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## Talking with objects

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*Both language and objects seem to play an important role in mathematics learning. In our research, we focus on their interplay: How do language and objects support students' development of mathematical ideas? In order to develop a framework of 'talking with objects', we draw on three approaches. First, we adopt the idea from Bauersfeld that learning is a domain specific process. It is always bound to a very specific situation and context. Second, Aukerman's approach of re-contextualization supports our insight in the link between language and context. Third, Latour's Actor-Network-Theory helps us to better understand, how concrete objects take part in the process of constructing social reality in mathematics lessons.*

*Keywords: Language, objects, recontextualization, domains of subjective experience (DSE).*

### Introduction

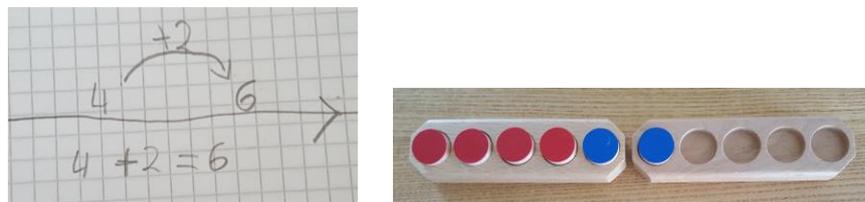
Children are supposed to learn what a 'number' is or what we mean by 'addition'. The challenge of mathematics learning is to construct abstract mental objects that can neither be touched nor seen. Even if we cannot see the mathematical objects themselves, there is a lot acting and handling of and with concrete objects to be observed in everyday mathematics classes: Children write, read, and work with different concrete objects like bead frames, hundred boards or Dienes blocks. And they speak: They ask questions, explain their ideas and discuss about different mathematical interpretations. Obviously, both objects and language play an important role, when students are learning mathematics. Children and teachers use physical representations of mathematical objects in order to clarify what they are talking about and what they are referring to. These physical objects help to coordinate children's mathematical communication and their learning processes (Sfard, 2008, p. 147).

As mathematics educators, we see already quite clearly that language is an important aspect of mathematics learning. But how do objects come into play? And how do language and objects interact as means of representation? These questions lead our main research interest. In our research project, we intend to reconstruct mathematical learning processes with a special focus on the interplay of language and objects (Fetzer & Tiedemann, 2015). To begin with, we concentrate on primary school children who learn arithmetic in different German primary schools. We collect data in several schools so that we cover different social and cultural backgrounds and get an impression of our research topic that is as broad as actually possible.

Theoretically, our study is based on two main assumptions. First, we assume, together with many other researchers, that *mathematics learning is a social process* (Bauersfeld, 1988; Jungwirth & Krummheuer, 2006; Miller, 1986). Children do not construct abstract mathematical objects without any suggestions from their environment, but rather in permanent exchange with it. In processes of social interaction and collective argumentation, mathematical objects are constructed, negotiated, and clarified. In this sense, children create abstract mathematical objects on the basis of social

processes. But who are the players in these social processes? Usually, mathematics educators think of students and teachers as actors. However, concrete objects influence the ongoing interaction, too. And, in our opinion, it will mean missing important opportunities to support mathematics learning if we neglect them. Especially in primary classes, objects play an important role in the process of abstraction.

Second, we conceptualize mathematical abstraction as a process of *becoming aware of similarities in different experiences* (Skemp, 1986). According to that assumption, children have to grasp the similarities in different representations, which they encounter in the context of arithmetic. To make this clear, we can consider children playing with little game figures. Four are sitting in a train and two more are getting on. What do these little figures have to do with the drawing of a number line, with six fingers of our hands, with the arithmetical task “4+2” written on a sheet of paper or with specific arrangements of didactical material in mathematics classrooms? (Compare Fig. 1)



**Figure 1: Different representations of 4+2**

In order to express what is similar in all those representations and to come to a social agreement on those similarities, children and teachers need language. It is a tool, which allows individuals to share their interpretations of reality with each other. They can express what they ‘see’ in a certain representation and can, in this way, develop a shared interpretation. Within that interplay of language and objects, children construct their concepts of addition or number. It is for that reason that children have to develop appropriate language skills in mathematics classes, i.e. that their language has to become suitable for describing similarities in different representations.

