



HAL
open science

Elliptic flow fluctuations in central collisions of spherical and deformed nuclei

Giuliano Giacalone

► **To cite this version:**

Giuliano Giacalone. Elliptic flow fluctuations in central collisions of spherical and deformed nuclei. Phys.Rev.C, 2019, 99 (2), pp.024910. 10.1103/PhysRevC.99.024910 . hal-01937770

HAL Id: hal-01937770

<https://hal.science/hal-01937770>

Submitted on 6 Dec 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

Elliptic flow fluctuations in central collisions of spherical and deformed nuclei

Giuliano Giacalone

Institut de physique théorique, Université Paris-Saclay, CNRS, CEA, F-91191 Gif-sur-Yvette, France

(Received 16 November 2018; published 19 February 2019)

Elliptic flow (v_2) fluctuations in central heavy-ion collisions are direct probes of the fluctuating geometry of the quark-gluon plasma and, as such, are strongly sensitive to any deviation from spherical symmetry in the shape of the colliding nuclei. I investigate the consequences of nuclear deformation for v_2 fluctuations, and I assess whether current models of medium geometry are able to predict and capture such effects. Assuming linear hydrodynamic response between v_2 and the eccentricity of the medium, ε_2 , I perform accurate comparisons between model calculations of ε_2 fluctuations and STAR data on cumulants of elliptic flow in central Au+Au and U+U collisions. From these comparisons, I evince that the most distinct signatures of nuclear deformation appear in the non-Gaussianities of v_2 fluctuation, and I show, in particular, that the non-Gaussian v_2 fluctuations currently observed in central Au+Au collisions are incompatible with model calculations that implement a quadrupole coefficient of order 12% in the ^{197}Au nuclei. Finally, I make robust predictions for the behavior of higher order cumulants of v_2 in collisions of nonspherical nuclei.

DOI: [10.1103/PhysRevC.99.024910](https://doi.org/10.1103/PhysRevC.99.024910)

I. INTRODUCTION

Elliptic flow is the dynamical response of a fluid to an elliptic deformation of its geometry. It is a salient feature of the hydrodynamic expansion of the quark-gluon plasma created in relativistic heavy-ion collisions, where elliptic anisotropy is generated, on the one hand, by the fact that the area of overlap of two nuclei at finite impact parameter looks like an ellipse [1], which explains why elliptic flow grows quickly with the impact parameter of the collision, and on the other hand, by density fluctuations in the fluid [2,3], that explain the striking emergence of elliptic flow in collisions at small impact parameter [4]. Following Teaney and Yan [5], the elliptic anisotropy of the medium, dubbed ε_2 , can be defined rigorously for a generic heavy-ion collision, and hydrodynamic simulations show that elliptic flow (v_2) is essentially a linear response to ε_2 , i.e., $v_2 = \kappa \varepsilon_2$, where κ is a constant [6–8]. This simple relation implies that v_2 and its event-by-event fluctuations can be used as direct probes of the fluctuating geometry of the quark-gluon plasma at the beginning of the hydrodynamic phase.

In central collisions, where linear hydrodynamic response is exhibited at its best [9,10], an important source of fluctuations that contributes to ε_2 is given by the random orientation of the colliding nuclei, if they are nonspherical. This explains the large magnitude of the rms elliptic flow measured in central collisions of nuclei that have a pronounced deformation, i.e., U+U collisions at the BNL Relativistic Heavy Ion Collider (RHIC) [11] and Xe+Xe collisions at the CERN Large Hadron Collider (LHC) [12]. In the current modeling of initial conditions for hydrodynamics, the nonspherical shape of the nuclei is obtained by adding a quadrupole deformation in the wave functions of the colliding bodies, the implementation of which is performed with the guid-

ance of tabulated data on nuclear ground-state deformations [13,14]. This kind of modeling allows hydrodynamic simulations to reproduce quantitatively the aforementioned large rms elliptic flow observed in U+U [15] and Xe+Xe [16,17] collisions.

In this paper, I argue that more interesting and nontrivial signatures of nuclear deformation can be observed in the non-Gaussian fluctuations of v_2 , and that such effects are currently visible in the experimental data, in particular, in the fourth-order cumulant of elliptic flow $v_2\{4\}$, accurately measured by the STAR Collaboration [11] in central collisions of nonspherical nuclei, i.e., $^{197}\text{Au} + ^{197}\text{Au}$ collisions and $^{238}\text{U} + ^{238}\text{U}$ collisions. Note that the present study fills an important gap in the literature: Theoretical studies devoted to the consequences of the prolate shape of ^{238}U nuclei for relevant observables are numerous in the literature [15,18–28], but a careful assessment of the impact of nuclear deformation on the non-Gaussianities of v_2 fluctuations, which are central to the phenomenology of flow in heavy-ion collisions [29–32], is still missing.

I first investigate how nuclear deformation affects the fluctuations of eccentricity in Monte Carlo simulations and whether these effects help explain the experimental data. To achieve this, I use state-of-the-art Monte Carlo models of initial conditions (described in Sec. II) to perform extensive calculations of ε_2 fluctuations, that I rescale and compare (Sec. III) to STAR data on cumulants of elliptic flow fluctuations, $v_2\{2\}$ and $v_2\{4\}$, in central Au+Au and U+U collisions. The outcome of these comparisons is eventually used (Sec. IV) to explain the prominent consequences of nuclear deformation for the non-Gaussian fluctuations of elliptic flow, and I predict how these effects can be observed in the higher order cumulants of v_2 .

II. MODELS OF INITIAL GEOMETRY

In this section, I aim at exhibiting models of initial conditions that are viable for hydrodynamic simulations of Au+Au and U+U collisions at $\sqrt{s} = 200$ GeV. A model of initial conditions is a prescription that provides, event by event, the energy or entropy density, say $S(x, y)$, deposited in the transverse plane (for simplicity, at midrapidity) by a given collision. This is by far the most crucial ingredient in the hydrodynamic framework, because it provides the medium with its spatial anisotropies,¹ that get eventually converted into momentum anisotropies (i.e., the Fourier coefficients v_n) through the hydrodynamic evolution.

In this paper, I shall employ two models of initial conditions that correspond to two parametrizations of T_RENTo [28], a model for the profile of entropy density deposited at midrapidity in heavy-ion collisions. Let me provide a detailed explanation of these models.

A. T_RENTo initial conditions

The starting point is the modeling of the colliding bodies, which one performs through a random sampling of nucleons. For a generic nonspherical nucleus, the sampling is done using the following Fermi distribution [33],

$$\rho(x, y, z) = \frac{\rho_0}{1 + \exp\left\{-\frac{1}{a}r(-R[1 + \beta_2 Y_{20} + \beta_4 Y_{40}])\right\}}, \quad (1)$$

where $r = \sqrt{x^2 + y^2 + z^2}$, a and R are the skin depth and radius of the nucleus, respectively, and the coefficients β_2 and β_4 multiplying the spherical harmonics, $Y_{20} = \sqrt{\frac{5}{16\pi}}(3 \cos^2 \theta - 1)$ and $Y_{40} = \frac{3}{16\sqrt{\pi}}(35 \cos^4 \theta - 30 \cos^2 \theta + 3)$, are the coefficients of quadrupole and hexadecapole deformation of the nucleus, respectively. Once the transverse coordinates of the nucleons, (x, y) , are sampled, the nuclei are overlapped at a random impact parameter, and nucleon-nucleon collisions take place. One simply assumes that a participant nucleon deposits entropy, s , according to a Gaussian distribution,

$$s_i(x, y) = \frac{\omega_i}{2\pi\sigma^2} \exp\left[-\frac{(x - x_i)^2 + (y - y_i)^2}{2\sigma^2}\right], \quad (2)$$

where I take $\sigma = 0.5$ fm for the width of each participant, and the index i refers to the i th participant nucleon. The normalization, ω_i , is randomly distributed according to the following gamma distribution,

$$P(\omega) = \frac{k^k \omega^{k-1} e^{-\omega}}{\Gamma(k)}, \quad (3)$$

which has mean value equal to unity and variance proportional to k^{-1} . Hence, the total entropy profile of a given nucleus, say A , is given by

$$S_A(x, y) = \sum_i s_i. \quad (4)$$

¹Note that for the definition of the eccentricity of the medium, ε_2 , it does not matter whether one considers anisotropy in the energy density or in the entropy density of the fluid [6].

