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# Tracing mathematics-related belief change in teacher education programs

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*Understanding mathematics related belief change in teacher education programs has been a concern due to the information it presents about the effectiveness of program experiences. The present study investigated how preservice mathematics teachers' (PMTs) mathematics related beliefs changed through a teacher education program by implementing a belief scale to a cohort of PMTs 12 times during 6 consecutive semesters. Comparison of mean scores for different time periods showed that the cumulative effect of the teacher education program is more detectable than year-long or semester-long effects. While such implementation is likely to provide a perspective for monitoring belief change and possible effects of courses, it might also address that the contents of the courses should be connected to ensure the best cumulative effect of the programs.*

*Keywords: Mathematics related beliefs, preservice mathematics teachers, tracing beliefs.*

## Beliefs and teacher education programs

Training teachers with knowledge, skills, and disposition to practice the reform-oriented teaching has been a major goal for teacher education programs as teachers are the key to the success of the reforms aiming constructivist and student-centred mathematics instruction at schools (Handal & Herrington, 2003). Hence, initiating and strengthening mathematics related beliefs parallel to the reform movements have been a concern for teacher education programs (Raymond, 1997). This study aimed to identify possible key courses and experiences in teacher education programs that influence PMTs' beliefs by monitoring the changes in beliefs for three years in a four-year middle grades mathematics education program. The other aim is to gain perspective on the information that continuous monitoring of beliefs can provide. We adopted the identification of beliefs which Muis (2004) used for students' mathematics related beliefs and considered PMTs' beliefs as *availing* if they had the potential to help PMTs to achieve the reform-oriented goals of the teacher education program and middle school mathematics curriculum, and *nonavailing* otherwise.

Preservice teachers generally start teacher education programs by nonavailing mathematics-related beliefs (Szydlik, Szydlik, & Benson, 2003) for their learning in the program. Although programs aim to initiate and strengthen availing beliefs to reach program outcomes and teach through reform when PMTs become teachers, availing beliefs are initiated and developed to a limited extent (Clift & Brady, 2005). However, nonavailing beliefs are carried to the student teaching and teaching profession and they influence teachers' practices and implementation of reform principles in classrooms (Szydlik, et al., 2003). Therefore, teacher educators face the challenge of understanding PMTs' beliefs in the beginning of programs and organize tasks and experiences to develop and strengthen the availing beliefs (Swars, Smith, Smith, & Hart, 2009). This requires monitoring PMTs' beliefs throughout programs and the possible influence of program experiences.

Several studies investigated the effects of teacher education program experiences on PMTs' mathematics related beliefs mostly by emphasizing a single course or set of courses such as the methods courses and student teaching. These studies have shown that course experiences had limited effect in terms of the extent and the duration of the developed beliefs (Szydlik, et al., 2003). Longitudinal studies which documented changes in PMT beliefs through a set of courses are rare. Swars, et al. (2009) investigated 24 preservice elementary teachers' mathematics teaching and self-efficacy beliefs, mathematics anxiety and specialized content knowledge through 3 semesters of a 4-semester teacher preparation program. They implemented the belief instruments at the beginning and end of the 2<sup>nd</sup> semester (1<sup>st</sup> methods course), at the end of the 3<sup>rd</sup> (2<sup>nd</sup> methods course) and 4<sup>th</sup> (student teaching) semesters. The analysis showed that preservice teachers' scores significantly increased after the 1<sup>st</sup> methods course, decreased non-significantly at the end of the 2<sup>nd</sup> methods course, and significantly decreased at the end of student teaching. Yet, there was a significant increase from the 1<sup>st</sup> implementation to the 4<sup>th</sup> showing that preservice teachers gained more cognitively aligned beliefs (as targeted by the program) at the end of the 4 semesters.

The present study attempted to trace PMTs' beliefs through a teacher education program beginning from the semester they were enrolled in the initial pedagogical content knowledge courses (beginning of the 3<sup>rd</sup> semester) until they graduated (end of the 8<sup>th</sup> semester). PMT beliefs were monitored by implementing a belief instrument for 12 times in 6 consecutive semesters. In this paper, we report and discuss the changes based on timing of the implementation through the following research questions: (1) How do PMTs' mathematics related beliefs change through the 2<sup>nd</sup>, 3<sup>rd</sup>, and the 4<sup>th</sup> year of a 4-year mathematics teacher education program (a) in the long term (between academic years and throughout the three years); (b) in the short term (within the academic years)? (2) What kind of information does timing of belief instrument implementation offer about mathematics related belief change for teacher educators?