We present the theoretical framework that we have developed so far for our research project. It consists of three parts. First, we refer to Bauersfeld’s (1988) framework of domains of subjective experience (DSE). He points out that learning is a domain-specific process, i.e. that children’s mathematical constructions are always bound to the situation in which they were developed. Second, we focus on the aspect of language and our fundamental assumption that every linguistic utterance, how concrete or abstract its content may be, always refers to a context (Aukerman, 2007). Third, there is the question of objects and the role that they may play in the process of mathematics learning. In this regard, we refer to Latour’s Actor Network theory (ANT) (2005) which offers a new perspective on objects and their contribution to mathematical communication.

### **On domain-specific learning: Bauersfeld (1988)**

Bauersfeld’s (1988) approach of Domains of Subjective Experience (DSE) elaborates how individuals organize their construction of mathematical knowledge. He assumes that children do not organize their remembrance of experiences in a hierarchical way, but rather accumulatively in separate domains. Each experience is stored with reference to the very specific and complex situation in which it was made and, accordingly, in its own domain. These different domains of

knowledge are called “domains of subjective experience (DSE)”. They include their own meaning, language, actions and objects. To illustrate this approach, Bauersfeld (1983, p. 3) reports from Ginsburg’s (1977) work about eight-year-old Alexandria. She is not able to solve the task “ $8:4=$ ” which is written on a piece of paper, She only suggests 0 or 1 as possible solutions. But, surprisingly, she can solve another task without any apparent effort: “Imagine you have 5 dollars and there are four children. How many dollars will each child get?” In fact, this second task is more difficult from a mathematical point of view. So we might ask why Alexandria did not transfer the initial task “ $8:2=$ ” to the money-world herself. Why did she not solve it with reference to the domain that is obviously much more familiar to her? Bauersfeld’s answer points to an important characteristic of DSEs: They are not linked automatically. Thus, from Alexandria’s point of view, two different DSEs are affected which are unconnected up to now. In the paper-world, you have to cope with mathematical signs that are written on a piece of paper. In the money-world, you have to cope with banknotes and coins and think about buying attractive goods. Language, actions, objects, but also interests, motivations and feelings are fundamentally different in both DSEs. For that reason, Bauersfeld (1983, p. 6) doubts fundamentally, whether Alexandria regards the number word ‘eight’ which appears in both domains as the same at all.

According to Bauersfeld, mathematics learning can be understood as a process of constructing, deepening and connecting DSEs. However, how can those separate domains be linked? How does mathematics learning proceed? Bauersfeld (1983, p. 31) describes that individuals cross the borders of a DSE by trying, creating and negotiating. In order to link two different DSEs, they have to build a third DSE that exclusively aims at comparing the two already existing ones. Solely in such comparative DSEs, it becomes reasonable to develop a comparative language. In fact, it is this comparative language that allows students to talk about similarities, which they ‘see’ in different representations. This means that all parts of a DSE, including language and objects, can help children to link DSEs and to get access to abstract mathematical objects. In the following paragraph, we focus on the language at first.

### **On language and context: Aukerman (2007)**

Aukerman (2007) points out that it is quite misleading to talk about a ‘decontextualized’ language because no “text, and no spoken word, ever exists without a context” (p. 630). This approach puts the main emphasis on the content level of a linguistic utterance: Every utterance refers to a context, no matter whether this context is concrete or abstract, close or far, accessible to observation or only hypothetical. It is important to notice that Aukerman does not make any statement about the setting in which language is used or how language is used in it, but rather about the point of reference. Utterances in mathematics classes may be produced in many different ways, e.g. with gestures or not, with a parallel action or not, with pointing at something in the closer environment or not, etc., but they are all produced with the intention of talking *about* something. Subsequently, we always talk and listen to others with regard to a specific context. We think about a specific context and produce an utterance. We hear an utterance and interpret it against a background that we deem appropriate. Thus, no matter whether we are the ones who speak or the ones who listen, we relate every utterance to a context that we regard as adequate at that very moment. Aukerman (2007) refers to the process of connecting utterances with contexts as *recontextualization*. In the process of recontextualizing, speaker and listener have to agree to a certain extent on the context of their

conversation: What are we talking about? Thus, when students are expected to talk about mathematical objects, they have to re-contextualize their language and match it with rather abstract contexts. Seen from that perspective, the question is no longer, whether a student is able to decontextualize his or her language, but the question is whether students and teachers succeed in finding a shared context: Do their *recontextualizations* fit together sufficiently?