Eventually, for two colliding nuclei, A and B , the total entropy profile of the system is given by a generalized mean, i.e.,

$$S(x, y) = \left(\frac{S_A^p + S_B^p}{2}\right)^{1/p}, \quad (5)$$

where p is any real number.

In this paper, I shall use $S(x, y)$ from two different T_RENTo parametrizations. I take a geometric mean,

$$S = \left(\frac{S_A^p + S_B^p}{2}\right) \Big|_{p=0} = \sqrt{S_A S_B}, \quad (6)$$

and an arithmetic mean,

$$S = \left(\frac{S_A^p + S_B^p}{2}\right) \Big|_{p=1} = \frac{S_A + S_B}{2}. \quad (7)$$

Let me provide a bit of motivation for these choices.

The case $p = 0$ is very successful in phenomenological applications and is the favored value of p resulting from the extensive Bayesian analyses of Refs. [34,35]. The reason for this success is likely the fact that S in this model is proportional to the product $S_A S_B$. This is reminiscent of a class of models inspired by high-energy QCD [9,36,37]. Taking $S(x, y)$ as a product leads typically to systems whose eccentricity follows closely the almond shape of the nuclear overlap [38], leading eventually to a very good description of elliptic flow data [28,39].

The case with $p = 1$ corresponds instead to a wounded nucleon model with participant nucleon scaling [40]. This class of models are variations of the original Monte Carlo Glauber [33] model and are typically the models employed in experimental analyses.² Taking $S(x, y)$ as the sum of two components leads to systems that are more scattered in the transverse plane and that have less enhanced eccentricity [38]. The $p = 1$ model is essentially ruled out by elliptic flow fluctuations data in Pb+Pb collisions at the LHC [39]. Nevertheless, such a model has never been compared to RHIC data, and RHIC data was not used in the Bayesian analyses of Refs. [34,35]. I deem, then, that one can not *a priori* state that the $p = 1$ model is ruled out as well at RHIC energies.

The bottom line, and I shall come back to this point later, is that these two parametrizations of T_RENTo capture, arguably, all the basic features of the widest classes of initial condition models for nucleus-nucleus collisions that are on the market.

B. Multiplicity

I discuss now the implementation of the parameter k that regulates the fluctuations of the entropy produced by each participant nucleon. This feature is important for a correct

²For instance, the model calculations shown by the STAR Collaboration in Ref. [11], i.e., a Glauber–Monte Carlo model with binary collision scaling, and a constituent quark Glauber model, are essentially mild variations of the $p = 1$ model for what concerns the medium geometry and should lead to the same kind of ε_2 fluctuations. This will be confirmed in Sec. III.

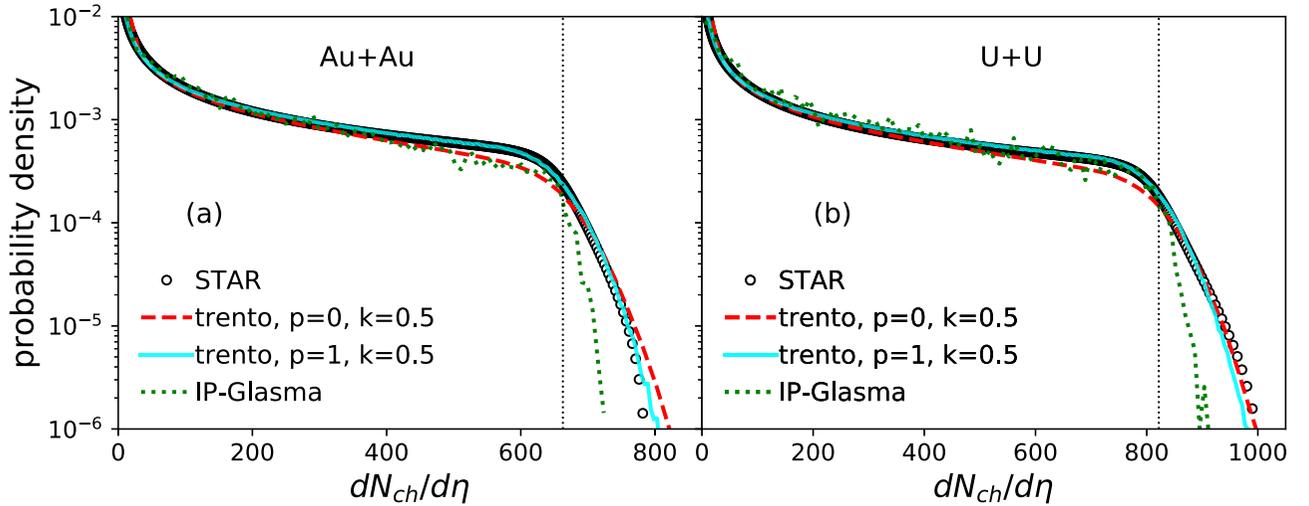


FIG. 1. Symbols: Distribution of multiplicity measured by the STAR Collaboration [11] in Au+Au collisions at $\sqrt{s} = 200$ GeV (a) and U+U collisions at $\sqrt{s} = 193$ GeV (b). Lines are model calculations: IP-Glasma simulations [26] (dotted line) and T_RENTo with $p = 1$ (solid line) and $p = 0$ (dashed line). The vertical lines indicate the knees of the histograms.

definition of collision centrality in the models. In experiments, centrality classes are defined from the histogram of the multiplicity of particles (or hits, or energy measured in a calorimeter). In hydrodynamics, one typically assumes that the total entropy at the initial condition is, in each event, proportional to multiplicity of particles in the final state. Therefore, in numerical simulations one uses the histogram of total entropy to define the classes of collision centrality. In order to exhibit meaningful model-to-data comparisons, I need to obtain in the models a distribution of total entropy which is in agreement with the distribution of $dN_{ch}/d\eta$ used by the STAR Collaboration [11] to sort their events into centrality bins. In particular, since I shall deal with central collisions, I want the high-multiplicity tails of the measured multiplicity distributions to be captured by the models. This can be achieved by a proper choice of the fluctuation parameter, k .

In Fig. 1, I display as circles the distributions of $dN_{ch}/d\eta$ in Au+Au and U+U collisions measured by the STAR Collaboration.³ Note that multiplicity distributions collected in the STAR detector present a high-multiplicity tail that is twice as broad as that measured in detectors at the LHC [41]. This is consistent with the fact that detectors at RHIC have a much smaller acceptance, and therefore they are more sensitive to statistical fluctuations. Since in T_RENTo the variance of fluctuations scales essentially as $1/k$, I expect that one needs to implement a lower value for the parameter k at RHIC than

at LHC. The choice of this parameter is, essentially, detector dependent.

