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# Educating to rationality in a narrative context: An experimentation

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*The focus of the paper is the analysis of written argumentation in solving logical-linguistic riddles by 6<sup>th</sup> and 7<sup>th</sup> grade students. This is part of a larger path dealing with the introduction of some logical contents, in which all the activities are immersed in a narrative framework. In analyzing students' productions, we pay great attention to the interplay between logical-scientific thinking and narrative thinking, with the awareness that a rigorous mathematical argumentation can be obtained only at the end of a path starting from different, often not rigorous, forms of reasoning.*

*Keywords: Language, written argumentations, logical riddles, narrative and scientific thought.*

## Introduction

In this paper we focus on the analysis of written argumentations produced by 6<sup>th</sup> and 7<sup>th</sup> grade students to solve logical-linguistic riddles. This kind of activities is part of a path carried out within a project in a secondary school near Salerno (Italy), during the year 2014-15. The aim of the whole project was to reconcile students with low level of mathematical skills with the subject. In accordance with the teachers of the school, the focus was on linguistic competences in a scientific environment, with particular attention to the development of the argumentative competence.

The starting point was the didactic path described in Tortora (2001), consisting of 15 structured worksheets. Its aim was to bring the contents of classical propositional logic to the students, through a fantastic and attractive way. The innovation with respect to the initial idea, favoured by the introduction of the logical-linguistic riddles (Smullyan, 1978), is the great attention devoted by us to students' reasoning. To give importance to students' answers, we have let them naturally emerge from a learning set in which discussion had a central role. In analysing students' productions, we can observe how their spontaneous reasoning is a first step toward the development of their argumentation skills. We are aware that a rigorous mathematical argumentation can be obtained only at the end of a long path that starts from different forms of reasoning, often not scientifically rigorous. This does not mean that different forms of rationality should be dismissed in favour of the specific mathematical rationality. We know that for each of us all the forms of rationality coexist more or less in our life, but what is important is the possibility given to all students as early as possible to acquire the special kind of mathematical rationality.

This is the specific purpose of this work, where we analyse some of students' productions in solving the riddles and we trace the development of their reasoning. Therefore, our main research question is to what extent, and by means of what specific didactic mediations, the use of logical riddles with their linguistic challenges, can favour the development of argumentative competences and of scientific language and thought.

## Theoretical background

Language and in particular linguistic competencies are considered very relevant issues in mathematics learning. For example, Sfard (2000), to quote just a single seminal work, interprets thinking as a form of communication and considers languages not only as vehicles of pre-existing

meanings, but as builders of the meanings themselves. These competencies are the basis of many cross abilities, argumentation, communication, problem solving and so on, recommended as essential in all the official documents (for example, MIUR, 2012, Italian Ministry of Education).

In our works (e.g., Coppola, Mollo & Pacelli, 2010) we have often used logic in educational contexts, just because, in addition to being an important learning goal in itself, it has a special role in relation to language. In fact, logic appears as a privileged field for analysing the relation between language and interpretation, for identifying, studying and using linguistic manipulation rules and especially for the dual role of object and tool of investigation that language plays within logic (Ferrari & Gerla, 2015). The attention to the distinction between language and metalanguage is evident in our study, where the language is in the logical riddles and the metalanguage occurs in the discussions and the written argumentations used to solve them. However, no educational use of mathematical logic can be exhausted in its strictly disciplinary or formal aspects. These aspects may at most be considered a point of arrival, bearing in mind that in any case the way leading to the formalization is long and arduous. Along this road, the language takes on different forms and levels and the argumentations meet various needs. The importance of the contexts in which communication occurs and of the different forms of language has been widely recognized by the research that has put into the foreground the pragmatic aspects of language (Ferrari, 2004).

In general, the topics of pragmatic are deeply connected to the critical points of the research on learning and teaching mathematics (Ferrari, 2004). In our study, we use these tools to interpret some of the students' behaviors, elsewhere classified as 'irrational'. On the contrary, according to (Zan, 2007), we believe that the behaviors of the subjects 'getting wrong' may appear consistent when considered in relation to contexts and purposes other than those strictly adhering to rigorous logical reasoning. For this reason, we prefer to speak of two forms of rationality, rather than counter the rationality of mathematics with other behaviors that obey to different pulses. For example, according to one of the central issues of the pragmatic (Grice, 1975), in a particular context it is possible to make interpretive inferences based on the belief that who speaks or writes respects the Principle of Cooperation, according to which the communication is a collaborative process among those who are involved. These inferences, called conversational implicatures, differ by the logical implication, which relies only on the semantic content. Moreover, in making inferences in a certain context, it is frequent and legitimate to resort to one's own *encyclopedic knowledge*, that is the general knowledge of a person about the world (Zan, 2007).