### **On objects as actors: Latour (2005)**

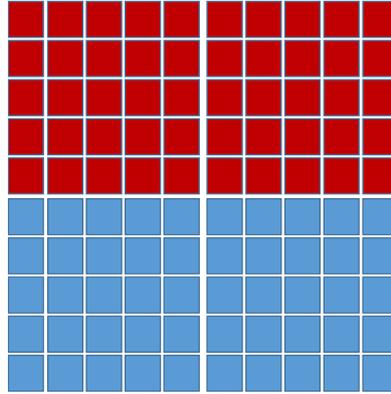
When students and teachers are negotiating a shared context for their constructions of DSEs, they can get help from concrete objects, which have a lot to offer. Objects as actors? This conceptualization appears to be unfamiliar at first sight. Nevertheless, we think that it can be very useful to adopt Latour's (2005) sociological proposal for accepting objects as actors in the course of action. According to him, they participate in the emergence of social reality.

Latour (2005) goes beyond the traditional understanding of the social, widens the perspective and redefines the notion of 'the social'. He takes a closer look on who and what assembles under "the umbrella of society" (p. 2). As a consequence, he defines sociology as „the tracing of associations“ and thus “reassembles” the social (p. 5). In his view, the social refers to any kind of networking: humans with humans, but also humans with any kind of things. Heterogeneous elements that are not necessarily social themselves associate in different ways. According to Latour, all these different associations create social reality. Thus, in his *Actor Network Theory* (ANT), he extends the list of potential actors in the course of action fundamentally and accepts all sorts of actors: “Any thing that does modify a state of affairs by making a difference is an actor” (p. 71). Consequently, objects *participate* in the emergence of social reality, too. In this sense, Latour asks for a broader understanding of agency. “Objects too have agency” (p. 63). They are associable with one another, but only momentarily. To say it with Latour's words, they “assemble” (p. 12) as actor entities in one moment and combine in new associations in the next one. Following Latour, there are no longer stable and pre-defined associations and actor entities.

Again, following Latour (2005), objects participate in the emergence of classroom reality. In fact, this is true for *all sorts* of objects: Paper and pencil, as well as manipulatives or even the bottle of water on the table. Should we as researchers in mathematics education not focus on a certain kind of object, on didactical material? From a theoretical as well as from a methodological point of view, we clearly deny that restriction. Just imagine that the bottle was open, and would drop. Not only the table, but also the paper would get wet, the pencil might fall on the floor. This would surely influence any process of social interaction. “Any thing” (p. 71), a human or non-human actor, might become associated with other actors in the course of action, but only momentarily. The association might be dissolved the next minute. However, in that very moment these actors, no matter who and what they are, contribute to the ongoing process of social interaction.

Looking through Latour's sociological glasses, we can see clearly that concrete objects do play a role in the emergence of social reality. This appears to be especially true for manipulatives and other didactical material. They participate in the negotiation of a shared context and, in this way, offer help in the social process of constructing and connecting DSEs. However, how *do* they contribute? Earlier research revealed different modes of participation that objects might take or have in ongoing classroom interactions (Fetzer, 2013). Our current research on the interplay of language and objects

goes one-step further. Now, we try to get hold of objects' contributions on the content level. In our opinion, their most important contribution is to offer various contexts for re-contextualisation from which students and teachers may choose. A short example that we could observe in a second grade class might illustrate this variety of possible offers. The students and the teacher talk about the question what *the* diagonal might be on their hundred board. On that special hundred board (compare figure 2), the numbers from 1 to 100 are covered with red and blue pieces of paper.



**Figure 2: Hundred board covered with red and blue pieces of paper**

Here are some of the offers that the students accept and express in linguistic utterances - always in association with the hundred board in front of the classroom:

- 1) "The diagonal runs from 10 to 91."
- 2) "The diagonal runs from one corner of the hundred board to the opposite one."
- 3) "The diagonal runs from one corner of a square to the opposite corner."

