

A LINEAR APPROACH OF INTERNAL WAVE FOCUSING

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Internal gravity waves in density-stratified fluids share with inertial waves in rotating fluids the property that, at a given oscillation frequency, propagation takes place at a fixed angle to the vertical or rotation axis. For annular forcing, such conical geometry gives rise to self-focusing of the waves on the axis, some distance away from the annulus. This mechanism, first noticed by Appleby & Crighton (*J. Fluid Mech.* 1987) and Simakov (*J. Fluid Mech.* 1993; *Wave Motion* 1994), has been re-discovered by Bühler & Muller (*J. Fluid Mech.* 2007), Grisouard & Bühler (*J. Fluid Mech.* 2012) and Le Dizès (*J. Fluid Mech.* 2015). In the ocean, owing to the horizontal curvature of the bottom topography, it is thought to account in part for the breaking and dissipation of internal tides, and ensuing mixing.

We propose an asymptotic theory of internal wave focusing for a circular annulus of vertical axis, major radius b and azimuthally varying minor radius $a(\varphi)$, on the assumption that the annulus is slender, namely of large aspect ratio $\varepsilon = b/a$. Each cross section admits of a two-dimensional representation, and two separate approximations are derived for the interference of the sections, valid at the global scale b of the annulus and local scale a of the section, respectively. Based on them, a uniform representation is proposed for the annulus and applied to two particular cases: a torus of circular cross section, and circular Gaussian topography at the ocean bottom. Both full axisymmetric and incomplete partial tori are considered, and both axisymmetric and horseshoe-shaped Gaussian topographies. The analysis is compared with the experiments of Ermanyuk, Shmakova & Flór (“Internal wave focusing by a horizontally oscillating torus”, submitted to *J. Fluid Mech.*) for the torus and with the ray calculations of Bühler & Muller and numerical simulations of Grisouard & Bühler for the topography.

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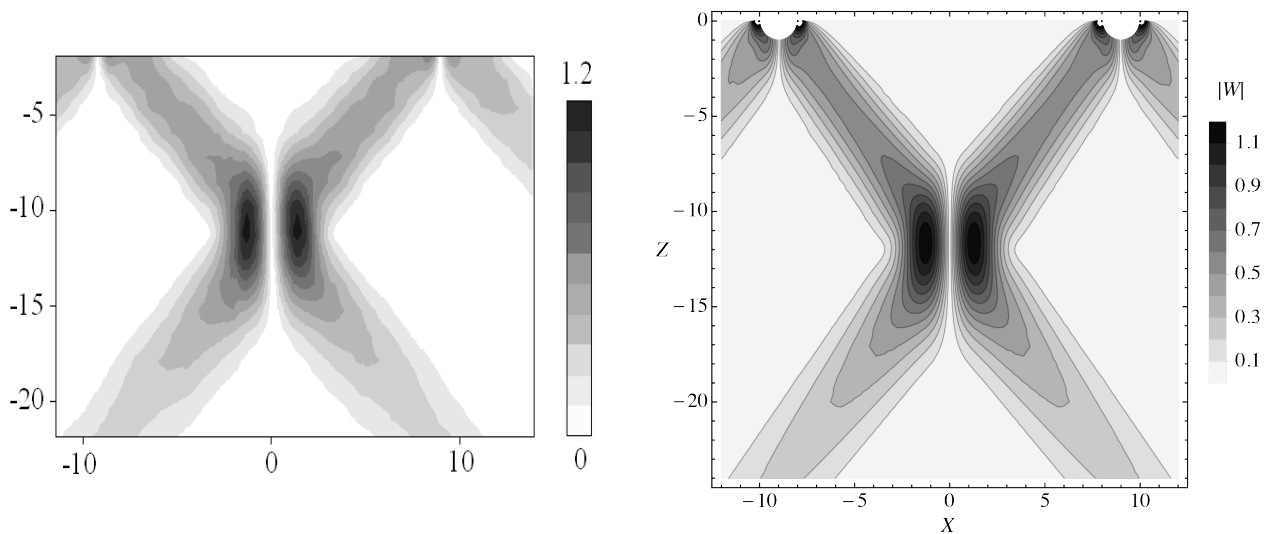


Figure 1: Vertical velocity amplitude in experiment (left, from Ermanyuk *et al.*) and theory (right) for a torus of aspect ratio $\varepsilon = 9$. The torus is of vertical z -axis and oscillates along the horizontal x -axis; its cross sections are visible at the top. Coordinates are non-dimensionalized based on the minor radius a and velocities based on the velocity of oscillation.