Algorithmics in secondary school: A comparative study between Ukraine And France

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This article is focused on the teaching and learning of Algorithmics, a discipline at the intersection of Informatics and Mathematics. We focus on the didactic transposition of Algorithmics in secondary school in France and Ukraine. Based on epistemological and didactical frameworks, we identify the general characteristics of the approaches in the two countries, taking into account the organization of the content and the national contexts (in the course of Mathematics in France and in the course of Informatics in Ukraine). Our results give perspectives on understanding the place that Algorithmics can hold in the teaching and learning of Mathematics and Informatics.

Keywords: Algorithm, algorithmics, didactic transposition, curricula comparison, France, Ukraine.

Introduction

Mathematics and Informatics[1], as disciplines, have strong links. On one hand, Mathematics gives theoretical basis and instruments to Informatics, and on the other hand, Informatics enriches Mathematics with new objects and problems, and brought some changes in the mathematical activity. Many disciplines (Discrete Mathematics, Algorithmics, etc.) developed at their interface. Nowadays, there is an international movement towards including Informatics and these subjects in secondary education. Algorithmics is more and more present in secondary school in many countries. It can be involved in programs of Mathematics (as in France) or Informatics (as in Ukraine). This situation questions the goals of teaching and learning Algorithmics in secondary school, the contents of the curricula as well as the approaches and activities proposed to pupils. To contribute to the study of these issues, we propose a comparative study regarding the concept of algorithm and the contents of Algorithmics in secondary school in France and Ukraine.

Algorithm is a central notion in Mathematics and Informatics. Algorithm in "classic" Mathematics is generally used with the meaning of a general effective procedure for solving problems. Since the origins of Mathematics, algorithms have been designed and used for solving problems (arithmetic operations with decimal numbers, solving equations, etc.). The use of computers and programming languages brought a new point of view on the notion of *algorithm* referring to the formalization of procedures, the automation of computations and the problem of data treatment. In this context, the notions of finiteness, iteration and recurrence play an important role (Chabert, 1999, pp. 6–7).

As a reference, we retained the following definition of an algorithm given by Modeste (2012, p. 25) and based on the academic literature on the subject: "a procedure for solving a problem which, in a finite number of constructive, effective, not ambiguous steps, gives a result for any instance of the

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1 We will use the term "Informatics", a more faithful translation from the French and Ukrainian than Computer Science.

problem". We will delimit the discipline Algorithmics as the field that deals with algorithms, the problems they can solve, their design, their use, their analysis and their comparison.

Research questions, theoretical framework and methodology

To formalize our problematic, we have formulated the following research questions:

- What is the place of Algorithmics in secondary school in France and Ukraine?
- In what way the learning contents related to Algorithmics are organized?
- What types of activities in Algorithmics are proposed to pupils in both countries?
- What common points and differences appear? How can they be interpreted?

Our study is based on the concept of didactic transposition (Chevallard, 1985) developed in the Anthropological Theory of the Didactic (Bosch & Gascón, 2014). The didactic transposition describes the processes of transformation between academic knowledge, the knowledge to be taught and the taught knowledge. It takes a step back from the curriculum and the content actually taught, to understand the role and the influence of the different institutions involved in these processes.

In order to analyse the content devoted to Algorithmics in the curricula, to measure their distance to the academic knowledge, and to develop our comparative study, we lean on an epistemological framework, in accordance with a classical methodology in Didactics of Sciences and Mathematics (Artigue, 1990). Concerning the concept of algorithm and Algorithmics we used the epistemological framework developed by Modeste (2012; Modeste & Ouvrier-Buffet, 2011). The epistemological model distinguishes five fundamental aspects of the concept of algorithm: ***problem*** aspect (the fact that an algorithm is a tool to solve all instances of a problem, the notions of input and output); ***effectiveness*** aspect (all elements referring to the fact that an algorithm solves problems effectively: the notion of operator; the finiteness of instructions and executions, etc.); ***complexity*** aspect (all elements referring to the notion of complexity of algorithms and problems); ***proof*** aspect (referring to the links between algorithm and proof); ***theoretical models*** aspect (referring to the theoretical works in Logic and Informatics concerning the concept of algorithm). Among these aspects, *problem* and *effectiveness* refer to algorithm as a tool, whereas *proof*, *complexity* and *theoretical models* refer to algorithm as an object. This tool-object dialectic of the concept of algorithm will be useful to understand the points of view in different institutions.