Let me first discuss the case of the T_RENTo model with $p = 0$. In Ref. [16], this model is found to provide an excellent description of ALICE data when $k = 2.0$. To describe RHIC data, i.e., a large-multiplicity tail which is twice as broad, I use $k = 0.5$, which is smaller by a factor of 4, consistent with the fact that the width of the Γ distribution is proportional to $k^{-1/2}$. The probability distribution of the total entropy in this model is reported as a dashed line, for both Au+Au and U+U collisions, in Fig. 1. To show a meaningful comparison between the distribution of entropy provided by T_RENTo and the measured multiplicities, I rescale the entropy in T_RENTo in order to have the *knee* of the histogram at the same coordinate as the knee of the histogram of experimental data.⁴ Agreement with the data is reasonable,⁵ especially in the high-multiplicity tail, although the comparison is not as excellent as the one observed with LHC data.⁶

Moving to T_RENTo with $p = 1$, I find that also in this case an excellent description of data is achieved using $k = 0.5$. The rescaled distributions of entropy in this model are shown as solid lines in Fig. 1. Note that, for most of the histograms, the description provided by this model is better than that observed with $p = 0$.

³These distributions can be obtained from the parametrizations of $dN_{ch}/d\eta$ vs centrality (c) provided at the beginning of Ref. [11]. Knowing $dN/d\eta(c)$, and using the fact that centrality is defined as the cumulative of the multiplicity, i.e., $c(dN/d\eta) = \int_{dN/d\eta}^{\infty} P(dN/d\eta)$, the plots of $P(dN/d\eta)$ shown in Fig. 1 can be simply obtained by plotting $(\frac{d(dN/d\eta)}{dc})^{-1}|_c$ vs $dN/d\eta|_c$, with $c \in [0, 1]$.

⁴The knee is defined as the mean value of multiplicity at zero impact parameter, and for the experimental data I calculate it through the fitting procedure of Ref. [41]. From the STAR parametrizations, I find $dN_{ch}/d\eta|_{\text{knee}} = 663$ in Au+Au collisions, and $dN_{ch}/d\eta|_{\text{knee}} = 821$ in U+U collisions.

⁵I have also checked that the model presents the same centrality of the knee, c_{knee} , of data, i.e., the area of the histogram on the right of the knee. From the STAR parametrizations, I find $c_{\text{knee}} = 0.81\%$ in Au+Au collisions, and $c_{\text{knee}} = 0.75\%$ in U+U collisions.

⁶I have actually tried several values of k , and agreement does not get better.

I would like to expand a bit further on this point, which is very striking. The model with $p = 0$ allows one to accurately reproduce LHC multiplicity data [16], but here it yields a poor description of RHIC data at multiplicities that are smaller than the location of the knee of the histogram. At such multiplicities, one cannot simply improve agreement with data via a change in k : The model is simply unable to reproduce the data. Another model that is able to reproduce with great accuracy the distributions of multiplicity measured at LHC is the IP-Glasma model [36], which was employed in calculations of Au+Au and U+U collisions in Ref. [26]. Rescaled multiplicity distributions in this model are shown as dotted lines in Fig. 1. Note that they are essentially compatible with the curves of T_RENTo $p = 0$, and consequently, this model provides a rather bad description of Au+Au data before the knee of the histogram.⁷ Therefore, either the difference between multiplicity distributions at RHIC and at LHC is due to some unknown issue due to the different detectors used or there might actually be some physical motivation for which entropy production within IP-Glasma-like models (e.g., T_RENTo $p = 0$) provides a better description of data at the TeV energy scale. Moreover, let me emphasize that the same issue might jeopardize future Bayesian analyses that, using the T_RENTo model, are aimed at a determination of the parameters k and p via a simultaneous fit of RHIC and LHC data. My results here indicate that it is very unlikely that, using the same values of k and p , one may be able to obtain a good description of both RHIC and LHC data.

That being said: I have exhibited two T_RENTo parametrizations whose distributions of total entropy in Au+Au and U+U collisions provide a good description of the multiplicity distributions measured by the STAR Collaboration. Therefore, these are viable models of initial conditions for hydrodynamic calculations, and I use them to compute the eccentricity of the medium in each event, as discussed below.

C. Eccentricity fluctuations

The elliptic anisotropy of a smooth profile in two dimensions, for instance, the entropy density $S(x, y)$ given in each collision by the T_RENTo model, can be computed as indicated by Teaney and Yan [5] (in polar coordinates)⁸:

$$\mathcal{E}_2 = \varepsilon_2 e^{2i\Psi_2} = -\frac{\int r^2 e^{i2\phi} S(r, \phi) r dr d\phi}{\int r^2 S(r, \phi) r dr d\phi}. \quad (8)$$

This complex quantity fluctuates in both magnitude (ε_2) and orientation (Ψ_2) in each event. Linear hydrodynamic response implies that \mathcal{E}_2 is linearly correlated with the complex elliptic

flow coefficient, $V_2 = v_2 e^{2i\Phi_2}$, that also fluctuates in magnitude (v_2) and orientation (Φ_2) event to event. If this is the case, the probability distribution of V_2 coincides then with that of \mathcal{E}_2 up to a factor, and the statistical properties of V_2 fluctuations provide direct information about the fluctuations of the initial \mathcal{E}_2 [43]. Let me recall the formulas of the first two cumulants of the v_2 distribution,

$$v_2\{2\} = \sqrt{\langle v_2^2 \rangle}, \quad v_2\{4\} = 4\sqrt{2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle}, \quad (9)$$

where brackets indicate an average over events in a centrality bin. Now, if $v_2 = \kappa \varepsilon_2$, then,

$$v_2\{2\} = \kappa \varepsilon_2\{2\}, \quad v_2\{4\} = \kappa \varepsilon_2\{4\}. \quad (10)$$

The ratio $v_2\{4\}/v_2\{2\}$ has been used in many studies to observe, in hydrodynamic simulations, the transition between the linear regime, where $v_2\{4\}/v_2\{2\} = \varepsilon_2\{4\}/\varepsilon_2\{2\}$, and the nonlinear regime where this equality breaks down [39,43–45]. In Ref. [39], in particular, the ratio $\varepsilon_2\{4\}/\varepsilon_2\{2\}$ was computed in extensive Monte Carlo calculations of ε_2 in very central collisions, and directly compared to LHC Pb+Pb data, in order to test the validity of different T_RENTo parametrizations.

In what follows, I repeat this game, although without taking any ratio: I compute $\varepsilon_2\{2\}$ and $\varepsilon_2\{4\}$ in the T_RENTo models, and then rescale them by an appropriate factor κ , in order to find the best possible agreement with experimental data on $v_2\{2\}$ and $v_2\{4\}$ in central Au+Au and U+U collisions. As anticipated, eccentricity fluctuations in central collisions are strongly sensitive to the event-by-event fluctuations of the spatial orientation of the colliding ¹⁹⁷Au and ²³⁸U nuclei, i.e., to the choice of the deformation parameters that enter in Eq. (1). The goal of performing Monte Carlo calculations and model-to-data comparisons is essentially twofold. First, I want to understand how ε_2 fluctuations are affected by the presence of deformed nuclei in the models. Second, I want to check whether these effects predicted by the models are in agreement with the features of v_2 fluctuations observed in the data.