## **Method**

A longitudinal survey design was employed and data were collected from the same group of PMTs for 6 consecutive semesters to capture the possible changes in their mathematics related beliefs.

### **Participants and context**

Turkish education system is centralized at the national level with national curricula implemented at all grade levels at all contents. All students have to take a national examination at the end of high school to attend 4-year degree programs at universities including teacher education programs. The context of the study was a four-year middle grades (grades 6-8) mathematics teacher education program (EME) at a Turkish public university. EME program had three mathematics education faculty members at the time of the study and was placed under the Department of Elementary Education with 10 faculty members. The program had mathematics courses offered by the Department of Mathematics in the first four semesters. Mathematics education courses started in the 3<sup>rd</sup> semester and were offered by the program faculty and pedagogical courses were offered by the Department of Educational Sciences. The participants of the study were a total of 33 female and 10 male PMTs who started the EME program in 2006, referred here as the "cohort".

EME program started in 1998, was renewed in 2006 and the cohort was the first to study the renewed program. The changes in the EME programs were due to a major constructivist curriculum reform in

the national mathematics curriculum in Turkey in 2005. Previous EME program offered a mandatory minor degree in science education which was removed in the renewed program. School Experience course in the 2<sup>nd</sup> semester and Textbook Analysis course in the 8<sup>th</sup> semester were removed, and 1-semester Methods of Teaching Mathematics course was renewed as a 2-semester course, which allowed the dense content be covered in more time and depth. Methods of teaching course content was combined with curriculum issues content in a new course. Two new courses on research methods and nature of mathematical knowledge, and two statistics courses from the Department of Statistics were added to the renewed program. Mathematics courses, field experience courses in the 7<sup>th</sup> and 8<sup>th</sup> semesters and most pedagogical courses were maintained. The previous program experiences were based on constructivist approaches, however, the renewed program provided more opportunities for widened and deepened experiences for PMTs.

Two studies investigated the belief change in the previous EME program. Haser and Star (2009) conducted a cross-sectionally longitudinal study through interviews with 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> year PMTs. Their findings revealed that PMTs' mathematics related beliefs did not change much throughout the program. However, methods of teaching mathematics course provided PMTs a different understanding of teaching and learning mathematics, which they did not experience in their pre-college education. Haser and Doğan (2012) investigated how mathematics related beliefs differed among PMTs in different year levels. They first surveyed a total of 100 PMTs who were at the beginning of the 2<sup>nd</sup>, 3<sup>rd</sup> and the 4<sup>th</sup> year. Their analysis showed that PMTs who just started the 4<sup>th</sup> year in the program had significantly higher belief scores. Then, they focused on the effect of the general methods of teaching course in the 3<sup>rd</sup> year of the program on PMTs' beliefs.

The major changes in the EME program and the opportunity to monitor the 2006 cohort from the semester they started to take courses from the Department enabled us trace the possible influence of the renewed EME program experiences on PMTs' mathematics related beliefs.

### **Data collection instrument**

The belief scale used in this study was developed and used in the previous study (Haser & Doğan, 2012) in order to investigate Turkish PMTs' beliefs about the nature of mathematics and teaching and learning mathematics. Mathematics-related belief scale (MBS) included 38 five-point Likert type items asking PMTs' agreement with belief statements with responses ranging from totally disagree (1) to totally agree (5). Higher scores in MBS indicated existence of more availing mathematics related beliefs. Some of the MBS items are as follows: "Problem solving should be used as a teaching method within mathematics education", "The aim of mathematics education is to obtain correct answer by using the ways previously shown in the course" and "Visual and concrete materials are used in order to set up an environment for students to investigate their ideas". The Cronbach's alpha coefficient for MBS was calculated as .85 in the earlier study.

### **Data collection and analysis**

MBS was implemented for 12 times at the beginning and the end of each semester in the 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> year in one of the courses PMTs attended. However, the number of PMTs who took the MBS in each implementation varied due to the number of PMTs present at the implementation time. PMTs completed the MBS in about 15 minutes in each implementation.

