclair, 2004). While recent research has attempted to characterize what exactly this aesthetic life consists of (Raman-Sundström, & Öhman, 2016) or what is meant by aesthetics in mathematics in the first place (Rota, 1997), evidence suggests that aesthetic reactions are common in, and perhaps even central to, the working lives of mathematicians. According to some, aesthetic reactions include pleasure, tension, surprise, and a sense of being compelled (or repelled) (Marmur & Koichu, 2016). Interestingly, neuroscientists suggest that the same region of the brain is involved for judging mathematical equations and works of art (Zeki et al., 2014).

Recently, there has been interest in studying aesthetics in mathematics education. While some of those studies have investigated school children's aesthetic reactions while working on mathematical problems (e.g., Sinclair, 2006), few have focused on young children. Yet, research has shown that young children are capable of sophisticated proof-like reasoning (Maher & Martino, 1996), justification, and argumentation (Tatsis, Kafoussi, & Skoumpourdi, 2008). Thus, it seems reasonable to ask whether young children could be capable of aesthetic experiences, and if so, at what age. Are young children able to experience the satisfaction of finding a good explanation? Do they find pleasure in coming to understand an explanation?

This paper presents an explorative study of the possible aesthetic experiences of young children. Mathematics, like other aesthetic subjects, provides experiences that have the potential to draw people in. It also provides a reward or a sense of satisfaction (Sinclair, 2004). What makes the experience "complete" is the presence of *both* a startup phase and a reward phase. We study the presence or absence of aesthetic reactions of young children (age 6-7) by comparing their behavior to an older cohort (ages 9-10). We find that the younger children, while lacking some of the insight of the older children, have what we might call "aesthetic dispositions" that allow them to enjoy and to be curious about fairly complex mathematical tasks.

## Theoretical background

One of the central questions of aesthetics is whether beauty is objective or subjective. According to Marmur & Koichu (2016), those that consider mathematical beauty as objective (such as Dreyfus and Eisenberg, 1986) list characteristics such as clarity, simplicity, brevity, and conciseness when judging theorems and proofs. In other words, beauty is an intrinsic property of the mathematical object. Those that consider beauty to be subjective, claim that mathematical beauty is in the eye of the beholder and that experience, age, knowledge, and culture contribute to aesthetical views. Marmur and Koichu (2016) integrate both views, concluding that when discussing school mathematics, we may hypothesize that a mathematical problem might elicit an aesthetic experience among students because of its simplicity or surprising result. Ultimately, however, students may or may not have an aesthetic experience depending on, among other things, the pedagogical setup of the problem.

In their study of university students, Marmur & Koichu (2009) found that surprise was integral to experiencing mathematical beauty. They found that students who referred to a solution as beautiful had first struggled with the problem, had put significant effort into finding the solution, and ultimately were surprised at the simple and unexpected solution. Struggle was also a factor in Brinkmann's (2009) study of middle and upper school students' appreciation of mathematical beauty. A problem was considered to be beautiful if it had a certain degree of complexity, yet felt solvable. Eberle (2014) investigated students' (ages 8-10) aesthetic attractions when evaluating geometric tessellations. Students referred to several characteristics of the geometric objects which contributed to their appreciation, such as real world connections, color, complexity, and uniqueness. Eberle (2014), as well as Sinclair (2001) also noted the generative role of aesthetics when students were involved in inquiry-based tasks. In both studies, aesthetics led students to engage and play with the mathematics, guiding them when deciding which direction to pursue.

The above studies related to aesthetics with regard to problems, solutions, and geometric objects. In our study, we focus on mathematical explanations and young children's appreciation of those explanations. Previously, Levenson (2010) found that fifth-grade students have preferences regarding different types of explanations. Students' preferences were based on clarity, brevity, relatedness, and because the explanation was perceived as fun. Although some of the reasons students mentioned for their preferences are reminiscent of aesthetic evaluations given by older students, and even mathematicians, the focus of that study was not specifically on aesthetic appreciation or satisfaction from an explanation. In this study, we draw on a theory of explanation developed by Gopnik (2000), which helps explain what makes certain explanations satisfying. Gopnik suggests that an explanation consists of two parts, the why? (the 'hmmm....' phase) and the because! (the aha! or wow! phase depending on how surprising the result is for the individual). An exploration can have a why? without a because! and vice versa. Both phases are needed for an explanation to be found satisfying. Moreover, Gopnik suggests that both the seeking and the satisfaction from finding a good explanation are part of our human nature.

