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# Preschool teachers' variations when implementing a patterning task

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*It is often recommended to engage young children with patterning activities. As part of a professional development program, nine preschool teachers were introduced to repeating patterns and were given the materials and instructions with which to implement an extension task with children. This study presents the various ways teachers implemented this task and investigates the impact of the various implementations on children's success in extending the repeating patterns.*

*Keywords: Repeating patterns, tasks, preschool teachers*

## Introduction and background

For several years, we have been providing professional development for preschool teachers guided by the Cognitive Affective Mathematics Teacher Education (CAMTE) framework (e.g. Tsamir, Tirosh, Levenson, Tabach, & Barkai, 2014). Our aims are to promote teachers' knowledge and self-efficacy for teaching mathematics to young children. An essential element of pedagogical-content knowledge is knowledge of tasks (Sullivan, Clarke, & Clarke, 2009). In Israel, where this study took place, there is a mandatory mathematics preschool curriculum, but few curricular materials are available. Thus, introducing preschool teachers to appropriate mathematical tasks is essential. However, studies have shown that even when providing teachers with a task, and with explicit instructions for carrying out that task, teachers may implement the task in different ways (Bieda, 2010). In turn, different implementations may affect the cognitive load of a task or a student's conceptualization of key mathematical ideas (Stein, Grover, & Henningsen, 1996). This paper investigates the various ways preschool teachers implement a given repeating pattern task.

Repeating patterns are patterns with a cyclical repetition of an identifiable 'unit of repeat'. For example, a pattern of the form ABBABBABB... has a (minimal) unit of repeat of length three. The importance of engaging young children with pattern activities is supported by mathematicians, mathematics education researchers, and curriculum developers (Sarama & Clements, 2009). Pattern exploration and recognition may support children as they learn a variety of mathematical skills developed at this age. For example, recognizing repeating patterns may help children develop skip counting, such as 5, 10, 15, 20, 25, 30 ... where the ones digit forms the pattern 5, 0, 5, 0, ... Recognition and analysis of patterns can also provide a foundation for the development of algebraic thinking and provide children with the opportunity to observe and verbalize generalizations as well as to record them symbolically (Threlfall, 1999).

Several studies have investigated ways in which young children engage with repeating patterns. For example, Seo and Ginsburg (2004) found that young children build block towers with an ABAB pattern. Fox (2005) observed young children painting stripes in ABAB patterns as well as one child who painted four sets of an ABC pattern and then said, "Look at my pattern" (p. 317). Waters (2004) observed a young girl who created a necklace out of game materials and described her necklace as "diamond, funny shape, diamond, funny shape" (p. 326). Papic, Mulligan, &

Mitchelmore (2011) found that some preschool children may be able to draw an ABABAB pattern from memory by recalling the pattern as single alternating colors of red, blue, red, blue, basically recalling that after red came blue and after blue came red. However, when shown a more complicated pattern such as ABBC, they could not replicate the pattern.

This study focuses specifically on the task of extending a repeating pattern. Pattern extension tasks mostly include showing the child a pattern and requesting the child to continue it. Papic, et al., (2011) reported that many children succeed at these tasks without necessarily recognizing the unit of repeat. Instead, they use the “matching one item at a time” strategy, also known as the “alternation strategy” which is especially successful with simple ABAB patterns. Rittle-Johnson, et al., (2013) found that some children reverted to producing an ABAB pattern while others could not produce more than one unit of repeat correctly when extending an ABB pattern. Similarly, Swoboda (2010) found that for some four-year old children, continuing a pattern means duplicating the unit of repeat once, and no more. In other words, both the complexity of the unit of repeat, as well as the number of times the unit is to be repeated, seemed to contribute to the difficulty of the task.

Another factor which may impact on the difficulty of extending repeating patterns is whether (or not) the given pattern ends with a complete unit of repeat. In one study, children were shown given repeated patterns and asked to consider extending the patterns by choosing between different possible continuations, some appropriate and some not appropriate. Children had greater success extending patterns which ended with a complete unit, than extending patterns which ended in a partial unit (Tsamir, Tirosh, Barkai, Levenson, & Tabach, 2015). Furthermore, several of the appropriate continuations would have extended the pattern in such a way as to end the pattern with an incomplete unit of repeat. Fewer children chose those possibilities as appropriate, even though they were correct extensions.

From the above studies, we see that there are several variables that may be taken into consideration when engaging with pattern extension activities: the structure of the pattern, the length of the unit of repeat, the number of times a unit appears in a pattern, and if the presented pattern ends in a complete unit of repeat or not. However, these variable all have to do with the repeating pattern. Might there be other variables that need to be considered when requesting children to extend a repeating pattern? Our first question is: Given an extension task and a set of repeating patterns, what are the various ways preschool teachers implement the task? Considering variation theory of learning and that learners may experience objects in various ways (Ling & Marton, 2012), our second question is: What can we learn about children’s patterning abilities from the different implementations?