Data were analysed to investigate both long-term and short-term changes in mean MBS scores, therefore, separate analyses were conducted. Long-term changes were investigated by comparing mean MBS scores of PMTs in the beginning of 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> year, and at the end of 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> year by one-way repeated measures ANOVA. Mean MBS scores in the beginning of 2<sup>nd</sup> year and at the end of 4<sup>th</sup> year were compared through paired-samples t-test. Short-term changes were investigated by comparing PMTs' mean MBS scores at the beginning and the end of each year and semester by paired-samples t-test. Cronbach's alpha coefficient was calculated for each implementation and ranged between .74 and .95.

## Results

The results are presented for long-term and short-term changes. First, for long-term changes, beginning of 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> year scores were compared to see the belief change based on the 2<sup>nd</sup> and 3<sup>rd</sup> year experiences. Then, end of 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> year scores were compared to see the change after 3<sup>rd</sup> and 4<sup>th</sup> year experiences. When there is a significant change in the MBS scores, it is interpreted as the effect of the EME program. MBS scores at the beginning of 2<sup>nd</sup> year and end of 4<sup>th</sup> year were compared to see the cumulative effect of the 3 years in the program. "Cumulative effect" refers to the effect of all program experiences until the mentioned implementation. Short-term changes were explored by comparing beginning of year/semester scores to end of year/semester scores. The aim was to detect possible influence of course experiences on PMT beliefs. The comparisons helped us discuss the information that the timing of the implementation might provide.

### Long-term changes

In order to identify possible change in PMTs' beliefs at the beginning of the academic years through the program a one-way repeated measures ANOVA was conducted to compare scores on MBS at the Time 1 (beginning of the 2<sup>nd</sup> year), Time 2 (beginning of the 3<sup>rd</sup> year) and Time 3 (beginning of the 4<sup>th</sup> year). A total of 19 PMTs were common at all Time 1, Time 2, and Time 3 of MBS implementation. The means and standard deviations are presented in Table 1.

Time (beginning of year)	N	M	SD
Time 1 (beginning of the 2 <sup>nd</sup> year)	19	3.81	.237
Time 2 (beginning of the 3 <sup>rd</sup> year)	19	4.00	.246
Time 3 (beginning of the 4 <sup>th</sup> year)	19	4.08	.318

**Table 1: Descriptive statistics for PMTs' MBS scores at the beginning of the 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> years**

There was a significant effect for time, [Wilk's Lambda = .550,  $F(2, 36) = 10.379$ ,  $p < .05$ , multivariate partial eta squared = .366]. Pairwise post-hoc comparisons with Bonferroni adjustment ( $p < .05$ ) showed that there was a significant mean difference between Time 1 and Time 2, and Time 1 and Time 3. The difference between Time 1 (beginning of 2<sup>nd</sup> year) and Time 2 (beginning of 3<sup>rd</sup> year) indicated that 2<sup>nd</sup> year experiences in the EME program had a significant effect on PMTs' mathematics related beliefs. Similarly, the difference between Time 1 (beginning of 2<sup>nd</sup> year) and Time 3 (beginning of 4<sup>th</sup> year) indicated that a possible cumulative of 2<sup>nd</sup> and 3<sup>rd</sup> year experiences in the EME program had a significant impact on PMTs' mathematics related beliefs.

We wanted to explore if the effect of the program differed for academic years through the program when the program experiences were rather recent for the PMTs by conducting another one-way repeated measures ANOVA to compare mean scores on MBS at the end of the academic years as Time 4 (end of the 2<sup>nd</sup> year), Time 5 (end of the 3<sup>rd</sup> year) and Time 6 (end of the 4<sup>th</sup> year). A total of 21 PMTs were common at all Time 4, Time 5, and Time 6 implementations of MBS. The means and standard deviations are presented in Table 2.

Time (end of year)	N	M	SD
Time 4 (end of the 2 <sup>nd</sup> year)	21	3.86	.197
Time 5 (end of the 3 <sup>rd</sup> year)	21	4.03	.251
Time 6 (end of the 4 <sup>th</sup> year)	21	4.01	.239

**Table 2: Descriptive statistics for PMTs' MBS scores at the end of the 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> years**

