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# How do students develop lexical means for understanding the concept of relative frequency? Empirical insights on the basis of trace analyses

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*In order to enhance students' proficiency in the academic language, building up more formal language on the basis of individual and everyday language is claimed as a fruitful approach. However, there is little empirical research on how students adopt and develop lexical means of the academic language. This paper addresses this field of research for the case of concept-specific lexical means for relative frequencies by presenting the applied design principles for learning opportunities as well as empirical insights into initiated concept- and language development processes.*

*Keywords: Academic language, relative frequency, scaffolding, design research, trace analysis.*

## Introduction

Due to the cognitive and epistemic function of language, the academic language can be challenging for many students (Schleppegrell, 2004). These challenges are also relevant for mathematical learning, as shown in various empirical studies. Thus, studies in mathematics education concentrate on designing and researching learning environments that integrate mathematics and language learning (Prediger & Wessel, 2013; Prediger & Pöhler, 2015).

For the field of understanding relative frequencies, the presented study relies on analyses of design experiments in small group settings focusing on one lesson within the larger intervention study of the DFG project MUM-MESUT (Grant PR 662/14-1 to S. Prediger). Detailed analyses of students' conceptual development and language development against the background of intertwined conceptual and lexical-discursive learning opportunities serve to structure the relevant lexical means and give insights into how students become proficient in the academic language for relative frequency.

## Theoretical background: Design principles for learning opportunities

Academic language proficiency has repeatedly been shown to influence achievement in mathematics and this general finding also applies for the mathematical topic of understanding fractions (Wessel, 2015). As a consequence, some current design research studies focus on developing and investigating content- and language integrated instructional approaches for fostering students with low language proficiency (Prediger & Wessel, 2013). The following paragraphs deal with the major design principles that were implemented in the presented design research study.

*Design principle "Macro-scaffolding".* The general structure of intended lexical learning trajectories is well described in the principles of *macro-scaffolding*, namely from students' everyday resources to academic and formal technical registers (Gibbons, 2002). However, its topic-specific realization is still an urgent need of research, as well as explorations of students' individual learning pathways (Prediger & Wessel, 2013). Previous research shows the relevance of phrases and syntactical constructions needed to express the meanings of a mathematical concept in view, which is also relevant for understanding the concept of fraction (ibid). Wessel (2015, p. 327) shows the importance of understanding how macro-scaffolding and interactional moves on the micro level relate to each other for moving students beyond their zone of proximal development. Here, the principle of *macro-*

*scaffolding* by coordinating conceptual learning opportunities with well-structured language learning opportunities on the lexical level is applied.

*Design principle “Pushing students’ output by realizing discursive practices”*. Given the sociocultural perspective on the learning of mathematics as participating in mathematical practices, mathematical activity is to a great extent mediated by language and interaction. In the context of mathematics learning of English language learners (ELLs) and with a perspective on extending academic language proficiency, Moschkovich (2013) stresses the relation between the lexical and discursive level of language: “The question is not whether students who are ELLs should learn vocabulary, but rather how instruction can best support students to learn vocabulary as they actively engage in mathematical reasoning about important mathematical topics” (Moschkovich, 2013, p. 46). This theoretical assumption leads to an extension of the design principle so that we use the principle of *macro-scaffolding* by coordinating conceptual learning opportunities with well-structured language learning opportunities on the lexical level *in addition to rich demands and language initiation on the discursive level*.

*Design principle “Relating registers”*. Pushing the students’ output and applying scaffolding strategies can be supported by the *design principle of relating registers*, according to which the graphical, the symbolic and the different verbal registers (everyday, academic, and technical register) are related systematically to achieve conceptual understanding (Prediger, Clarkson & Bose, 2016). For the lessons of the presented intervention, activities of relating registers have been realized with the fraction bar and bar board as a prominent graphical representation. In order to activate students’ individual and everyday language resources, typical contexts of downloads, fair share and soccer competitions have been implemented (for detail see Prediger & Wessel, 2013).