III. COMPARISON WITH STAR DATA

A. Setup

Using the T_RENTo parametrizations with $p = 0$, $k = 0.5$, and $p = 1$, $k = 0.5$, I simulate Au+Au and U+U collisions at $\sqrt{s} = 200$ GeV. The nuclear shape parameters that enter in Eq. (1) are implemented as follows: I take ²³⁸U nuclei with $R = 6.80$ fm, $a = 0.60$ fm, $\beta_2 = 0.236$, and $\beta_4 = 0.098$. Concerning ¹⁹⁷Au nuclei, I use $R = 6.40$ fm and $a = 0.53$ fm, and I shall run calculations for both spherical nuclei, i.e., $\beta_2 = \beta_4 = 0$, and deformed nuclei with $\beta_2 = -0.125$, $\beta_4 = -0.017$,

A couple of comments are in order. The chosen values of radii and skin depths are rather standard. I do not follow the suggestion of Ref. [27] and do not set $a \sim 0.4$ in both ²³⁸U and ¹⁹⁷Au nuclei. As suggested by the results shown in that same reference, this choice has essentially no impact on eccentricity fluctuations in central collisions. Moving on to the deformation parameters, for ²³⁸U nuclei I implement a value of β_2 that is smaller than in previous studies, as I

⁷Note that the IP-Glasma results shown in Fig. 1 present as well a large-multiplicity tail which is twice as steep as the STAR data, suggesting that they miss the detector-dependent part of the fluctuations of multiplicity. In the IP-Glasma formalism, this issue could be solved via the inclusion of fluctuations of the saturation scale at the level of the colliding nucleons [42].

⁸The expression for the anisotropy of order 3 (triangularity) is completely analogous [5].

take it from the most recent table of nuclear deformations [14]. Moving to the deformation of ^{197}Au nuclei, I cautiously stress that the parameter β_2 for this nucleus is not a measured quantity. The data tables resulting from the model calculations of Refs. [13,14] yield $|\beta_2| \sim 0.13$, whereas, in Ref. [46], a value of 0.15 is guessed from the measured deformations of neighbor nuclei. My choice $\beta_2 = -0.125$ is taken again from Ref. [14].

For these T_RENTo setups ($p = 0$ and $p = 1$), and using both spherical and deformed ^{197}Au nuclei, I simulate $\mathcal{O}(10^7)$ U+U and Au+Au collisions, and I compute the fluctuations of ε_2 as function of collision multiplicity that I compare to experimental data.

When rescaling the results for ε_2 fluctuations, I shall take the same value of κ [Eq. (10)] for both Au+Au and U+U systems: This is a good approximation, as the value of κ is reduced solely by viscous corrections. Dimensional analysis indicates that viscous corrections scale like $1/R$, where R is the radius of the system. A good approximation for R is given by $A^{-1/3}$. Therefore, moving from U+U collisions to (smaller) Au+Au collisions, I expect a reduction of κ of order $3\sqrt{238/197}$, which is a negligible 5% correction. Let me also stress that, in view of this argument concerning the system size, the value of κ is expected to decrease as I move from central to peripheral collisions. This effect should be very small in the centrality range I am interested in, as also indicated by hydrodynamic calculations [10]; therefore, I simply assume that κ is a constant.

B. Model vs data

I show STAR data [11] on cumulants of elliptic flow fluctuations as empty symbols in Fig. 2. Figures 2(a) and 2(b) show STAR data in Au+Au collisions, whereas Figs. 2(c) and 2(d) show U+U collisions. In each panel, the minimum multiplicity corresponds to roughly 20% centrality, whereas the maximum multiplicity is around 0.1% centrality.

1. Spherical ^{197}Au nuclei

I start by showing results from the T_RENTo calculations implementing spherical ^{197}Au nuclei. These results are reported as full symbols in Fig. 2.

$p = 0$: The calculations using T_RENTo with $p = 0$ are shown as red symbols in Figs. 2(a) and 2(c). The value of the constant $\kappa = 0.165$ is chosen in order to provide the most accurate possible description of the cumulant $v_2\{4\}$ in Au+Au collisions [Fig. 2(a)]. I choose to tune the model to $v_2\{4\}$ because 4-particle cumulants are not affected by non-flow contributions [32]. The agreement between the model and the data in Au+Au collisions in Fig. 2(a) is excellent. The description of $v_2\{4\}$ provided by this T_RENTo parametrization is perfect. It captures the trend of the data all the way up to 20% centrality, and it correctly reproduces the change of sign of the cumulant $v_2\{4\}^4$ observed around 2.5% centrality in experiment. Agreement is equally impressive for $v_2\{2\}$, all the way up

to 20%. The slight shift of the experimental data toward larger values with respect to the model is easily understandable as due to nonflow. Indeed, the nonflow subtraction performed by the STAR Collaboration is not perfect, as two-particle correlations were calculated implementing a small gap of 0.1 units of pseudorapidity in the analysis [11]. Moving to the red symbols shown in Fig. 2(c), the description of U+U data provided by this model is overall good, but less satisfactory. I see that, although the comparison with data is not as quantitatively as good as in Au+Au collisions, the model captures nicely the qualitative features due to the deformation of ^{238}U nuclei: $v_2\{2\}$ is larger in U+U than in Au+Au and, for reasons that I shall discuss in detail in Sec. IV, $v_2\{4\}$ is observed to be nonzero all the way to the highest multiplicities.

$p = 1$: I look now at the red symbols shown in Figs. 2(b) and 2(d), where I show results for T_RENTo with $p = 1$. Again, the value of $\kappa = 0.27$ is chosen such to yield the best description of $v_2\{4\}$ in Au+Au collisions.⁹ Starting with Au+Au collisions in Fig. 2(b), I see that the description of experimental data provided by this T_RENTo parametrization is much worse than for the $p = 0$ case: $v_2\{2\}$ is overestimated and $v_2\{4\}$ reaches zero at too large centrality with respect to the experimental data. This is consistent with the comparisons between this model and LHC data shown in Ref. [39]. Agreement with data is also bad if I look at U+U collisions in Fig. 2(d), where a much smaller value of κ would be needed to match the experimental data. The qualitative features of U+U collisions are, on the other hand, reasonably captured, in particular, the fact that the cumulant $v_2\{4\}$ is nonzero all the way to the highest multiplicity.

2. Deformed ^{197}Au nuclei

I discuss now the results obtained in T_RENTo with deformed ^{197}Au nuclei presenting $\beta_2 = -0.125$ and $\beta_4 = -0.017$ [14]. These results are reported as lines in Fig. 2.

$p = 0$: The results for T_RENTo with $p = 0$ are the green lines in Figs. 2(a) and 2(c). The constant κ that I need to tune the model to $v_2\{4\}$ in Au+Au collisions turns out to be smaller than previously with spherical ^{197}Au nuclei, since both cumulants increase if I implement $\beta_2 > 0$. Now, in Fig. 2(a) I see that the description of $v_2\{2\}$ is still very good all the way up to 20% centrality, and so is the description of $v_2\{4\}$ essentially above 5% centrality. But switching on the nuclear deformation has a dramatic effect of the fourth-order cumulant: It prevents $v_2\{4\}^4$ from going negative, and one

⁹Note that the coefficient κ varies a lot moving from $p = 0$ to $p = 1$. This is due to the fact that the eccentricity grows much faster with centrality for $p = 0$ than for $p = 1$.