In this study, we analysed the official instructions, the official resources for teachers and particular textbooks (we focused on the knowledge to be taught, but our analysis of textbooks also informs about the taught knowledge). For this purpose (addressing the didactic transposition in a given institution), it is not sufficient to study the discourse about algorithms: it is essential to examine the algorithms selected by the institution, their representations and the activities involving these algorithms: this can reveal differences (and even contradictions) with the general discourse. To do this, for each resource, we answered to the following questions:

- What kind of definition of algorithm is proposed?
- What algorithms (or types of algorithms) have been selected?
- What representations of algorithm are used?
- What tasks (or types of tasks) in Algorithmics are proposed?

Answering these questions brings elements that permit to address our principal question:

- What aspects of algorithm are presented (according to the five fundamental aspects)? Do this aspect relate to algorithm as a tool or as an object?

Our corpus of resources is described in Table 1. In the following, we present the results of our analyses and the answers to the research questions. As we cannot provide all details in this paper, we present only the main results of the comparison. Before that, it is necessary to give an overview of the situation in France and Ukraine, regarding the teaching of Algorithmics.

Resources	France	Ukraine
Official instructions and documents	Official program of Mathematics for middle school; Official curricula of Mathematics for high school: grade 10, grades 11 and 12 (all paths); Official accompanying resources in Algorithmics for grade 10; Official program for the ISN option.	Official program of Informatics for middle school (5-9 grades); Official programs of Informatics for high school (grades 10 and 11): standard, academic, professional and advanced levels.
Textbooks	Three collections of Mathematics textbooks (Indice, Bordas ; Math'x, Didier ; Transmath, Nathan) for grades 10, 11 and 12 (scientific, economic and humanities paths) ; Textbooks for middle school are not available yet.	One collection of textbooks in Informatics (Ryvkind J.Ya. et al., 2011) for grade 11, standard and academic levels, part 'Algorithmics'; two collections of textbooks in Informatics (Morze N.V. et al., 2014, 2015, Ryvkind J.Ya. et al., 2014, 2015) for grades 6 and 7, part 'Algorithmics'.

Table 1: Analysed resources.

Presentation of the contexts and evolutions of curricula in Ukraine and France

Situation in Ukraine (organization, history and recent evolutions)

The Ukrainian school system consists of primary school (grades 1-4, age of pupils – 6-9 years), middle school (grades 5-9, 10-15 years) and high school (grades 10-11, 16-17 years). In high school pupils make a choice between general or professional-oriented paths. Informatics has been taught since 1985 in secondary school in USSR. From the beginning, Algorithmics was a part of it. At that time, the course was mainly dedicated to writing algorithms in pseudo-code and executing them manually. Only two programming languages were used: Rapira (especially elaborated in USSR for teaching) and Basic. The major part of the first manual of Informatics in USSR is devoted to solving algorithmic problems. An algorithm is defined as “a clear and precise instruction destined to an operator to carry out a sequence of actions in order to reach a goal or solve a problem”. In this manual, the notion of *operator* plays a central role. It is also underlined that an operator executes an algorithm formally, i.e. it can carry out operations one by one in defined order without understanding the goal. A *scheme for solving a problem with a computer* is presented: in brief, it consists of modelling the problem, constructing an algorithm, writing it in a programming language, executing it, and analysing the results. Many tasks require constructing algorithms for solving mathematical and physical problems with a computer, such as Horners' method or the approximation of the area under the graph of a positive function.

Since 1985, Informatics has always been a mandatory subject in high school. In the programs published in 2008, Informatics can be taught at the following levels: standard (for general and humanitarian paths), academic (for science path), professional and advanced (for informatics and mathematics paths). The difference is the number of hours of Informatics per week (between 1 and

5) and the contents. As a part of the subject, Algorithmics is studied at every level with a total amount of hours that varies a lot: Standard (5), Academic (28), Professional (175), Advanced (191). In 2013, Informatics also became a mandatory subject in primary (from grade 2) and middle school. Thus, at the moment, Algorithmics is also present in middle school (from grade 6).