The aspects of language brought to the fore by the studies on pragmatics are intertwined with the Bruner's distinction between two kinds of language or thought, the narrative and the scientific ones (Bruner, 1986). The scientific thought categorizes reality, recognizes the order of things, and produces demonstrative argumentations. It comes up in linguistic forms which are typically impersonal and timeless. Narrative thought, instead, interprets human facts: actions, intentions, desires, beliefs and feelings. It comes up in linguistic forms in which actions are performed by individuals and are accomplished in time. The acquisition of the first kind of thought and language, necessary for the understanding of science and mathematics in particular, is slow and it requires a careful didactic mediation, whereas the narrative way is more spontaneous and within everyone's means. For this reason, in many researches there are several suggestions for using narrative forms or even invented stories as a way to present mathematical contents (see, e.g. Zazkis & Liljedahl,

2009). Hence our decision to use a fantastic setting to introduce abstract concepts and present logical tasks. Our choice also depends on other reasons. It prepares the students themselves for using narrative modes. This establishes a working setting in which teachers and researchers can use and interpret students' answers in order to guide them through the gradual acquisition of forms of scientific language. Moreover, a third reason, regarding the logical contents we introduce, led us to design a fantasy narration. In fact, we agree with what Eco (2009) says about the relationship between narration and the notions of true and false:

“[...] every statement in a novel draws and constitutes a possible world whence all our judgments of truth or falsity will refer not to the real world but to the possible world of that fiction [...] The epistemological function of such fictional statements is that they can be used as a litmus paper for the irrefutability of any other statement. They are the only criterion that we have to define what the truth is” (Eco, 2009, translated by the authors).

It is only in the context of an invented story that true and false are incontrovertible: for example, Rome could stop being the capital city of Italy, but Juliet will never stop loving Romeo.

## Methodology

The study involved eighty 6<sup>th</sup> and 7<sup>th</sup> grade students of the same school with medium-low level of mathematical skills for about two months. The activities were carried out outside school time, in the presence of a mathematics teacher and a researcher (one of the authors of this paper). In a Vygotskian perspective, according to which the reasoning ability increases in the interaction among peers under the guidance of an expert (Vygotsky, 1934), the children participated in the activities working in small (2, 3 or 4 people) cooperative groups. Moreover, in accordance with the notion of *didactical cycle* (Bartolini, Bussi, & Mariotti, 2009), the activities were carried out with the alternation of different phases: exploration of the artefact, problem solving and collective discussion guided by the researcher. In our case, the artefact is the text of the riddle, so a linguistic artefact.

The students alternated their work with structured worksheets and with logical riddles. All the texts are adapted from the tales of “the knights and knaves island” (Smullyan, 1978): an imaginary island populated by two kinds of inhabitants, the knights who always tell the truth, the knaves who always lie. The activities on the structured worksheets, already used in the original path (Tortora, 2001), were proposed in the first part of every lesson. Their aim was to introduce in each lesson some of the basic elements of logic, e. g. the notion of proposition, the truth values, the distinction between simple and compound sentences, the logical connectives. The worksheet activities also gave to the researcher the opportunity to involve students in reflections about the differences between mathematical logic and common sense, as well as about the relativity of the notions of true and false and their dependence on the available information, the context and in some cases the judgment of the evaluator. The second part of every lesson was devoted to the solution of logical-linguistic riddles, as an application of the notions and the abilities acquired. From a formal point of view, the resolution of this kind of riddles requires that the students succeed in determining the only model<sup>1</sup> coherent with the dialogues in the text of the riddle.

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<sup>1</sup> We use here the term ‘model’ to mean a correspondence that assigns to each character in the story the category he belongs to (knaves or knights).