In their combination, the formulated design principles allow to integrate theoretical aspects on developing learning opportunities on the conceptual, lexical and discursive level. However, while the integrated analysis of initiated learning processes on conceptual and discursive levels are well-established in mathematics education research, only rarely empirical studies reconstruct lexical learning processes (exceptions e.g. Prediger & Pöhler, 2015, for the field of percentages). That is why Schleppegrell (2010, p. 107) demands more respective research which goes beyond analyzing short interactional sequences: “More research is needed that takes a developmental approach (...). We need rich studies of how language and ways of talking about mathematics evolve over a unit of study, focusing on more than brief interactional episodes and fragments of dialogue”. The presented study aims at minimizing this research gap for the field of relative frequency.

## **Research questions**

On the basis of the theoretical background and the research gaps listed above, the developmental work and analyses of the learning processes are guided by the following two questions:

- 1) *On the level of design outcome*: How can conceptual and lexical-discursive learning opportunities for understanding relative frequency be intertwined and designed in a sequence of rich mathematical activities?
- 2) *On the level of initiated learning processes*: Which lexical means do students activate and how are those lexical means intertwined with individual conceptual development when working on the learning opportunities towards relative frequency?

## Methodological framework and research context

The research was conducted in the methodological framework of topic-specific didactical design research (Prediger & Zwetzschler, 2013) in which the analysis of teaching-learning processes takes place in carefully designed teaching experiments. The design outcome, namely the consolidated intertwining of conceptual and lexical-discursive learning opportunities, is next described.

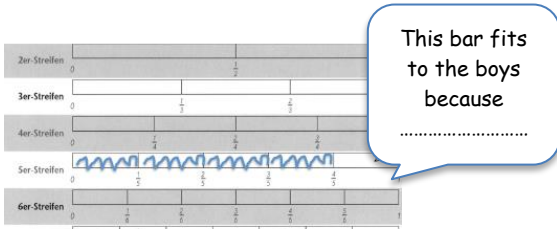
### Design outcome: Learning opportunities towards relative frequency (research question 1)

In order to combine conceptual, lexical and discursive learning opportunities according to the design principles described above, the larger intervention with five lessons for fostering conceptual understanding of students with diverse language proficiency in the language of instruction aiming at enhancing understanding of fractions was designed. For answering research question 1 the designed learning opportunities towards relative frequency as a design outcome are presented in the following section.

The intended *conceptual learning opportunities* were adapted from Prediger (2013). It starts with students' individual approaches and everyday experiences to compare three groups with different relative frequencies in the context of a soccer competition (see Table 1, Task 5). It then proceeds to constructing meaning of the given relative frequencies by introducing the bar board (Task 6). At this point, students' informal strategies for comparison are elaborated by focusing the need for normed referent wholes (here fraction bars of normed length) and the necessity of including every group's number of shots (not only number of strikes) to refine the concept of relative frequency which finally aims at the flexible use of relating number of shots and number of strikes.

The intended *lexical-discursive learning opportunities* focus on the vocabulary required for the conceptual learning process of thinking in relative frequency which is mainly the prepositional "of- or thereof construction" ("to score ... of ... shots", "... shots, thereof ...") (see Table 1), which can be conceptualized from the so-called 'basic meaning-related vocabulary' (Wessel, 2015). Students are asked to give reasons in the setting of discussing ways of fair or unfair strategies to rank the three groups. It starts from students' individual resources as well as with offering the relevant "of-construction" already in Task 5.