The aim of this study is to begin an exploration of young children's appreciation of mathematical beauty. Specifically, we ask: Do young students' aesthetic reactions (or non-reactions) to mathematical inquiry and explanation differ from that of older students? Are younger and older

students capable of an experience that contains both the hmmm... and aha! phase of a mathematical explanation?

## Methodology

Data was collected using semi-structured interviews with two cohorts of children, one aged 6-7 years old, and one aged 9-10 years old. The older children worked with explanations about triangular numbers, namely that the number of dots in the  $n^{\text{th}}$  triangular number is  $n(n+1)/2$ . This cohort included four fifth grade girls, all from the same class, who sat together in a group with the interviewer in one of the girl's houses. The discussion began with introducing the girls to triangular numbers, discussing the number of dots in the first five triangles, and then asking them to come up with the number of dots in the 100<sup>th</sup> triangle. After giving them time to work on the problem, the girls were shown two solutions and asked to evaluate each solution.

The younger cohort of three children worked individually with the interviewer with explanations of the claim: the sum of two even numbers is always even. Two interviews were conducted in the house of the child and one was conducted in the house of the child's grandmother. Each interview began in the same way, asking the child to say if he or she could give examples of even numbers and to say why those numbers were even. After confirming that the children were familiar with even numbers they were given the following question: what would happen if you add two even numbers, would the answer be even or odd? Children were given time to think and reply. Although the interviewer had several explanations on hand for the children to evaluate, as will be shown in the next section, only one child was asked to evaluate explanations.

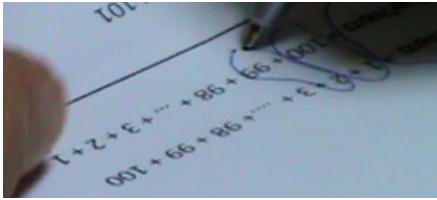
One of the difficulties of studying aesthetics in children, or even with mathematicians, is how to detect an aesthetic experience. While there may be bodily clues, such as changes in eye-dilation or neural correlates (e.g., Zeki et al., 2014), a natural place to start is by simply listening to what people say (see Wickman, 2006) and watching for engagement (or disengagement) during the experience. In this study, we take this approach as a first approximation, using key words (taken from the background studies) such as "Wow!" and "Funny!" as markers for a general aesthetic experience.

## Findings

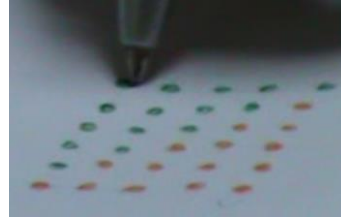
Below we present four episodes, one with older girls and three with younger children. What is striking about the young children is that there is no sense of surprise. The children seem to lack the 'hmm....' needed to build the wow! or to even warrant an explanation.

### Fifth-grade girls, age 11

Trying to figure out how many dots will be in the 100<sup>th</sup> triangle proves challenging to the girls. When they realize that they would have to sum all of the numbers from 1 to 100, the interviewer gives them some time to work this out and then explains to the girls the Gaussian method for summing an arithmetic sequence. She lines up the numbers from 1 to 100 in one row and on top of that row, lined up the numbers from 100 to 1 (see Figure 1), explaining that this shows how many dots are in each row of the 100<sup>th</sup> triangle.



**Figure 1: Summing the numbers from 1 to 100**



**Figure 2: Combining two triangles**

To explain why one multiplies 100 by 101 and then divide by 2, the following discussion ensues:

Esther: Two triangles of 100, right? Now look what we have here. (Esther circles the 100 and 1 and the 99 and 2).

Girls: Ahh!

Esther: 1 and 100, 2 and 99, 3 and 98.

Trina: Wow! It's great. It's the same thing.

The girls are surprised that the sums all add to the same number, 101. Their remarks of "Ahh" and "Wow" indicate their pleasure in this simple conclusion. After showing the girls this explanation, the interviewer shows them a second method, that of drawing two congruent triangles, inverting one and placing it next to the first, thus creating a rectangle. The number of dots in the triangle is then equal to the area of the rectangle divided by two (see Figure 2). After establishing that the girls understand both explanations, the girls are asked to compare the two methods.

Esther: Which explanation of the method gives you more satisfaction?

(All of the girls point to the second explanation with the dots.)