Situation in France (organization, history and recent evolutions)

The secondary French school system consists of middle school (*collège*, grades 6-9, 11-15 years) and high school (*lycée*, grades 10-12, 16-18 years). In high school, professional, technical and general orientations are proposed, and the general orientation is divided into humanities, economic and scientific paths[2]. In this study, we will concentrate on the general orientation. Recently, many reforms happened in the French curricula, the last one was in 2016 and concerned middle school. Gueudet, Bueno-Ravel, Modeste and Trouche (to appear) give more details about the evolution of French mathematics curricula, including Algorithmics. Informatics appeared for the first time in the French curricula in the 1980's (Baron & Bruillard, 2011), with an introduction to Programming and Algorithmics in high school. It disappeared in the 1990's, replaced by the use of computer tools and new technologies. Recently, Informatics came back in secondary school. In 2012, an optional course was created in grade 12 in the scientific path (ISN: “Informatics and digital sciences”, 2h/week), and in 2015, an optional course appeared in grade 10 (“Informatics and Digital Creation”, 1h30/week). Starting from 2016, Informatics will also be taught in middle school (principally in grades 7-9) in the mathematics and technology classes. Few years before that (from 2009 for grade 10 to 2012 for grade 12), some contents of Algorithmics were introduced in the curricula of Mathematics in high school (Modeste & Ouvrier-Buffet, 2011).

Comparison at high school level (grades 10-12, ages 15-18)

Algorithmics in high school in Ukraine

Through all grades and levels, a common approach to Algorithmics can be identified. It includes the presentation of *the steps for solving problems using a computer* and the role of algorithms in this process, distinguishing algorithms from programs, with an emphasis on the notion of *operator*. The activities involve various representations of algorithms (common language, flowchart and program). In the analysed textbooks the term *algorithm* is defined as a finite sequence of instructions that determines what operations and in which order to carry out for obtaining a goal. In this definition, as well as in the description of the properties of an algorithm (discreteness, certitude, feasibility, finiteness, effectiveness) given explicitly at all levels, we can identify the aspect *effectiveness*. The *problem* aspect is expressed in the property of “generality” of an algorithm, which says that an algorithm applies to a set of similar problems, which have the same question and solving procedure and differs only by the values of initial data. At the same time, the specific term “instance algorithm” is used in the textbooks to define an algorithm that solves only one case of a problem. Most part of the proposed instance algorithms are: algorithms of daily life (e.g., preparing meal), algorithms from others disciplines (e.g., geometrical constructions), algorithms implementing a

[2] For details about French system: http://eduscol.education.fr/cid66998/eduscol-the-portal-for-education-players.html

strategy (e.g., the wolf, goat, and cabbage problem). The expected competence is to represent the solution of a problem in an algorithmic way (as a sequence of instructions) rather than to solve it.

At **standard level**, many tasks concern the construction of algorithms for a given operator. The activity is centred on identifying the system of commands of an operator and writing an algorithm using only these commands. Although, most of the problems are quite easy and the goal of the tasks is to find the best strategy and present it in the required form. This concerns principally instance algorithms and refers to the *effectiveness* aspect. Generic algorithms (algorithms with many instances) mostly relate to solving mathematical problems (solving equations, evaluating the area of a polygons.) and computations (evaluation of simple expression). Pupils construct algorithms, describe them in the requested form (principally flowcharts) and execute them manually.

At **academic level**, Algorithmics is based on Delphi, an object-oriented programming language: pupils get accustomed to Algorithmics by learning the instructions of Delphi. 65% of the tasks are about writing programs, executing and modifying them, 19% of tasks are devoted to object-oriented programming. Algorithms are mostly verified by testing the programs. *Generic algorithms* are more present than *instance algorithms*. Most of them concern computations (e.g., evaluation of simple expressions) (33 %) and solving mathematical problems (e.g., solving equations, primality test) (31%). A bit less tasks concern the computation of sequences, products, sums (11%) and data treatment (e.g., sorting, searching in an array) (17 %). The textbook includes many tasks where an algorithm is only used for formulating a procedure before programming it. One can also found tasks where algorithms plays the role of *tool* for problem solving (e.g., finding the divisors of an integer). In this case, the focus is more on the construction of algorithms than on writing and debugging programs. Although we found many generic algorithms, only the *effectiveness* and *problem* aspects are strongly present. The *complexity* aspect is only evoked concerning the binary search algorithm.