We see that the hundred board suggests a wide variety of contexts, which might be suitable for re-contextualization. Most important to us is the fact that those offers range from "concrete" to "abstract". Thus, on the one hand, objects support the opportunity to construct new DSEs because they make a very concrete offer. They are concrete in nature so that students can associate with them and refer to their rather concrete offer: The 10 is at the top right, the 91 is at the bottom left. On the other hand, they make offers that appear to be suitable for comparing. The hundred board has properties that other geometrical forms have as well. It is a square and in every square, "the diagonal runs from one corner [...] to the opposite corner." However, in order to grasp that similarity in different representations, you have to construct a DSE of a specific square at first and then a comparing DSE, which aims at comparing different geometrical forms. Those offers of objects may support the construction and connection of DSEs and may support the language development which goes along with it, too.

### **Integrating the concepts: Talking with objects**

According to Bauersfeld (1988, p. 178), the subjective realization of a mathematical object remains always bound to the context of experience, i.e. to the objects and language used in the situation of construction. This approach gives us a clue that we will understand mathematics learning better, if we concentrate not only on language or on objects but on the interplay of both. How do language

and objects interrelate in the process of mathematics learning? How can we talk with (the help of) objects? Aukerman's (2007) approach of re-contextualization and Latour's (2005) Actor Network Theory seem to be useful background theories to tackle these questions.

According to Aukerman, every spoken word in mathematics classrooms refers to a context and is re-contextualized by the recipient. In order to achieve a shared understanding, the interlocutors have to agree on the 'right' context. What can serve as a solid ground, which a linguistic utterance can be related to? At this point, objects come into play. According to Latour, humans and non-humans associate with one another and create social reality. In terms of mathematical learning processes, children and objects interact in the social process of learning. Objects make offers that students and teachers can accept and refer to in order to coordinate their mathematical communication. The "steely quality" (Latour, 2005, p. 67) is a solid ground that allows individuals to experience reality. Objects are not a mere tool in students' hands that can easily be manipulated. Objects are participants in their own social right and contribute to the ongoing classroom interaction: Objects make offers and students 'listen' to those offers. Students talk *to* objects, and become associated with one another. At this point students 'talk' together *with* objects in a combined action.

But how does that work? Objects are concrete in nature. Nevertheless, in their concreteness they prove to be not a limitation, but a chance for development of (mental) mathematical ideas. Indeed, objects offer a variety of possible contexts for re-contextualization ranging from concrete to abstract. Sometimes, objects may provide the context for very specific experiences. In these cases, objects can help to construct a new DSE or to deepen already existing DSEs. At this stage, students try to find words with which they can express the particularity of this specific context and to negotiate it with others. Language is probably the most important tool for such a negotiation: What do I 'see' in that object? What do you 'see'? In a second step, students have to become aware of similarities in different experiences. They are in permanent exchange with their social environment. They listen to as well as talk to and with participating actors. In doing so, they construct new DSEs that aim at comparing already existing DSEs. Again, objects profoundly contribute. They offer a context for comparisons: What do I 'see' as the same in different objects (or in different actions with objects)? What do you 'see' as the same? Where are differences? Do we agree? In this sense, objects help not only to coordinate mathematical communication, but also to develop language more and more. Students are challenged to match their language with a concrete experience at first and with a comparison afterwards.

On closer inspection, we see that objects are actors that students can talk *to* and talk *with*. In fact, objects contribute to the process of negotiating mathematical meaning. In most interactional situations, it is not only the child who is responsible for a linguistic utterance. Words are not the only means to negotiate mathematical meaning. Instead, students and objects often associate and convey a mathematical idea together. In these cases, the object actor takes over part of the act of re-contextualization (Fetzer & Tiedemann, 2016). Students talk together *with* objects. The boundaries between language and objects almost seem to merge.

## Discussion

The theoretical framework that we have sketched in this paper raises awareness of some aspects that are not new in mathematics classes, but that are new in our thinking about content-related language

use. When students want to express their mathematical interpretation of reality, they are not restricted to the words they have at their disposal. Thus, they can accept one of many offers that the objects in their close environment make. In this process of assembling, the objects achieve two things. They offer their help, but at the same time, they challenge the children to move further in their mathematical development and in their improvement of content-related language use.

For that reason, the framework does not only make us sensitive to the importance of objects in the process of language development, but it points in a direction that might be productive for our further research. We have not only to analyze objects that we use in mathematics classes, but we have to analyze children's associations with them, too. Which offers do the children accept? Moreover, how do these offers support their language development in mathematics classes? These are the questions, which will lead our further steps in that project about the interplay of language and objects.

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