Conceptual learning opportunities	Tasks and mediator bar board	Lexical-discursive learning opportunities
Initiation of <i>individual</i> approaches for comparing relative frequency conceptualized as strike rates in soccer competition	<p><b>5. Who scored best?</b> In class 7c three groups took part in a soccer competition. <i>The group of boys scored 4 of 5 shots.</i> <i>The group of girls scored 8 of 10 shots.</i> <i>The group of teachers shot 20 times and didn't score 4 times.</i></p> <p>a) Who won the competition? Write your answer on a card. b) Put your cards in the middle of the table. Do you agree? Give reasons for your answer.</p>	<p>Initiation of <i>discussing</i> individual approaches and <i>giving reasons</i></p> <p>Introduction of <i>lexical</i> means "to score / not score ... of ... shots"</p>
Investigation of individual hypotheses in the bar board:  Comparing with fraction bars of normed length	<p><b>6. Who scored best?</b> Use the bar board in order to commonly find out whether one group scored better. The boys have already been marked. Add the results of the girls and the teachers as well as the speech bubbles.</p>	<p><i>Reflecting and discussing</i> fitting of fraction bars and groups ("This bar fits to the boys because ...")</p>

<p>Necessity of including number of shots (not only number of strikes) to refine concept of relative frequency</p>		<p>Activating lexical means for marking strike rates in the bar board focusing number of shots as a referent whole, number of strikes and strike rate</p>
<p>Systematize and deepen understanding by giving reasons for all groups scoring equally well</p>	<p><b>7. And the winner is...</b>          In the bar board you have found out how well the different teams scored.          Which group won the competition?          Give reasons for your answer.</p>	<p>Written reasoning on equivalence of relative frequency in the three groups          Applying introduced lexical means</p>

**Table 1: Conceptual and lexical-discursive learning opportunities (not necessarily strictly sequenced)**

The tasks in Table 1 illustrate how conceptual and lexical-discursive aspects are intertwined. On the discursive level, students are encouraged to verbalize and discuss their own ideas and structures. The vocabulary for these discussions is bound to the bar board as well as the context of the scoring situation, which always allows students to relate the vocabulary to its meaning. In Task 7 the students are free to note their reasons either with reference to the bar board, to the context or to the formal level of expanding and reducing fractions.

### Methods for data gathering and selection: Design experiments

Design experiments were conducted and video-taped within the larger research project MuM-MESUT with  $N = 343$  mathematically low-achieving mono- and multilingual students in grade 7. For the detailed analyses in this paper, a group of three students was selected according to their German language proficiency (measured with a German C-test) and language background (mono- or multilingual, operationalized by “speaks at least one other language than German with a parent or grandparent”), with the aim to have a linguistically heterogeneous sub-sample for conducting case analyses (in total the below presented method of analysis was applied in detail to  $n=16$  students). Due to the larger study, we can also draw on fraction test scores of the students (Wessel, 2015).

### Methods of data analysis for reconstructing conceptual and lexical development

In order to qualitatively reconstruct the students’ lexical pathways and how their lexical means relate to the initiated discourse and individual concept development (research question 2), the following three steps were applied:

*Step 1. Conceptual analysis.* For reconstructing the students’ conceptual development, strategies for comparing the given three groups of girls, boys and teachers and steps on the pathway to understand the concept of relative frequency have been identified by analysing transcripts and video data.

*Step 2. Trace analysis.* Concept-specific lexical means (words and phrases) which the students activated were inventoried and coded whether the students used them in oral or written language and whether they self-initiated the use or whether they adopted them from the material, the teacher or another student (for detail of the method “trace analysis” see Prediger & Pöhler, 2015). In this paper the focus is on oral language.

*Step 3. Relating conceptual development and language.* On the basis of step 1 and 2, the results of conceptual and language analysis were related and contrasted to reconstruct prototypical learning pathways and critical steps on the pathway under the perspective of different language backgrounds.

### **Empirical insights into the initiated conceptual and language learning processes**

On the level of initiated learning processes, research question 2 asks for lexical means that students activate and how these lexical means are intertwined with individual conceptual development. By contrasting the inventory of lexical means of two students the first part of the research question is addressed in the next paragraph.