At **professional** and **advanced levels**, an algorithm is presented not only as *tool* but also as an *object*. At **professional** level we found many “rich” algorithms, such as recursive algorithms, algorithms on graphs, algorithms of treatment of stacks and lists, etc. Expected competences for pupils are not only to understand some algorithms and write programs, but also to analyse algorithms’ efficiency and compare them. The *complexity* aspect is also evoked. At **advanced** level the *theoretical models* aspect is present (e.g., NP-complete problems). Expected competences concern the abilities to choose an algorithm appropriate to a problem, to compare algorithms according to their complexity, to analyse and compare algorithms. Algorithm is present as an object.

Algorithmics in high school in France

Programs of Mathematics for high school, for all levels, contain the same Algorithmics part, with a list of expectations for the end of high school: pupils must be able to “describe algorithms in natural and symbolic languages”, to “carry out some of them using a spreadsheet or a small program written in a calculator or a software” and “interpret more complex algorithms”. It is mentioned that “algorithmics has a natural place in all the mathematical subjects”. Pupils must learn elementary instructions, conditional instructions and loops. At each level, few specific algorithms are mandatory, e.g. plotting a curve (grade 10); solving equations of the type $f(x)=0$; simulating random experiments (grade 12, scientific path). Most of the algorithms in the programs deal with sequences, numerical methods and simulations in probability and statistics. Algebra and geometry are just mentioned as fields for algorithmic activities. Discrete Mathematics have a very little presence. The priority seems to be given to the implementation of algorithms in a programming language.

The **accompanying resource for grade 10** – that seems to have driven the approach to Algorithmics in high school (Modeste, 2012) – does not define the term *algorithm*, and does not even distinguish it from the term *program*. The activities are focused on language and rigorous expression of operations, and often aim at writing programs. It results in a confusion between *program* and *algorithm* that indicates a focus on the *effectiveness* aspect. A specific language to describe algorithms is implicitly developed, mixing pseudo-code and technical programming constraints – that we called *paper-program* (Modeste, 2012). Many instance algorithms are present, which confirms a confusion between writing algorithms and describing step-by-step operations.

In the studied textbooks for all levels, we can see the strong influence of this accompanying resource. Most algorithms are described as “paper-programs” before being implemented (generally as immediate translation). Many exercises deal with interpreting, writing or translating algorithms in a given language. Algorithm is only shown as a tool, even the problem aspect has little presence. In the program for the **ISN** option, the approach differs. The program explicitly defines the notion of algorithm and mentions that it must be distinguished from the notion of program. Algorithmics is presented as a branch of Informatics and algorithms are not restricted to programming. The concept of algorithm appears as a tool and as an object (*complexity* and *proof* aspects are present).

Comparison in high school

In Ukraine as in France, the effectiveness aspect is central. In all levels, algorithm is used as a tool, but the *problem* aspect is more developed in Ukraine. Algorithm is treated as an *object* only at professional and advanced levels in Ukraine, and in the ISN option in France. We could have expected them to appear in the French scientific paths but it is not the case. In Ukraine, the approach to Algorithmics seems to be guided by the development of algorithmic thinking whereas in France the focus is on the programming and language skills. It appears clearly in the texbooks: in Ukraine, the concept of algorithm is defined and a list of its properties is given, whereas, in France, an algorithm is defined by the language that describe it. Although in France, Algorithmics is taught in the Mathematics class, the focus on programming seems to be stronger than in Ukraine for standard level, where programming is not required and the focus is on elementary algorithmic thinking. In Ukraine, two features can be highlighted, probably inherited from historical context of teaching of Informatics in USSR: significant role of the scheme of problem solving (presumably influenced by the problem-solving theories) and the emphasis put on the notion of operator. In France, in Mathematics, algorithms are used to solve mathematical problems and are considered as a mean to deal with the mathematical concepts. The important presence of programs for simulations in probability or for embodying properties of mathematical objects attests to this point of view. The approach developed in ISN, in France, is close to the approach proposed at professional and advanced levels in Ukraine. They involve advanced concepts and aim at developing advanced algorithmic thinking, but we suspect a difference between the programs and the taught knowledge.

