Geometric focusing of internal waves – A linear theory

Bruno Voisin*1

The conical geometry of three-dimensional monochromatic internal waves gives rise to self-focusing as soon as the wave generator exhibits horizontal curvature. Focusing is all the more pronounced as the generator is close to axisymmetric. A linear theory is proposed for generation by an oscillating circular annulus of vertical axis, with or without azimuthal variations. The theory assumes that the annulus is slender, namely of large aspect ratio, and exploits the associated separation between a local scale at which each cross section interferes only with the opposite section in the same azimuthal plane, and a global scale at which the annulus reduces to a circle. Two annuli are considered, one super-critical (a torus of circular cross section) and the other sub-critical (flat circular Gaussian topography at the ocean bottom). Both complete annuli are considered, and horseshoe-shaped annuli of which one half has been removed either abruptly (for the torus) or through a cosine-squared factor (for the topography). Focusing is observed in all cases, irrespective of criticality or completeness, implying that horizontal curvature is indeed the key ingredient. Good quantitative agreement is obtained for the torus with the laboratory measurements of Ermanyuk, Shmakova & Flór (to be submitted to JFM).

¹ Laboratoire des Écoulements Géophysiques et Industriels (LEGI) – CNRS, Université Grenoble Alpes – France

^{*}Speaker