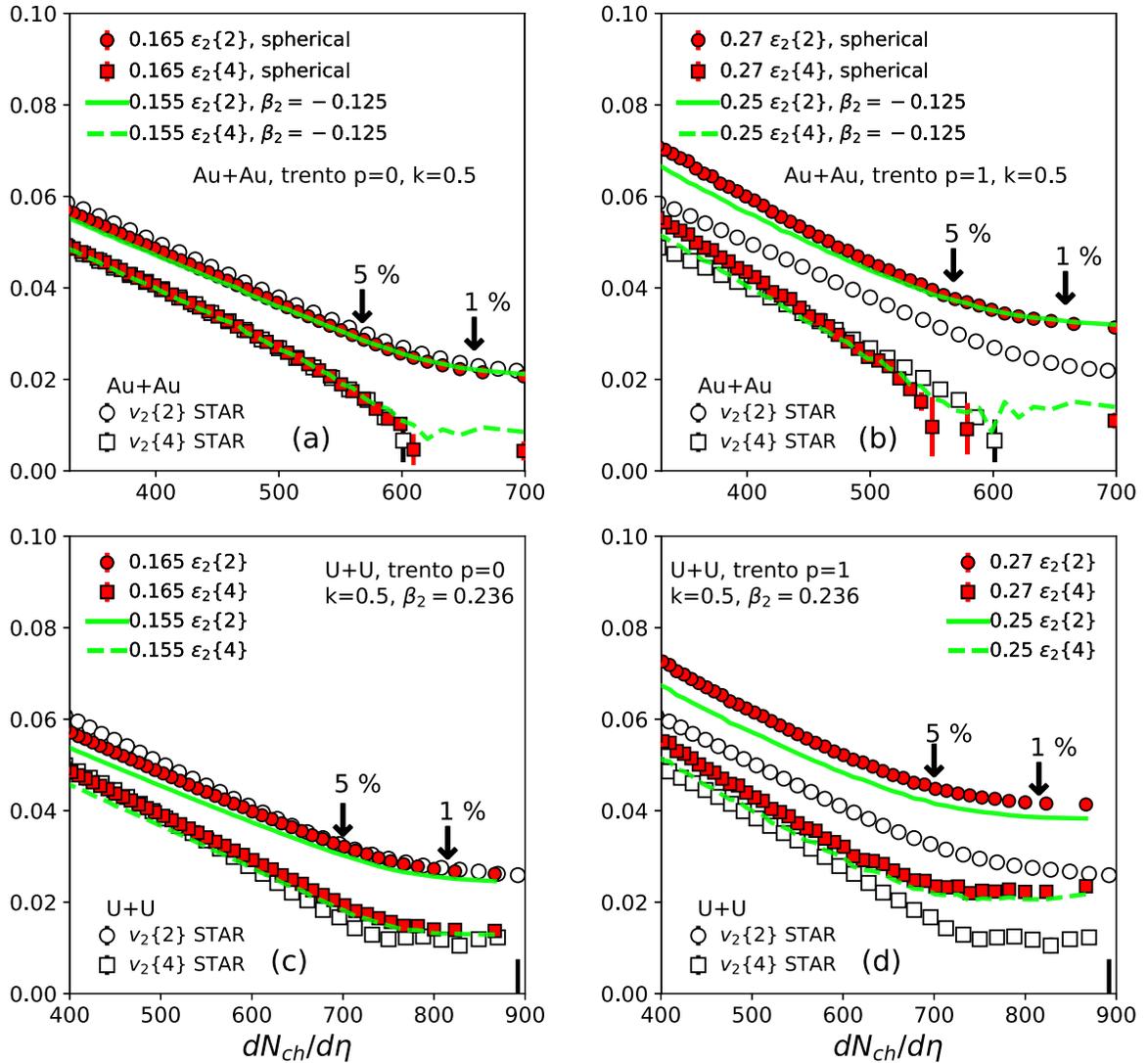


FIG. 2. Empty symbols: Cumulants of flow fluctuations measured by the STAR Collaboration in central Au+Au collisions at $\sqrt{s} = 200$ GeV [panels (a) and (b)] and central U+U collisions at $\sqrt{s} = 193$ GeV [panel (c) and (d)], as function of collision multiplicity. Full symbols: Cumulants of initial eccentricity fluctuations in the T_RENTo setup that implements spherical ^{197}Au nuclei, with $p = 0$ [panels (a) and (c)] and $p = 1$ [panels (b) and (d)]. Lines: Cumulants of initial eccentricity fluctuations in the T_RENTo setup that implements deformed ^{197}Au nuclei, with $p = 0$ [panels (a) and (c)] and $p = 1$ [panels (b) and (d)]. The arrows indicate the correspondence between multiplicity and collision centrality.

observes a nonzero $v_2\{4\}$ all the way to the highest multiplicity, much as in the case of U+U collisions. Nothing notable occurs for U+U collisions in Fig. 2(c).

$p = 1$: As for the results with $p = 1$ with deformed ^{197}Au nuclei, i.e., the lines shown in Figs. 2(b) and 2(d), I would simply like to point out is that I find the same striking result observed in Fig. 2(a): $v_2\{4\}$ is always nonzero in Au+Au collisions.

C. Discussion

Let me draw my conclusions from the comparisons shown in Fig. 2.

First, I confirm that the $p = 1$ model is ruled out by elliptic flow fluctuations data at RHIC. This is not a surprise: First, this model is currently ruled out by LHC data [39]; Second, as anticipated, this model is similar to the models used by the STAR Collaboration in their analysis, and those models provide a bad description of data, in the sense that they present a κ that varies with centrality even at the highest multiplicities.

Second, my results imply that a value $\beta_2 \sim -0.12$ in ^{197}Au nuclei is essentially ruled out by experimental data. The description of data achieved with the T_RENTo $p = 0$ parametrization implementing spherical ^{197}Au nuclei is remarkably good and is spoiled by the inclusion of the quadrupole parameter reported in the literature. Note that this is not a small effect, but on the contrary, it is a very visible change in the fourth-order cumulant that does not reach zero

even in the most central collisions, as clearly observed in STAR data. This occurs for both $p = 0$ and $p = 1$. I would like to stress, once more, that the analysis reported here is exhaustive, in the sense that one cannot easily argue that negative values of $v_2\{4\}^4$ can be obtained with $\beta_2 \sim -0.12$ in a different model setup. As an example, a slightly different class of models are those models that can be fitted by a negative p in T_RENTo, for instance, the Monte Carlo KLN model [47] or the Monte Carlo rcBK [48] model. But there the problem would remain, as these models present both a larger eccentricity [28] and a larger $\varepsilon_2\{4\}$ [39] than the $p = 0$ case.

The bottom line of this section is that the most notable effects of nuclear deformation are visible in $v_2\{4\}$ rather than in $v_2\{2\}$. The typical statement is that nuclear deformation yields larger fluctuations of v_2 and thus a larger $v_2\{2\}$. My analysis clearly indicates that experimental data are now precise enough to show distinct signatures of nuclear deformation in the details of the flow fluctuations: $v_2\{4\}^4$ in positive U+U collisions and negative in Au+Au collisions. Note that LHC data seems to point at the same phenomenon. Indeed, preliminary ATLAS data show that $v_2\{4\}$ in Xe+Xe collisions is positive, all the way down to the most central events [49]. This is in contrast with Pb+Pb data, where a change of sign of the fourth-order cumulant of elliptic flow is currently observed [50], and ^{208}Pb nuclei are perfectly spherical.¹⁰

How is it, then, that $v_2\{4\}$ is so sensitive to the deformed nuclear shapes?

IV. NON-GAUSSIAN FLUCTUATIONS

Flow fluctuations are to a good approximation Gaussian, in the sense that the elliptic flow vector (v_x, v_y) has a two-dimensional Gaussian distribution in a given centrality bin. The breakdown of Gaussian behavior leaves observable consequences in the higher order cumulants of elliptic flow. I refer to Refs. [52–54] for exhaustive calculations explaining how the non-Gaussian nature of fluctuations manifests in higher order cumulants and to the detailed analyses of Refs. [29–32] for experimental confirmations of such results.