Comparison at middle school level (grades 6-9, ages 11-15)

Algorithmics in middle school in Ukraine

In the programs for grades 6 and 7, the part devoted to Algorithmics is similar to the program for standard level of high school. Although the program declares programming as one of the pupils' activities, it does not specify any programming language to use. In the textbooks for grades 6 and 7, an algorithm is defined as a finite sequence of instructions to be carried out for solving a problem. As we can see, in the given definition the effectiveness and problem aspects are on the first plan. At the same time, the fact that an algorithm solves all instances of a problem is not presented. The main part of proposed problems concerns the construction of instance algorithms for different operators. Both textbooks propose to program in Scratch. Pupils' activity is focused on developing programs and projects, using this programming environment. In grades 8 and 9, the Algorithmics part of the program is devoted to object-oriented programming. In grade 8, the notion of variable and different types of data are introduced. In grade 9, search algorithms in arrays are studied. The expected competences of pupils refer mostly to writing, modifying and debugging programs. The aspects related to algorithm as an object are not present.

Algorithmics in middle school in France

In 2016, Informatics appeared in middle school. Algorithmics contents essentially appeared in the Mathematics course, in the cycle 4 (grades 7-9), in the theme "Algorithmics and Programming". Textbooks for this reform were not available at the time of the study, so we only analysed the programs. One general competence guides the program: "write, elaborate and execute a simple program". Then, more specific competences are listed (decomposing a problem into sub-problems, designing a program to solve a problem; writing programs driven by events; and writing parallel programs) and contents are specified: notions of algorithm and program; variables in Informatics; event-driven action, sequence of instructions, loops, conditional instructions; exchanged messages between objects. These contents are strongly oriented towards programming and, even if it is not declared, the software Scratch must be used to teach these notions. The chosen approach implies developing projects and games (in order to develop pupils' reasoning) and does not focus only on mathematical concepts. Effectiveness aspect of algorithm is present and problem aspect is more notable than in the high school curricula. Contrary to the approach proposed in Mathematics in high school, Algorithmics is introduced by programming (independently from mathematical contents).

Comparison in middle school

In France and in Ukraine, in middle school, algorithm is presented as a tool. The *effectiveness* aspect is dominant. Although the *problem* aspect is mentioned, the role of algorithms for problem solving and the place for generic algorithms are not clear. At this level, the *complexity* and *proof* aspects are not proposed. Contrary to the curricula for high school, there are more similarities between the two curricula for middle school. Particularly, the notions of algorithm and program are distinguished; the introduction to Algorithmics includes event-driven programming in Scratch, and objects and variables are introduced later. The approach is based on solving concrete problems and developing projects in Informatics and in other disciplines. This could be explained by the influence of an international movement towards the teaching of Informatics in primary and middle school. Nevertheless, there are also important differences. In France, the most part of the proposed projects are in Mathematics (maybe because it will be taught by Mathematics teachers, not well trained yet in Informatics), and there is a strong focus on programming (in a different way from high school). In Ukraine, the notion of *operator* is still highlighted, representations of algorithms with schemes and flowcharts are requested and many examples of algorithms are taken from everyday life. This is directly inherited from the didactic transposition proposed in the 1980's and today in high school.

Conclusions and perspectives

On the one hand, this comparative analysis of the didactic transposition of the concept of algorithm and Algorithmics in secondary school in France and Ukraine brings out differences that reveal the impact of institutions, traditions and historical contexts on the curricula. The comparison of two contexts where Algorithmics is not a part of the same course (Mathematics versus Informatics) shows the influence of these disciplines on the contents, on the points of view on Algorithmics and on the algorithmic activity. On the other hand, in the two countries, we see general orientations in middle school that seem to be part of an international movement towards the teaching and learning of Informatics. This study contributes to understand and improve curricula, by taking into account the points of view of Mathematics and Informatics on Algorithmics. It gives perspectives to study the development of algorithmic thinking, and the teaching and learning of Algorithmics' concepts.

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