All the collected data, that is, the students' written argumentations and the audio-recordings of their interaction within the group, have been analyzed. Here we refer only to the analysis of some written protocols, produced as answers to a single riddle. The task we examine is the solution of the first riddle, proposed at the end of the first lesson, after having introduced the notions of logical proposition and true values. We find this task the most interesting in order to reflect on how the students switch from one kind of thought and language to another, since in this first phase they were totally free from the influence of any didactic contract in solving linguistic riddles. Protocols have been examined on the basis of the awareness that different contexts and aims activate different forms of rationality and different linguistic styles (narrative vs. scientific). This aspect is crucial since we required students to logically solve linguistic riddles situated in a narrative environment. The analysis was carried out recognizing students' behaviors just on the basis of these categories.

The task that we examine is the solution to the riddle described in Figure 1.

Team:

Riddle 1

*Oreste is in the knights and knaves island and he meets two persons, Alberto and Bernardo.*

*Alberto claims: "One of us is a knave, at least"*

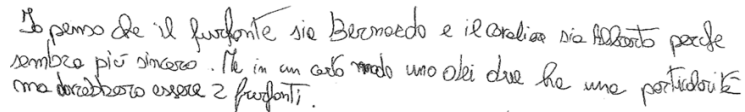
What can we say about Alberto and Bernardo? Can we establish which kind of inhabitant is Alberto? Can we know what kind of inhabitant is Bernardo? Discuss about this with your team mates. Then, write your reasoning.

**Figure 1: Riddle 1 - The right answer is: Alberto is a knight and Bernardo is a knave**

## Analysis of protocols

We report four protocols<sup>2</sup>, which seem to be meaningful and representative. We have selected them among the others, to show a spectrum of resolutions starting from a completely narrative approach until a prevalently scientific one. In them we have found also many interesting examples of conversational implicatures.

Protocol G1 (Fig. 2) gives us an example of completely narrative resolution of the riddle, without any explicit argumentation. In the first part, the students attach to Alberto the identity of a knight: probably, since Alberto speaks with Oreste, they affirm that "he seems sincerer". From that they deduce that Alberto is a knight and Bernardo is a knave.



Io penso che il fuorsente sia Bernardo e il cavaliere sia Alberto perché sembra più sincero. Ma in un certo modo uno dei due ha una particolarità, ma dovrebbero essere 2 fuorsenti.

**Figure 2: Protocol G1 - Answer: Alberto is a knight and Bernardo is a knave**

G1: [First part] I think that Bernardo is the knave and Alberto is the knight because he seems sincerer. [Second part] In a certain way one of the two has got a particularity, but they should be 2 knaves.

<sup>2</sup> The labels G1, G2,... indicate the protocols of the different groups (Group 1, Group 2,...). In Figures 2 to 5 we show the original protocols (including some erasures), and then just below we report our English translations.

Many groups attribute conversational purposes to knaves and knights. In particular, here students seem to believe that the knaves prefer not to intervene in the dialogues, because they do not want to risk, exposing themselves to reveal their nature, whereas the knights speak freely, because they have nothing to hide.

In Protocol G2 (Fig. 3), the possibility that Alberto be a knave is excluded on the basis of encyclopedic knowledge: in fact, the group imagines that in that case Alberto would have said something different. Thus we have an example of narrative thinking, with an argumentation.

Alberto è un conduttore perché, se lui fosse stato un furfante avrebbe detto al cavaliere che il cavaliere non era un furfante.

**Figure 3: Protocol G2 - Answer: Alberto is a knight**

G2: Alberto is a knight because, if he were a knave he would have said the opposite i.e., that ~~both~~ neither of them was a knave.

Very often the students' priority in solving their first linguistic riddle seems to be to preserve the coherence of the story, based on their daily life experience. In this protocol, for example, we note a *change of script*, which is one of the most frequent phenomena we found in the first approaches to the riddles. By this we mean an argumentation used to exclude cases that appear to the students inconsistent with the narrative. When the students judge a case as inadmissible, they try to make examples of what the characters would have said in a case coherent with the narration (*"this case is not possible, because otherwise the character would have said so..."*). This change of script is in accordance with the cooperative purpose often attributed to the knaves.