Amanda: You simply see that you do this times this, and then divide by 2 because you have 2 triangles.

Hailey: Because instead of computing it all, this is easier and simpler and in front of your eyes.

Amanda: Also, it draws attention more. It's more fun, but not just more fun, it's like, it goes more into your head.

In the second segment, the girls claim to like the second explanation better because it is simpler and because they can "see it." This hints at their appreciation for the aesthetic value of efficiency, which might have been enhanced by their struggle to find the solution.

### **Zev, age seven**

Zev is seven years old, attends first grade in Israel, where learning about even and odd numbers is part of the curriculum in school. Zev is able to list several even numbers as well as several odd numbers. When asked why eight is an even number he says, "because four is even and ... because four is even and it's... and also... the second four is even." Note that he does not stress that 8 could be written as the sum of two equal whole numbers, but rather that both of the addends, in this case fours, were both even numbers. When asked why 10 is an even number, the following discussion ensues:

Esther: OK. Is ten an even number?

- Zev: Yes.
- Esther: How do you know that ten is even?
- Zev: Because odd plus odd is even.

Here Zev was probably thinking that 10 results from  $5 + 5$ , both odd numbers. Realizing that Zev seems already familiar with summing odd numbers, the interviewer asks about the sum of two even numbers:

- Esther: And what about an even number plus an even number?
- Zev: Even.
- Esther: Always?
- Zev: Yes.
- Esther: Can you tell me how you know that even plus even is always even?
- Zev: Four plus four, eight. Eight plus eight, sixteen.
- Esther: How do you know that sixteen is even?
- Zev: Because ten is even, and six is even.

For Zev, it seems that his working definition of an even number is of a number that can be written as the sum of two other even numbers. Although this is a recursive definition, it does not seem to bother him and the outcome of this conception is that Zev does not even recognize the question of what might be the sum of two even numbers. Zev has no sense of *hmm...* When Zev is shown another explanation for why the sum of two even numbers is always even (that every even number can be written as the sum of twos and thus the sum of two even numbers can also be written as the sum of twos), Zev says that the explanation is boring because he already knew that.

### **Anna, age six**

Anna is six years old and attends kindergarten in Israel. Although even and odd numbers are not part of the kindergarten curriculum, Anna was able to list several even and odd numbers and claimed to have learned about them from her teacher. When asked why eight is an even number she responded, "Because each one has a partner." When she was asked to explain what an even number is, she said, "That the two of them have a partner. That each of them has a partner." What Anna is alluding to in her own language is that an even number may be written as the sum of twos. After talking about even numbers for a few minutes, we discuss the sum of two even numbers:

- Esther: What would happen if I added an even number with another even number?
- Anna: It would be even.
- Esther: How do you know?
- Anna: Because both of them are not separate, it never separates from the second.
- Esther: What do you say! Are you sure that this is always this way?
- Anna: Yes. It will never be that, uhmm, a partner doesn't run away from the pair.

Anna has a very specific conceptualization of even numbers, that an even number represents an inseparable pair. She draws on this conceptualization to explain why the sum of two even numbers

must always be even. Engaging further with the problem, Anna takes out a bunch of wooden blocks from a basket (without counting) and proceeds to pair up blocks. When the interviewer asks if she can say if she took out an even or odd number of blocks she readily says that she took out an even number of blocks because on the table, every block is paired off. To summarize, Anna is drawn in to the problem, has an explanation, but she does not struggle with the claim.

### **Leila, age seven**

Leila is from Sweden, has just turned 7, and attends kindergarten. Although learning about even and odd numbers is not part of the curriculum, she is able to list several even numbers (perhaps hearing about them from her parents or friends). Although prompted to think about the possible sum of any two even numbers, Leila shows no interest and instead, with the use of colorful cubes, begins to explore a self-generated conjecture involving pairs of twos. She puts together six pairs of cubes to represent an even number and then wonders if seven pairs will still be even.

Leila: I have a question. You can't have an odd number of twos.

Manya: How are you thinking?

Leila: Like, if I have these (pointing to the six pairs of cubes), you can never have an odd number. Look here. I have 1, 2, 3, 4, 5, 6. If I have this many (she takes another pair), now I have enough.... odd numbers of twos... because there's seven... does it get an odd number or an even? I think it is actually funny. If you have an odd number of twos, it even gets an even number. That I think is really funny.

Manya: I see. Why is that?

Leila: I don't know, because it feels like it's actually pretty funny.