In this section, I show that nuclear deformation can yield pronounced non-Gaussianity in the event-by-event distribution of elliptic flow, and I discuss the observable consequences of such an effect.

A. Deformation as a source of non-Gaussianity

Let me start by looking at the sensitivity of the cumulants $\varepsilon_2\{2\}^2$ and $\varepsilon_2\{4\}^4$ under variations of the parameter β_2 , using the T_RENTo model with $p = 0$ and $k = 0.5$. I simulate U+U collisions at zero impact parameter, $b = 0$, and I systematically vary the quadrupole deformation of the nuclei. Note that colliding at $b = 0$ implies that $\varepsilon_2\{2\}^2$ is equal to the variance of the distribution of eccentricity, whereas $\varepsilon_2\{4\}^4$ measures the

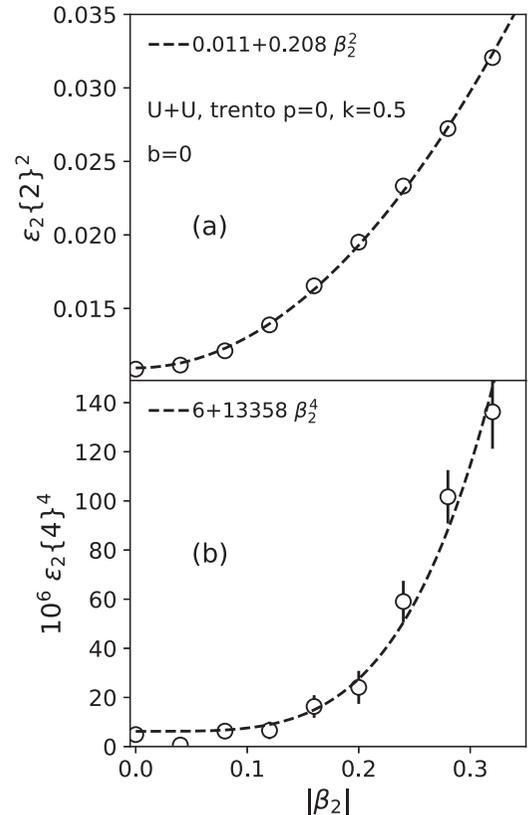


FIG. 3. Cumulants of eccentricity fluctuations in U+U collisions at zero impact parameter. The cumulants are plotted as function of the quadrupole coefficient, β_2 . I simulate collisions using the T_RENTo model with $p = 0$ and $k = 0.5$. (a) Variance of eccentricity fluctuations. (b) Kurtosis of eccentricity fluctuations. Lines are polynomial fits.

kurtosis, i.e., whether the distribution has heavier or lighter tails than a Gaussian. In particular, at $b = 0$ one expects $\varepsilon_2\{4\}^4 = 0$ if the distribution of the eccentricity vector is a two-dimensional Gaussian.

In Fig. 3(a), I display $\varepsilon_2\{2\}^2$ as function of β_2 for collisions at zero impact parameter. By the symmetry properties of the spherical harmonic multiplying β_2 in Eq. (1), any effect that is linear in β_2 should cancel when averages over events are taken. Therefore, the value of $\langle \varepsilon_2^2 \rangle$ is expected to grow with the square of β_2 . To check this, I perform a parabolic fit of $\varepsilon_2\{2\}^2$, reported as a dashed line in Fig. 3(a). The fit is of excellent quality, and it shows how the variance of the distribution varies with the quadrupole coefficient.

Figure 3(b) shows instead $\varepsilon_2\{4\}^4$ as function of β_2 . I note a great enhancement of this quantity with increasing β_2 . This implies that nuclear deformation does not simply make the distribution of eccentricity broader, but also less Gaussian, as it makes the kurtosis grow by essentially orders of magnitude. The dashed line is a quartic fit, which again confirms the symmetry argument.

The previous result is very intuitive. The kurtosis enters in $\varepsilon_2\{4\}^4$ with a negative sign [54], which means that nuclear deformation makes the kurtosis of ε_2 fluctuations less negative. Negative kurtosis for an azimuthally symmetric eccentricity

¹⁰I should stress that the precise mechanism that leads to the change of sign of this cumulant in Pb+Pb collisions is not understood yet. It seems to be a generic feature of systems presenting small eccentricity driven by impact parameter fluctuations [51].

distribution can be simply understood as a consequence of the fact that the eccentricity is bounded by unity [55]. Nuclear deformation, then, causes eccentricity fluctuations to get closer to their bound, and thus to present more negative kurtosis.

I conclude that nuclear deformation yields a slight increase in the variance of elliptic flow fluctuations and a dramatic increase in their kurtosis. This explains intuitively why $v_2\{4\}^4$ is much larger in U+U collisions than in Au+Au collision, provided, as experimental data seem to suggest, that ^{197}Au nuclei are much more spherical than ^{238}U nuclei.

In the following, I argue that this phenomenon has robust, observable consequences that go beyond the simple positive value of $v_2\{4\}$ in central collisions.

B. Higher order cumulants

The bottom line of the previous discussion is that nuclear deformation yields stronger non-Gaussianity in the distribution of v_2 . Here I discuss the implication of this phenomenon for the splitting between higher order cumulants of elliptic flow.

The third cumulant of elliptic flow is of order 6 and is equal to

$$v_2\{6\} = 6\sqrt{\frac{1}{4}((v_2^6) - 9(v_2^2)(v_2^4) + 12(v_2^2)^3)}. \quad (11)$$

Again, in the regime of linear hydrodynamic response, one can write

$$v_2\{6\} = \kappa \varepsilon_2\{6\}, \quad (12)$$

that, using Eq. (10), leads to [39,56]

$$\frac{v_2\{6\}}{v_2\{4\}} = \frac{\varepsilon_2\{6\}}{\varepsilon_2\{4\}}. \quad (13)$$

For collisions of spherical nuclei, e.g., ^{208}Pb nuclei, the ratio $v_2\{6\}/v_2\{4\}$ is very close to unity (~ 0.99) in noncentral collisions [29,30]. This is because the probability distribution of elliptic flow, and consequently of the initial eccentricity, is well approximated by a two-dimensional Gaussian. Gaussian fluctuations imply that all higher order cumulants of ε_2 are equal to the mean value of the eccentricity projected along the reaction plane [57], which I dub, in a standard notation, ε_{RP} ,¹¹

$$\varepsilon_2\{4\} \approx \varepsilon_2\{6\} \approx \varepsilon_{\text{RP}}. \quad (15)$$

This is the famous degeneracy of cumulants of the eccentricity and of elliptic flow. Departure from Gaussian behavior is expected starting from semicentral collisions, due to the negative skewness of ε_2 fluctuations in the reaction plane [52], as recently confirmed by LHC data in Pb+Pb collisions [30,31].

¹¹Following Eq. (8), the mean eccentricity along the direction of the reaction plane, which is customarily taken as the x axis, is given by

$$\varepsilon_{\text{RP}} = \left\langle \frac{\int r^2 \cos 2\phi S(r, \phi) r dr d\phi}{\int r^2 S(r, \phi) r dr d\phi} \right\rangle, \quad (14)$$

where the average is over events.