Nevertheless, already within the activity of resolution of the first riddle, it is possible to notice the emergence of a different form of rationality. In many protocols, there is a transition from a first response, corresponding to an involvement of only the narrative thinking, to subsequent responses, in which the students bring into play simple forms of logical-scientific thinking. This evolution was supported by collective discussions, which took place during the activity. For example, in Protocol G4 (Fig. 4), we can read three successive different kinds of resolutions: exactly what we intend for a complete spectrum of different approaches to the riddle resolution.

Alberto e Bernardo, sono entrambi ~~conduttori~~ furfanti perché dopo aver detto questo, Bernardo non ribadisce, quindi vuol dire che la proposizione è vera.  
 Alberto è un cavaliere e Bernardo è un furfante, perché dicendo questo Alberto dice che uno dei loro è un furfante e che quindi, tra loro dice, c'è perfino un furfante.  
 Supponiamo che Alberto sia un furfante.  
 Diciamo questa frase la proposizione sarà vera. Ma dato che lui è un furfante, non dovrebbe dire cose vere. Quindi Alberto è un cavaliere, e Bernardo è un furfante.

**Figure 4: Protocol G4–First version answer: Alberto and Bernardo are both knaves.  
 Second and third versions answers: Alberto is a knight and Bernardo is a knave**



to be both knights; ~~if Alberto was a knave~~ it cannot be that Alberto is a knave otherwise he would have said to be a knight. Therefore by exclusion Alberto is a knight having told the truth and consequently Bernardo is a knave.

In both versions the answer is correct. In the first argumentation, we can notice a clear predominance of the narrative thinking over the logical one, leading to a change of script, coherent with the conversational purposes attributed to the characters. On the contrary, in the second one, which comes after the long collective discussion, we notice the use of a scientific language, while the content is at the same time narrative and logical. In fact, the possibility that Alberto and Bernardo are knights is ruled out by means of a logically correct argumentation. The other two non-admissible cases, instead, are excluded on the basis of narrative argumentations, through changes of script. For example, the change of script “*they cannot be both knaves otherwise they would have said to be both knights*”, is based on the conversational purpose that the knaves team up to hide.

### **Discussion and conclusions**

It is well known that the logical formalism, although necessary, to a certain extent, for a full acquisition of mathematical notions, may constitute a difficult obstacle for students, due to its distance from common sense and to its exasperated exactness. The awareness of this risk was the starting point of our research and experience. For this reason, a first decision was to introduce the didactic activities by means of riddles, which are a sort of game. Secondly, these puzzles were immersed in a fictional context, using explicitly a narrative mode. But in the experimentation analysed in this study, the role of narrative has been twofold. On the one hand, as we have said, following a well-established research trend, we have benefited from the context of an invented story and its appeal to introduce some not easy logical-mathematical concepts; on the other hand, we have paid special attention to the narrative mode adopted by students in their oral and written productions. It seems to us that our experimentation has brought some interesting results. First, the path supported the students toward a strengthening of the metalinguistic control over the texts, spurring a reflection on the relationship between language and metalanguage. In our context, the object language corresponds to the sentences pronounced by the knights and the knaves, whereas the metalanguage is the one used in group discussions and in the production of written argumentations. Thus the students became aware of the dual role of language, as a communication tool and an object of manipulation. In addition, our way of introducing the activities allowed the students to grasp a first sense of the logical formalism, although deliberately not rigorous. Addressing the proposed activities, the students were gradually able to experience directly how a rational management of statements, at a first glance uninformative, could be very efficient. The narrative dimension has played a key role in providing a criterion of truth, which was naturally accepted by the students and which allowed to (partially) approach what logicians call a ‘model’.

During the steps towards the resolution of the riddles, alternated with the collective discussions about the students’ argumentations, it seemed to emerge a gradual evolution from a purely narrative approach toward an approach where some form of scientific thinking appears. Even in the solution of the first riddle, where the two kinds of rationality are intertwined, it emerges a shift towards a more conscious management of the two forms of thinking, spurred by the resolution of the logical tasks. This kind of evolution is supported by the emergence of a more and more rigorous language.



Our further step will include a deeper analysis of the oral discussions among the students, in order to try to better understand if and how the peer discussion, the comparison of different views and the necessity to write down the shared conclusions favour the transition toward a more and more sophisticated use of a scientific language. If this will be the case, there will be room for designing and experimenting further didactic proposals also in order to observe whether, in a longer period of time, there are positive repercussions on mathematical competences.

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