Manya: Do you think it's funnier to have an even number made of an odd number of twos than an even?

Leila: What do you mean – an even number of odd number ...? Yeah. It gets an even number if you have an odd number of twos. Because I figured it out of these.

This discussion suggests that autonomy might play a role in having an aesthetic experience. Leila generates her own question (What is the sum of two odd numbers?) and the resolution of the question (that an odd number of twos can be even). She thinks that this conclusion is funny. While she might have had a similar reaction to a given statement, the affect seems to be closely correlated to ownership of ideas. In the last two lines, the researcher asks her to compare two statements. Leila replies “what do you mean”? The statement is funny to her because she “figured it out of these.”

### **Comparing the older cohort to the younger cohort**

The following table summarizes the observations from the data presented above and the children's aesthetic experiences (AE). A full circle represents that an AE took place, a dotted circle reflects a partial experience, and an empty circle that no AE was detected. In all of these cases, we can see that the path to having an aesthetic experience consists of several distinct phases. One needs to be engaged; there should be some build-up, some crucial moment, and then some release. Only the 10 year olds seem to have a complete experience.

Children	Age	Task	AE	Behavior
Trina, Hailey, Amanda	10	Triangular numbers		Have an aesthetic experience, marked by both surprise and satisfaction. They appreciate a solution which is simple and “goes into their head”.
Zev	7	Even numbers		Has a conception of even numbers, but is uninterested in explaining why the sum of evens is even. He “already knows that”.
Anna	6	Even numbers		Has a conception of even numbers which goes along with the claim about the sum of even numbers. Is engaged and involved, but is lacking a “hmmm....”
Leila	7	Even numbers		Takes control. Explores her own hypothesis. Tests if an odd number of twos can be even. Finds the result “funny”.

**Table 1. Types of aesthetic experiences for the triangular number and even number tasks**

Build-up takes place when they explore the question, and their interest increases as the interviewer shows the explanation involving pairs of numbers. After having time to digest this information, the interviewer shows another explanation which further increases their interest. Each time a new explanation is understood, the girls say words like “wow!” and “aha!”. The fact that they have this kind of reaction, we claim, is because they had time to explore and to start generating their own explanations. We also cannot rule out that there might be some developmental issues, such as the children being old enough to abstract and/or take in the explanations given.

In contrast to the fifth-graders, the younger students had limited or no aesthetic experience. Zev is strikingly uninterested in any explanation at all. We suspect that his disinterest came from the fact that he had been told in school that even + even is even, so there was no tension left to resolve. Leila has some interest in explanation, but not for the question given to her. Rather, she generates her own question about whether an odd number of pairs can result in an even number. She finds this result funny, indicating some level of surprise, which she quickly believed despite her initial expectation. Anna is drawn in to the explanation activity, but did not seem to have a full aesthetic experience. Unlike the fifth graders who could “see” why the explanations held, Anna simply states her conception of even numbers in terms of pairs or partners, and claims that any sum of pairs will still be even. She does not give an actual reason, which might be because she did not experience the hmmm... phase of explanation. She is not bothered by any alternative, so no relief or satisfaction is expressed.

## Conclusion

One of the challenges of this study was to find tasks that might elicit an aesthetic reaction. We attempted to find tasks that would be suitably challenging, yet accessible to each of the age groups. In the end, the older children worked on a new task, presented not only in general manner, but with an iconic illustration, while the younger children worked on familiar (at least for two children) general characteristics of numbers. Thus, it might be that the different conditions affected the aesthetic experiences. Taking these limitations into consideration, there is still the possibility of developmental differences in aesthetic experiences. In the naïve view, children have and rely on concepts, but are not yet puzzled. Because of this lack of puzzlement (an essential ingredient



according to several researchers (e.g., Gopnik, 2000; Marmur & Koichu, 2009)), there is no tension in their mathematical exploration, nothing to be resolved, and so no aesthetic experience is possible. In contrast, in the mature view, children are engaged and puzzled. They are more open to explanations because they themselves have struggled with the questions. This kind of behavior is possible among quite young children (Leila, at age 7, has a very small amplitude aesthetic experience when she generated her own conjecture), but might be more likely to occur the more autonomy is given to the students, the more challenging the task, and the more supported the students are to not to give up when they think they already have the answer. As an exploratory study, this paper has begun a discussion regarding young children's possible aesthetic experiences when working on mathematics. Additional research is needed to continue this discussion.

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