I test the validity of Eq. (15) in the T_RENTo model with $p = 0$. I show in Fig. 4(a) the values of $\varepsilon_2\{4\}$ and ε_{RP} in central Au+Au and U+U collisions. I choose Au+Au collisions implementing spherical nuclei, in order to highlight how results change between collisions of spherical nuclei and collisions of nonspherical nuclei. For Au+Au collisions, I see, as expected, that $\varepsilon_2\{4\}$ is essentially equal to ε_{RP} already at 5% centrality. This is the onset of Gaussian fluctuations. Moving on to U+U collisions, I observe that the splitting between $\varepsilon_2\{4\}$ and ε_{RP} is much larger than in Au+Au collisions. This nicely illustrates how this cumulant, in the most central events, becomes fully dominated by the enhancement of the kurtosis of the distribution, which is due to the fluctuations of the orientation of the colliding nuclei. In these collisions, then, I do not observe any onset of Gaussian fluctuations, at least up to 10% centrality. I conclude that nuclear deformation breaks the degeneracy of cumulants, Eq. (15), in central events.

Therefore, the robust, model-independent prediction I can easily draw is the following: In collisions of deformed nuclei, I expect a large splitting between $v_2\{4\}$ and $v_2\{6\}$ between $\sim 5\%$ centrality and semicentral collisions. In Fig. 4(b), I show predictions for the ratio $v_2\{6\}/v_2\{4\}$, up to 20% centrality. Note that for U+U collisions, Gaussian fluctuations, i.e., $v_2\{6\}/v_2\{4\} \sim 0.99$, are not observed below 15% centrality. This confirms my expectations: The ratio is significantly lower in U+U collisions than in Au+Au collisions and the effect is very visible. Experimental verification of this feature would provide additional confirmation of the great robustness of the hydrodynamic modeling.

Note that, from the inset in Fig. 4(b), I also predict that $v_2\{6\}^6$, much as $v_2\{4\}^4$, is positive in U+U collisions all the way to the ultracentral events.

V. CONCLUSIONS

I have shown that the largest effects of nuclear deformation are hidden into the tails of the distribution of elliptic flow in central collisions. Nuclear deformation yields broader distribution of elliptic flow, and, for central collisions, it engenders the mechanism of enhanced negative kurtosis presented in Sec. IV, which allows me to predict and explain the behavior of the higher order cumulants of v_2 . I have thus explained in very simple terms the striking observation that $v_2\{4\}^4$ is much larger in U+U collisions than in Au+Au collisions and predicted that the splitting between $v_2\{6\}$ and $v_2\{4\}$ is larger in collisions of deformed nuclei.

My results imply that there is no room in the experimental data for a β_2 of order 0.12 in the ^{197}Au nuclei, as it would lead to too large values of the cumulant $v_2\{4\}^4$ in a model-independent way. Clearly, one would rather trust the numbers provided in established nuclear physics literature, instead of the crude models of initial conditions for heavy-ion collisions. But, arguably, this would just amount to rejecting evidence. Signatures of nuclear deformation are observed in elliptic flow data in U+U collisions and, more remarkably, in collisions of ^{129}Xe nuclei that present a β_2 of order 16%, which is very close to the reported 12% of ^{197}Au nuclei. As I have shown in this paper, in the hydrodynamic framework one can understand all these observations as simply due to generic and

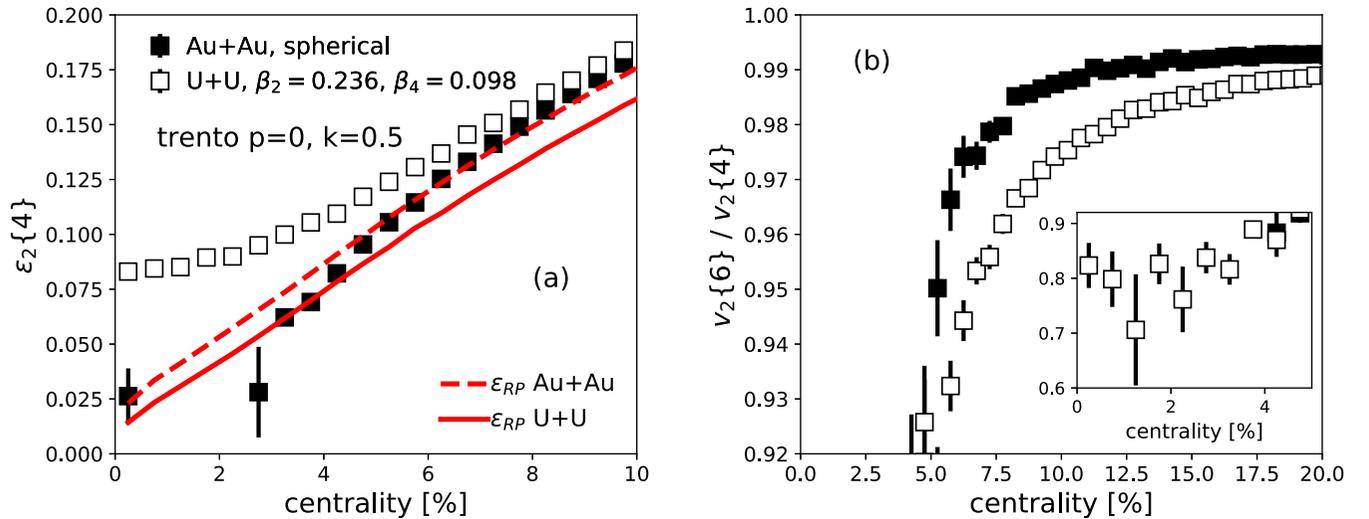


FIG. 4. (a) $\varepsilon_2\{4\}$ in central Au+Au (full symbols) and U+U (empty symbols) collisions in the $T_{\text{r}}\text{ENTo}$ model with $p = 0$. Lines indicate the mean eccentricity in the reaction plane, ε_{RP} , in both Au+Au (dashed line) and U+U (solid line) collisions. (b) Model predictions for the ratio $v_2\{6\}/v_2\{4\}$ as function of centrality percentile, in central Au+Au (empty symbols) and U+U (full symbols) collisions. The inset is an enlargement below 5% centrality.

model-independent features of the fluctuations of the initial geometry. Therefore, any such effects should show up as well in Au+Au data.

This is an interesting puzzle, that, I think, nicely underlines the close link between flow fluctuations in central heavy-ion collisions, whose origin is purely geometric, and the actual shape and structure of the colliding nuclei. This may lead to interesting developments in the future, aimed at matching these two very different areas of nuclear physics. It would be useful, for instance, to collide nuclei

whose structure and deformation is precisely determined experimentally.

ACKNOWLEDGMENTS

I thank Jean-Yves Ollitrault for useful input and illuminating discussions. I thank Björn Schenke and Prithwish Tribedy for providing me with the results of IP-Glasma simulations and for useful comments on the manuscript. I also thank Jamie Nagle for useful comments on the first draft of this paper.