#### **Concept-specific language production: Qualitative overview and comparison**

Makbule and Kiran (working in a group of three together with Vehbiya) are multilingual learners in year 7 of a German secondary school. In a German C-test Makbule's score is at percentile rank 37 and Kiran's at 84. In the fraction test Makbule's score is at percentile rank 7 and Kiran's at 15 (percentile ranks for both tests for full sample of N=1124 seventh graders). While Kiran is the more language proficient student according to the C-test results, Kiran and Makbule started at comparable low levels of fraction proficiency.

In Table 2 the actual orally activated concept-specific lexical means in the analyzed transcript (23.46 minutes of video data) of Kiran and Makbule are contrasted. While Makbule activates 26 different concept-specific lexical means in the course of the process and 76 in total, Kiran activates 15 different concept-specific lexical means and 21 in total. As Makbule generally talks the most in this lesson, relating these numbers to each student's individual rate of participation will be a further step in the data analysis.

It becomes apparent that Kiran uses all lexical means correctly, which fits to his high percentile rank in the German C-test and which is not always the case for Makbule. Also, while Makbule uses many of the lexical means various times (which leads to the high number of lexical means in total), the list of Kiran can give a hint at the possibly *sufficient* language for working on the given tasks and developing the concept of relative frequency.

Makbule		Kiran	
Concept-specific lexical means in oral language production in chronological order of first use in process, (frequency in brackets, semantically not correct lexical means in <i>italics</i> )			
#14 best (1)	#143 stripe (9)	#26 had ... shots (2)	#182 divided in the middle (1)
#15 won (5)	#151 <i>similar won</i> (1)	#26 did not score (1)	
#25 because (9)	#151 this big (1)	#28 score (2)	
#25 had ... shots (4)	#151 to divide in the middle (1)	#34 won (2)	
#25 did not score ... times (3)	#151 <i>divide small</i> (1)	#60 fraction (2)	
#25 scored (2)	#153 <i>shoot similar</i> (1)	#67 took ... shots and scored ... of them (1)	
#58 scored ... times (3)	#180 similarly big (2)	#104 score ... times (1)	
#76 ... of ... (1)	#180 normal big (4)	#124 tie (1)	
#95 bar (13)	#180 separated in the middle (5)	#128 the same (1)	
#101 ... <i>times shots</i> (1)	#186 separated (1)	#175 bar (3)	
#101 shoot ... times (2)	#188 line (1)	#175 fits to (1)	
#103 took ... shots and scored ... of them (1)	#188 the same (1)	#175 because (1)	
#125 tie (1)		#175 as good as (1)	
#141 similar (2)		#177 as long as (1)	

**Table 2: Variety of concept-specific lexical means in comparison**

However, the transcript analysis of Makbule's conceptual development indicates that the additional lexical means like "to divide in the middle", "normal big" and "separated in the middle" in Makbule's inventory are of great importance for her learning process towards understanding the idea of expanding fractions as refining the structure in the fraction bar. As a first conclusion, the comparison of the results from Makbule and Kiran shows that the question of the required language seems to vary between the students and demands further analyses of different cases.

### **Makbule's process of adopting concept-specific lexical means when relating registers**

As a conceptually relevant step in the learning process, it is important to move from the strategy of comparing the three groups' results (girls, boys and teachers) on the basis of the absolute number of strikes to experiencing the necessity of and applying the concept of relative frequency as a fair strategy for comparison (Prediger, 2013). How this pathway can be related to the activated and required lexical means becomes apparent in the following two excerpts taken from the corresponding learning process initiated by Task 6. The transcripts were translated from German and shortened to relevant utterances of Makbule which are needed for tracing those concept-related language means in focus (// indicates interruption).