## Methodology

This study took place within the context of a professional development course for preschool teachers, focusing on patterning for young children. Twenty-three preschool teachers participated in the program. All had a first degree in education and between 1 and 38 years of teaching experience in preschools. The entire program was planned for 21 hours. The teachers met seven times over a period of about four months in the local professional development center in their area. Approximately five of the seven sessions were devoted to repeating patterns with the other two focusing on number concepts. All lessons and tasks were planned by the four authors of this paper.

During the program, teachers were introduced to different patterning tasks as a tool for promoting their mathematical and pedagogical knowledge for teaching patterns in preschool. For the final project of the program, teachers were instructed to choose two of the tasks that were presented and analyzed during the course, and implement and video those chosen activities with one child. Those videos were then analyzed and discussed together in terms of children's solutions. In this paper we investigate teachers' implementations of one task (see Figure 1). Nine teachers (T1-T9) implemented this task, each with one child from their preschool (C1-C9). It is important to note that the task, along with the explicit instructions, was presented to the teachers by the teacher educator, who demonstrated how the task should be implemented. Furthermore, this task was not meant to be an instructive task, but instead an evaluation task in the sense that it was meant to assess children's ability to extend various repeating patterns.

Present the child with one pattern at a time. For each pattern prepare two or three separate containers, each container containing cutouts of triangles, squares, or circles. For example, when presenting the first pattern, place before the child two containers, one with blue squares and one with red triangles. For each pattern ask: What comes next? This question is repeated three times so that in the end, the child will have added three elements to the pattern.

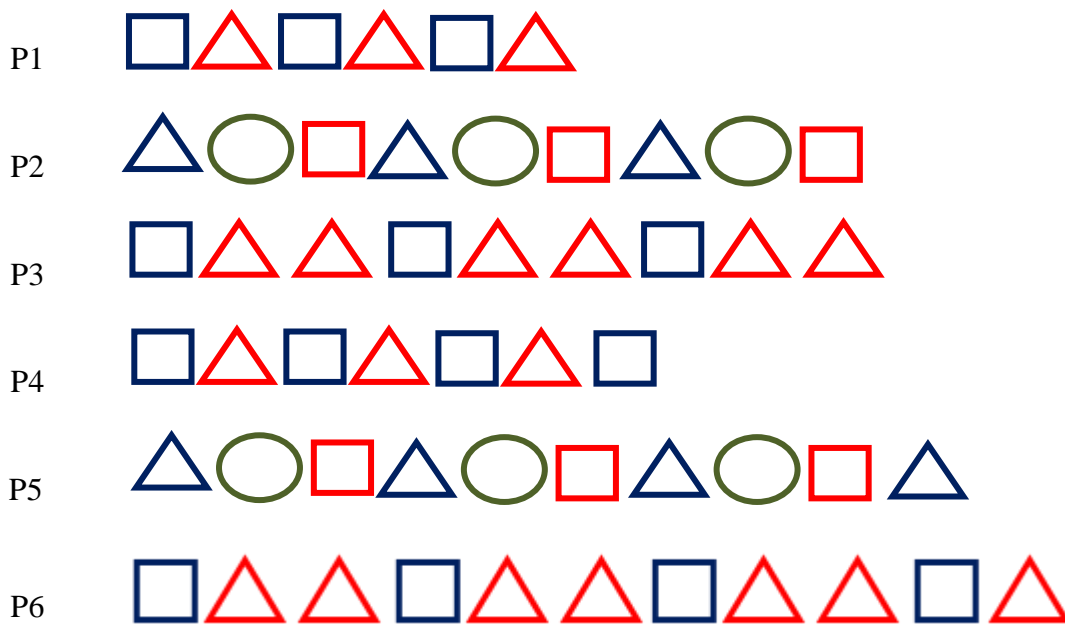


Figure 1: What comes next?

Note that there are basically three different structures, from the simpler AB, to the more complex ABC, and the even more complex ABB (e.g., Rittle-Johnson, et al., 2013). In addition, the first three patterns end in a complete unit of repeat and the last three do not. In other words, the sequencing of patterns goes from the simple to the more complex.

### Various ways of implementing the task

We first note that none of the teachers changed the given patterns. Some variations came about from not implementing the tasks according to the instructions. For example, although teachers were told

to prepare five separate containers for each possible cutout, according to shape and color, and only present to the child those containers which contained cutouts for that pattern, only one teacher actually followed this instruction. Four teachers did separate the elements into five containers according to shape and color, but then kept all of the containers on the table, no matter which pattern was being extended. Four other teachers separated the elements into only three containers according to shape (e.g., putting blue and red squares in one container), and then placed all three containers on the table, no matter which pattern was being extended. Another explicit instruction which was not followed was the sequencing of the patterns. Two teachers did not present the patterns in the order given above. One teacher showed the patterns in the following order: P4, P6, P3, P4, P1, and P2. The second teacher used the following order: P4, P3, P2, P1, P6, and P5.