There was a significant effect for time [Wilk's Lambda = .543,  $F(2, 40) = 5.919$ ,  $p < .05$ , multivariate partial eta squared = .228]. Pairwise post-hoc comparisons with Bonferroni adjustment ( $p < .05$ ) showed that there was a significant difference between Time 4 and Time 5, and Time 4 and Time 6. The difference between Time 4 (end of 2<sup>nd</sup> year) and Time 5 (end of 3<sup>rd</sup> year) indicated that 3<sup>rd</sup> year experiences in the EME program had a significant impact on PMTs' mathematics related beliefs. Similarly, the difference between Time 4 (end of 2<sup>nd</sup> year) and Time 6 (end of 4<sup>th</sup> year) indicated that a possible cumulative of 3<sup>rd</sup> and 4<sup>th</sup> year experiences in the EME program had a significant effect on PMTs' mathematics related beliefs. There was no significant difference between Time 5 (end of 3<sup>rd</sup> year) and Time 6 (end of 4<sup>th</sup> year), which might indicate that the 4<sup>th</sup> year experiences did not have a significant effect on PMTs' beliefs. Indeed, mean MBS scores were slightly lower at Time 6.

The effect of the teacher education courses through the three years of the program was investigated by comparing the MBS scores between Time 1 (beginning of 2<sup>nd</sup> year) and Time 6 (end of 4<sup>th</sup> year) by a paired-samples t-test. A total of 25 PMTs were administered the MBS at Time 1 and Time 6. There was a statistically significant increase in mean MBS scores from the beginning of 2<sup>nd</sup> year ( $M = 3.76$ ,  $SD = .174$ ) to the end of 4<sup>th</sup> year ( $M = 4.05$ ,  $SD = .235$ ),  $t(24) = 5.868$ ,  $p < .001$  (two-tailed). The eta squared statistics (.59) indicated a very large effect size. This showed that a possible cumulative of 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> year experiences in the EME program had a significant effect on the mathematics-related beliefs of PMTs.

### Short-term changes

Three paired-samples t-tests were conducted to investigate the possible effects of year-long experiences on PMTs' mean MBS scores by comparing the beginning-of-year and end-of-year scores for each year. Table 3 presents paired-samples t-test results and the number of PMTs who were common for in both implementations of MBS for each year.

Year	N	Paired-samples t-test results
2	27	No significant difference between the beginning ( $M = 3.82$ , $SD = .256$ ) and end of 2 <sup>nd</sup> year ( $M = 3.88$ , $SD = .222$ ), $t(26) = 1.035$ , $p > .05$ (two-tailed).
3	25	<u>Statistically significant increase</u> from the beginning ( $M = 3.93$ , $SD = .230$ ) to the end of 3 <sup>rd</sup> year ( $M = 4.07$ , $SD = .262$ ), $t(24) = 2.755$ , $p < .05$ (two-tailed). The eta squared statistic (.24) indicated a large effect size.
4	27	No significant difference between the beginning ( $M = 4.07$ , $SD = .302$ ) and end of 4 <sup>th</sup> year ( $M = 4.05$ , $SD = .240$ ), $t(26) = -.381$ , $p > .05$ (two-tailed).

**Table 3: Paired-samples t-test results for MBS scores and the number of PMTs for each year**

Comparisons of beginning-of-year and end-of-year mean MBS scores showed that only 3<sup>rd</sup> year experiences had a significant effect on PMTs' MBS scores. The scores did not significantly change from the beginning to the end of the 2<sup>nd</sup> and 4<sup>th</sup> year of the program. However, the mean MBS scores increased in each implementation until the beginning of the 4<sup>th</sup> year.

A series of paired-samples t-tests were conducted to investigate the possible effects of semester experiences by comparing the PMTs' mean MBS scores at the beginning and end of the semester for each semester. Table 4 presents paired-samples t-test results and the number of PMTs who were common in both implementations of MBS for each semester.

Semester	N	Paired-samples t-test results
3	29	<u>Statistically significant increase</u> from the beginning ( $M = 3.83$ , $SD = .216$ ) to the end of the semester ( $M = 3.95$ , $SD = .246$ ), $t(28) = 3.027$ , $p < .01$ (two-tailed). The eta squared statistic (.25) indicated a large effect size.
4	26	No significant difference between the beginning ( $M = 3.86$ , $SD = .233$ ) and end of the semester ( $M = 3.92$ , $SD = .211$ ), $t(25) = -1.849$ , $p > .05$ (two-tailed).
5	28	No significant difference between the beginning ( $M = 3.96$ , $SD = .225$ ) and end of the semester ( $M = 4.00$ , $SD = .259$ ), $t(27) = -.973$ , $p > .05$ (two-tailed).
6	32	No significant difference between the beginning ( $M = 4.02$ , $SD = .230$ ) and end of the semester ( $M = 4.05$ , $SD = .263$ ), $t(31) = -1.041$ , $p > .05$ (two-tailed).
7	32	No significant difference between the beginning ( $M = 4.07$ , $SD = .284$ ) and end of the semester ( $M = 4.05$ , $SD = .271$ ), $t(31) = -.747$ , $p > .05$ (two-tailed).
8	26	No significant difference between the beginning ( $M = 4.06$ , $SD = .253$ ) and end of the semester ( $M = 4.03$ , $SD = .244$ ), $t(25) = .656$ , $p > .05$ (two-tailed).