When answering Task 5 (see Table 1), Makbule focuses on the three groups' absolute numbers of strikes and claims that the teachers won the competition. When working on Task 6, the following process was initiated by reflecting on why the chosen fraction bar of fifths fits to the boys group:

#### *Excerpt I: Kiran stresses the idea of relative thinking*

- 56 Teacher: So why does this fraction bar fit to the boys, the one that is marked?  
 58 Makbule: Because they, because they took yes, ehm, five shots **and** have only scored four times.  
 59 Teacher: And why does the fraction bar fit?  
 61 Makbule: Because it's **four fifths**.  
 66 Makbule: Because they//  
 67 Kiran: //took five shots and scored four **of them**.

On the lexical level, Makbule uses the coordination "and" to relate the number of shots and the number of strikes to each other (#58). As the teacher again asks why the fraction bar fits, Makbule focuses on the representation of rates as fractions, namely "four fifths" (#61). When starting an additional explanation ("Because they", #66) she is interrupted by Kiran, who finishes the sentence with "took five shots and scored four of them" (#67). This utterance in #67 is assumed to be a relevant trigger for the following discourse in the group, which becomes clear in the next excerpt.

#### *Excerpt II: Makbule adopts Kiran's "of them" construction*

- 70 Makbule: *reads her written answer:* because they took five shots and scored only four **of them**.  
 73 Teacher: So now the girls and the teachers. Where do we mark them?  
 74 Vehbiya: The girls in the bar of tens.  
 75 Kiran: Eight tens.  
 76 Makbule: so 8 of, **8 of 10**.  
 99 Makbule: Because they took ten shots **and** have scored eight times.

101 Makbule: So they have yes, ehm, they had 20 times to shoot, so could shoot 20 times. And they only, so they had, so they didn't score four of them.

In #70 Makbule reads out her written answer in the speech bubble next to the bar of fifths. She adopts Kiran's mathematically more adequate construction for the relation of shots and strikes by using the prepositional sentence structure ("take ... shots and score ... of them"). Further in the process, Makbule also adopts the "of-construction", which had been introduced by the material in Task 5, in order to reason the fitting of the bar of tens to the results of the girls (#76). Considering Kiran's utterance of the fraction in #75, it can be assumed that Makbule purposefully links the fraction with its meaning-related conceptualization "8 of 10". In #99 and #101, when reasoning the fitting of the bar of tens and bar of twentieths, she once again uses the coordination "and" as well as the prepositional "of-structure". However, in her written products she constantly applies the mathematically preferred "of-structure". Thus, it can be assumed that thinking relatively as well as having lexical constructions for expressing relative frequency meaningfully anchored in her mental lexicon was successfully achieved for Makbule. It is assumed that Kiran's introduction of the sentence structure "take ... shots and score ... of them" was supportive for Makbule's conceptual and lexical learning pathway.

## Conclusion

To summarize, the empirical insights show how rich and demanding discourse practices can be initiated in small group settings by means of the design principles of macro-scaffolding and relating registers. The dual focus of the applied macro-scaffolding on the conceptual learning opportunities intertwined with language learning opportunities on the lexical level has to be emphasized as this builds the basis for the analysis of the initiated lexical learning processes. On the developmental level the presented design outcome thus helps to answer the question of how instruction can support students to learn vocabulary as they engage in mathematical reasoning (Moschkovich, 2013), here with a focus on relative frequencies. Moreover, the case of Makbule implies that offering and relating various mathematically intended lexical constructions could be supportive for becoming more proficient in the formal language of schooling. This can be implemented more prominently in the material by activities of reflecting and discussing concept-specific lexical means, which again would be an intertwined conceptual and lexical-discursive learning opportunity.

So far, analyses of learning processes on the lexical level are quite rare in mathematics education research. Applying the method of trace analysis (Prediger & Pöhler, 2015) reveals details of students' language production and development on the lexical level. For Makbule and Kiran differences with respect to which and how concept-specific lexical means have been activated and adopted became apparent. However, further insights into the processes of the other groups are necessary to ensure the first empirical results and formulated hypotheses.

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