Some variations in implementation seemed to come about because no explicit instructions were given as to what to say to the child before beginning the task. That is, teachers were instructed to ask for each pattern “What comes next?”, but were not told what to say when sitting down with the child and introducing the task. Six teachers stated at the beginning of the task, as they placed the pattern down on the table, “Here is a pattern.” Five teachers asked the children to say out loud each element from the beginning of the pattern. It might be that the teachers thought that saying out loud the elements would allow children to hear the repetition of the unit of repeat and thus enable the children to pick out the correct next element. For example, at first, T5 did not ask C5 to read out loud each element of the pattern. She told her that there was a pattern and then asked her to pick out the element that should come next. After waiting a bit and seeing that the child sat still and did nothing, she then requested the child to say out loud each element of the pattern from the beginning. After that, C5 continued with the task and picked out the next element (correctly). T5 then continued with this instruction for each additional pattern (and answered each one correctly). Other teachers did not wait to see what would happen, but from the beginning requested that the child say out loud each element. T7 began her interview with C7 by saying, “Let’s read the pattern together, let’s read.” At that point, C7 did not read the pattern but stretched out her arm to take a blue square (the correct element for extending the pattern) from one of the containers and place it at the end of the given pattern. T7 stopped her, despite that C7 chose the correct way of extending the pattern, and said, “No, sweetie. Wait a minute. First, let’s read it.” Three of the five teachers who requested children to read out loud the patterns, and an additional two teachers, asked the child they interviewed to say what was in the containers.

Variations in task implementation also occurred while the child was actively engaged with the task. Some of those variations were queries into why the child chose one or another element. For example, when engaged in the second pattern (P2), one child mistakenly took a blue square, but then immediately switched it with a correct red square. The teacher then inquired, “Why didn’t you put down the blue square and why did you put down the red one?” This type of intervention did not interfere with the child’s performance, but was instead a way for the teacher to listen to the child’s way of thinking. In this case, the child answered, “because here (pointing to the pattern), the square is red.” In other instances, the teacher’s intervention came about even before the child took action. For example, when placing on the table P4 (the first pattern that did not end with a complete unit of repeat) T4 said to the child, “Now pay attention.” After placing on the table the last pattern, T4 said, “Now look closely at the pattern, and also look carefully where it ends.” This type of intervention has the potential to alternate a child’s performance. In this case, despite all these warnings, C4

incorrectly extended all of the last three patterns (those that did not end with a complete unit). Some teachers intervened when the child chose an incorrect way of extending the pattern. For example, when T6 asked C6 to extend P3 (the pattern with an ABB structure), C6 incorrectly added ABA. The teacher then pointed to each element in the unit of repeat and said, “Look closely. Square, triangle, and ...” C6 then responded correctly, “triangle.” Interestingly, C6 had previously extended P2 in an incorrect manner. Although placing the correct shapes to extend the pattern, the child chose incorrect colors. In that case, T6 did not intervene, and instead said, “Good.” Perhaps the teacher was satisfied that at least the child had chosen the correct shapes. However, when it came to placing incorrect shapes for P3, the teacher (T6) intervened.

### Children’s performances

Results of children’s performances on the task, for each pattern, are shown in Table 1 according to structure and if the pattern ended in a complete unit (Comp.) or an incomplete (Inc.) unit. An extension of the pattern was only considered correct if the child successfully extended the pattern by three elements. As can be seen, children performed better extending a pattern that ended in a complete unit of repeat than a pattern which did not end in a complete unit.

Structure	P1 (AB Comp.)	P4 (AB Inc.)	P2 (ABC Comp.)	P5 (ABC Inc.)	P3 (ABB Comp.)	P6 (ABB Inc.)
Frequency	8 (89)	6 (67)	6 (67)	6 (67)	2 (22)	4 (44)

**Table 1: Frequency (%) of successfully extending each pattern (N=9)**

We now address the question of whether different implementations affected the children’s success in extending the pattern. Comparing results of children who read out loud the elements of the pattern before extending the pattern, and those who did not, the relative frequency of success was slightly greater for those children who did not first read out loud the pattern (see Table 2).