- [1] J. Y. Ollitrault, *Phys. Rev. D* **46**, 229 (1992).
- [2] B. Alver, B. B. Back, M. D. Baker, M. Ballintijn, D. S. Barton, R. R. Betts, R. Bindel, W. Busza, V. Chetluru, E. Garcia *et al.*, *Phys. Rev. C* **77**, 014906 (2008).
- [3] J. P. Blaizot, W. Broniowski, and J. Y. Ollitrault, *Phys. Lett. B* **738**, 166 (2014).
- [4] B. Alver *et al.* (PHOBOS Collaboration), *Phys. Rev. Lett.* **98**, 242302 (2007).
- [5] D. Teaney and L. Yan, *Phys. Rev. C* **83**, 064904 (2011).
- [6] F. G. Gardim, F. Grassi, M. Luzum, and J. Y. Ollitrault, *Phys. Rev. C* **85**, 024908 (2012).
- [7] H. Niemi, G. S. Denicol, H. Holopainen, and P. Huovinen, *Phys. Rev. C* **87**, 054901 (2013).
- [8] F. G. Gardim, J. Noronha-Hostler, M. Luzum, and F. Grassi, *Phys. Rev. C* **91**, 034902 (2015).
- [9] H. Niemi, K. J. Eskola, and R. Paatelainen, *Phys. Rev. C* **93**, 024907 (2016).
- [10] J. Noronha-Hostler, L. Yan, F. G. Gardim, and J. Y. Ollitrault, *Phys. Rev. C* **93**, 014909 (2016).
- [11] L. Adamczyk *et al.* (STAR Collaboration), *Phys. Rev. Lett.* **115**, 222301 (2015).
- [12] S. Acharya *et al.* (ALICE Collaboration), *Phys. Lett. B* **784**, 82 (2018).
- [13] P. Moller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, *Atom. Data Nucl. Data Tabl.* **59**, 185 (1995).
- [14] P. Möller, A. J. Sierk, T. Ichikawa, and H. Sagawa, *Atom. Data Nucl. Data Tabl.* **109–110**, 1 (2016).
- [15] A. Goldschmidt, Z. Qiu, C. Shen, and U. Heinz, *Phys. Rev. C* **92**, 044903 (2015).
- [16] G. Giacalone, J. Noronha-Hostler, M. Luzum, and J. Y. Ollitrault, *Phys. Rev. C* **97**, 034904 (2018).
- [17] K. J. Eskola, H. Niemi, R. Paatelainen, and K. Tuominen, *Phys. Rev. C* **97**, 034911 (2018).
- [18] P. F. Kolb, J. Sollfrank, and U. W. Heinz, *Phys. Rev. C* **62**, 054909 (2000).
- [19] U. W. Heinz and A. Kuhlman, *Phys. Rev. Lett.* **94**, 132301 (2005).
- [20] A. J. Kuhlman and U. W. Heinz, *Phys. Rev. C* **72**, 037901 (2005).
- [21] A. Kuhlman, U. W. Heinz, and Y. V. Kovchegov, *Phys. Lett. B* **638**, 171 (2006).
- [22] C. Nepali, G. Fai, and D. Keane, *Phys. Rev. C* **73**, 034911 (2006).

- [23] H. Masui, B. Mohanty, and N. Xu, *Phys. Lett. B* **679**, 440 (2009).
- [24] T. Hirano, P. Huovinen, and Y. Nara, *Phys. Rev. C* **83**, 021902 (2011).
- [25] M. Rybczyński, W. Broniowski, and G. Stefanek, *Phys. Rev. C* **87**, 044908 (2013).
- [26] B. Schenke, P. Tribedy, and R. Venugopalan, *Phys. Rev. C* **89**, 064908 (2014).
- [27] Q. Y. Shou, Y. G. Ma, P. Sorensen, A. H. Tang, F. Videbæk and H. Wang, *Phys. Lett. B* **749**, 215 (2015).
- [28] J. S. Moreland, J. E. Bernhard, and S. A. Bass, *Phys. Rev. C* **92**, 011901 (2015).
- [29] G. Aad *et al.* (ATLAS Collaboration), *Eur. Phys. J. C* **74**, 3157 (2014).
- [30] A. M. Sirunyan *et al.* (CMS Collaboration), *Phys. Lett. B* **789**, 643 (2019).
- [31] S. Acharya *et al.* (ALICE Collaboration), *J. High Energy Phys.* **1807**, 103 (2018).
- [32] A. Adare *et al.* (PHENIX Collaboration), *Phys. Rev. C* **99**, 024903 (2019).
- [33] C. Loizides, J. Nagle, and P. Steinberg, *SoftwareX* **1–2**, 13 (2015).
- [34] J. E. Bernhard, J. S. Moreland, S. A. Bass, J. Liu, and U. Heinz, *Phys. Rev. C* **94**, 024907 (2016).
- [35] J. S. Moreland, J. E. Bernhard, and S. A. Bass, [arXiv:1808.02106](https://arxiv.org/abs/1808.02106) [nucl-th].
- [36] B. Schenke, P. Tribedy, and R. Venugopalan, *Phys. Rev. Lett.* **108**, 252301 (2012).
- [37] J. L. Nagle and W. A. Zajc, [arXiv:1808.01276](https://arxiv.org/abs/1808.01276) [nucl-th].
- [38] L. Yan, J. Y. Ollitrault, and A. M. Poskanzer, *Phys. Lett. B* **742**, 290 (2015).
- [39] G. Giacalone, J. Noronha-Hostler, and J. Y. Ollitrault, *Phys. Rev. C* **95**, 054910 (2017).
- [40] M. Rybczyński, G. Stefanek, W. Broniowski, and P. Bozek, *Comput. Phys. Commun.* **185**, 1759 (2014).
- [41] S. J. Das, G. Giacalone, P. A. Monard, and J. Y. Ollitrault, *Phys. Rev. C* **97**, 014905 (2018).
- [42] L. McLerran and P. Tribedy, *Nucl. Phys. A* **945**, 216 (2016).
- [43] R. S. Bhalerao, M. Luzum, and J. Y. Ollitrault, *Phys. Rev. C* **84**, 034910 (2011).
- [44] P. Alba, V. M. Sarti, J. Noronha, J. Noronha-Hostler, P. Parotto, I. P. Vazquez and C. Ratti, *Phys. Rev. C* **98**, 034909 (2018).
- [45] C. Chattopadhyay, R. S. Bhalerao, J. Y. Ollitrault, and S. Pal, *Phys. Rev. C* **97**, 034915 (2018).
- [46] C. Nair, M. Erhard, A. R. Junghans, D. Bemmerer, R. Beyer, E. Grosse, J. Klug, K. Kosev, G. Rusev, K. D. Schilling, R. Schwengner, and A. Wagner, *Phys. Rev. C* **78**, 055802 (2008).
- [47] H.-J. Drescher and Y. Nara, *Phys. Rev. C* **75**, 034905 (2007).
- [48] J. L. Albacete and A. Dumitru, [arXiv:1011.5161](https://arxiv.org/abs/1011.5161) [hep-ph].
- [49] ATLAS Collaboration, ATLAS-CONF-2018-011, <https://cds.cern.ch/record/2318870>.
- [50] ATLAS Collaboration, ATLAS-CONF-2017-066, <https://cds.cern.ch/record/2285570>.
- [51] M. Zhou and J. Jia, *Phys. Rev. C* **98**, 044903 (2018).
- [52] G. Giacalone, L. Yan, J. Noronha-Hostler, and J. Y. Ollitrault, *Phys. Rev. C* **95**, 014913 (2017).
- [53] N. Abbasi, D. Allahbakhshi, A. Davody, and S. F. Taghavi, *Phys. Rev. C* **98**, 024906 (2018).
- [54] R. S. Bhalerao, G. Giacalone, and J. Y. Ollitrault, *Phys. Rev. C* **99**, 014907 (2019).
- [55] L. Yan and J. Y. Ollitrault, *Phys. Rev. Lett.* **112**, 082301 (2014).
- [56] L. Ma, G. L. Ma, and Y. G. Ma, *Phys. Rev. C* **94**, 044915 (2016).
- [57] S. A. Voloshin, A. M. Poskanzer, A. Tang, and G. Wang, *Phys. Lett. B* **659**, 537 (2008).