**Table 4: Paired-samples t-test results for MBS scores and the number of PMTs for each semester**

Results showed that semester-long changes in mean MBS scores were not significant except for the 3<sup>rd</sup> semester, while mean MBS scores generally increased at the beginning each semester.

## Summary of the analyses and information given by the timing of implementation

Analyses showed that there was a general trend of increase from the beginning of the 2<sup>nd</sup> year to the end of the 4<sup>th</sup> year of the EME program with slight decrease in the 4<sup>th</sup> year. This increase was significant for long-term comparisons and showed that EME program courses and experiences seemed to help PMTs develop or strengthen availing beliefs through the years.

When the analyses focused on short-term differences, the increase in mean MBS scores was not statistically significant in most of the comparisons. The comparison of mean MBS scores for the beginning and end of each academic year revealed significant increase only for the 3<sup>rd</sup> year of the program. On the other hand, semester-long comparisons of mean MBS scores addressed significant results only for the 3<sup>rd</sup> semester. These results showed that belief change might not always be significantly detectable in shorter periods. PMTs might not fully internalize course experiences only in one semester. The significance of the 3<sup>rd</sup> semester comparisons might show us that the first course on mathematics education was effective on beliefs, probably because it included methods of teaching PMTs had never experienced. The significance of the 3<sup>rd</sup> year comparisons pointed the effect of the 2-semester Methods of Mathematics Teaching courses, whose effects on preservice teachers' beliefs have often been investigated in the literature. Indeed, studies conducted in the previous program showed that courses on methods of teaching and mathematics teaching have influenced PMTs' beliefs in a more availing way (Haser & Doğan, 2012; Haser & Star, 2009). Although the decrease in MBS scores from the beginning of the 4<sup>th</sup> year to the end was not significant, it might signal for the rather undesired effect of the student teaching experiences on soon-to-be-teacher PMTs' beliefs due to the reality of classroom environment, differences in students, and lack of support from program instructors at the classrooms (Swars et al., 2009). These findings showed that detecting belief change for shorter time periods provided rather limited knowledge, but it raised issues about the effects of the program experiences for semesters or years.

The analyses reported here were conducted based on the number of common PMTs in the analysed implementations. When we compared the mean MBS scores at each point of time to the mean scores in the repeated measures ANOVA (Table 1 and Table 2) as well as the paired-samples t-tests (Table 3 and Table 4), we observed minor mean score differences between the mean scores of the PMTs who were common across the implementations and all the PMTs who were administered the survey at that implementation. These results are not given here due to space limitation. Hence, we concluded that missing cases did not impact the results of the study. Yet, it should be kept in mind that the analyses were not conducted with all PMTs for all implementations.

In summary, the results of the analyses showed that change in PMTs' mathematics-related beliefs were more detectable when the change was investigated in the long-term, throughout the program. The nature of the increase in MBS mean scores suggested a cumulative effect of the program as PMTs progressed. The short-term investigations did not give much significant results, yet they might give us clue about how courses might influence PMTs' beliefs. The significant results have addressed the possible influences of certain courses that should be investigated in detail.

## Discussion

The long-term and short-term change analyses results showed that while PMTs seemed to benefit from program experiences and develop more availing beliefs through the years in the program, 3<sup>rd</sup>

year experiences seemed to contribute to the belief change the most. Course experiences were not investigated in-depth in this study, therefore how PMTs made sense of these experiences and how these experiences helped them in forming rather availing beliefs were remained unexplored in this study.

The results suggest that teacher educators should investigate change in beliefs through the teacher education programs in different ways. The first teaching related course in the program, methods of mathematics teaching courses, and student teaching courses might have relatively more weight (either positive or negative) within the cumulative effect of the teacher education programs. Considering the significant cumulative effect of the program, it is possible that this cumulative effect might get stronger when the program experiences are meaningfully related to each other to support the availing beliefs and related practices.

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