	Reads out loud the pattern (N=5)	Does not read out loud the pattern (N=4)
P1 (AB Comp.)	4(80)	4(100)
P2 (ABC Comp.)	3(60)	3(75)
P3 (ABB Comp.)	3(60)	3(75)
P4 (AB Incomp.)	3(60)	3(75)
P5 (ABC Incomp.)	1(20)	1(25)
P6 (ABB Incomp.)	1(20)	3(75)

**Table 2: Frequency (%) of success per variations in reading out loud the pattern**

Regarding the placing of elements in containers, results (see Table 3) indicated that in general, the way the elements were presented made little difference to the children’s ability to extend the pattern. Taking a closer look, for the first three patterns that ended in a complete unit of repeat, there was a higher success rate when the elements were separated by shape and color. However, when the patterns did not end with a complete unit of repeat, there was either no difference or there was a higher success rate when the elements were separated only by shape.

	3 containers separated only by shape (N=4)	5 containers separated by color and shape (N=4)	2 or 3 containers, only necessary elements (N=1)
P1	4(100)	4(100)	-
P2	2(50)	3(75)	1(100)
P3	2(50)	3(75)	1(100)
P4	4(100)	2(50)	-
P5	2(50)	-	-
P6	2(50)	2(50)	-

**Table 3: Frequency (%) of success per variations in containers.**

Regarding other differences in implementations, few affects were noticed. For example, among the six children who were told explicitly before beginning the task that there was a pattern which needed to be extended, three children extended correctly only two of the six patterns; the other three correctly extended three, five, and six of the patterns. Among those who were not explicitly told that there was a pattern (three children), a similar variance in success rates was found. The same variance in success was noted regarding children who were requested to say which elements were in each of the containers.

When analyzing the errors made by children, we found that the most prevalent mistake when attempting to extend a pattern that did not end with a complete unit of repeat was to continue the pattern as if it had ended in a complete unit, i.e., adding the first three elements from the beginning of the pattern. For example, C2 continued P4 by adding a square, triangle, and then a square. Another type of mistake which occurred for patterns both that ended and did not end with a complete unit of repeat, was to continue the pattern with ABAB despite there being a different structure to the given pattern. This occurred for C6 who added BA to an ABC structure, and also an ABB structure. Likewise, C9 continued P6 by adding BABABABA. Another type of mistake was taking the correct shape, but with the wrong color, as was demonstrated above by C2. This last type of mistake was directly related to the way the task was implemented. Obviously, if only the correct colors of shapes would have been on the table, this type of mistake could not occur.

## Summary and discussion

As part of the professional development program, teachers were supplied with the materials for implementing the repeating pattern task. They were given laminated strips of paper with the patterns printed on them in color. They were given the matching pictures of colored squares, triangles, and circles to be cut out and placed in containers. They were even told what to ask each child. Yet, many variations occurred when implementing the task. Some of the variations occurred in the setup of the task, specifically with placing the elements in containers. Some of the variations occurred in the midst of implementing the task. When reflecting with the teachers on their implementations, it became apparent that these variations occurred spontaneously, without planning for them ahead of time. And yet, several of the teachers had the same ideas, such as having the children read out loud the pattern. Knowledge of tasks includes knowing the affordances and constraints of that task (Watson & Mason, 2006). It could be that the teachers saw this task as affording the opportunity to

review with the children names of two-dimensional figures. According to Zaslavsky (2008), teaching tools include not only materials, but other kinds of resources, such as language and time. It could be that teachers were incorporating the tool of language into this given task, having children say the names of the shapes in the pattern. It could also be, similar to other studies (Stein, Grover, & Henningsen, 1996) that teachers were attempting to lighten the cognitive load of the students by telling them that there was a pattern and having them say out loud the names of the shapes which repeated themselves. In any case, as teacher educators it is important to be aware that teachers may implement a given task in various ways. In fact, in our program, the teachers brought the videos of their implementations back to the program, and as a group, we viewed them together. This, in turn, enhanced the teachers' knowledge of tasks, including their knowledge of the way children engage with repeating patterns tasks.

Despite the variations in implementations, most of the outcomes were consistent with previous research. For example, children had greater success when extending patterns which ended in a complete unit of repeat than those which did not (Tsamir et al., 2015). Children in this study made similar errors as children in other studies, such as extending an ABB pattern with ABA (Rittle-Johnson, et al., (2013). Can we conclude then that variations seen in this study had no impact? Certainly, a study with nine children is not enough to make such a conclusion, but it does leave us with an interesting question. How come the variations seen in this study (e.g., telling the children that there was a pattern, reading out loud the elements of a pattern, placing the elements in various containers) did not seem to impact on children's performance? The answer to this question perhaps lies in acknowledging the essence of repeating patterns, which is the unit of repeat and its structure. None of the variations in implementations focused the child on the unit of repeat. What seemed to impact on results was the complexity of the structure as well as if the pattern ended in a complete unit of repeat. By reviewing these results with preschool teachers, noticing the variations as well as the little affect they had on children's performances, we strengthen teachers' appreciation for the structure of a pattern, and promote their knowledge for teaching repeating patterns in preschool.

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