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Dimension Reduction for Shape Optimization

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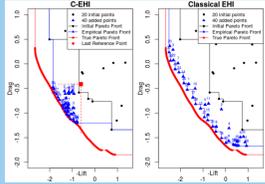
Context and past work

Multi-objective optimization of high dimensional systems

$$\min_{\mathbf{x} \in X \subset \mathbb{R}^d} (f_1(\mathbf{x}), \dots, f_m(\mathbf{x}))$$

Very tiny budget (≈ 100 - 200 evaluations), many objectives ($m \approx 6$ - 8) \Rightarrow impossible for classical MO-EGO approaches to uncover the Pareto Front (growing size of \mathcal{P}_y with m)

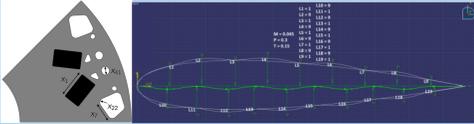
\Rightarrow Target well-chosen parts of \mathcal{P}_y [1]



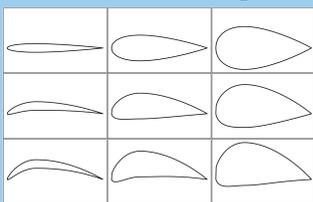
But what when d is large ($\gtrsim 50$)?

CAD parameters and shapes

- $\mathbf{x} \in \mathbb{R}^d$ design parameters



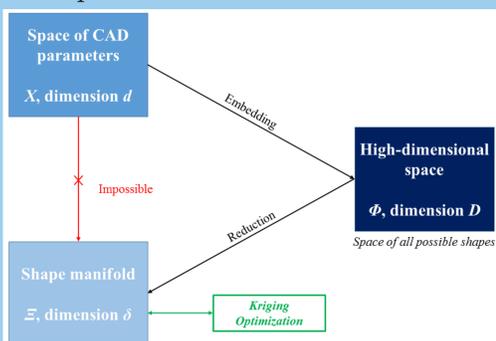
- Associated shapes $\Omega_{\mathbf{x}}$ approximately live in a $\delta < d$ dimensional space



- Computation time of $\Omega_{\mathbf{x}}$ and $\phi(\mathbf{x})$ negligible compared to evaluation of $\mathbf{f}(\mathbf{x})$

High-dimensional mapping

- Analyze many possible shapes $\Omega := \{\Omega_{\mathbf{x}}, \mathbf{x} \in X\}$ in a high-dimensional space $\Phi \subset \mathbb{R}^D$, $d \ll D$
- Retrieve the δ dimensional manifold embedded in \mathbb{R}^D
- Build the kriging surrogates and perform the optimization in this manifold

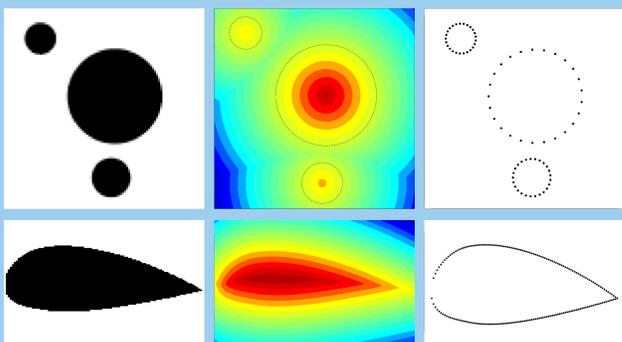


High dimensional representation of $\Omega_{\mathbf{x}}$

$$\phi: X \rightarrow \Phi$$

$$\mathbf{x} \mapsto \phi(\mathbf{x})$$

- Characteristic function $\chi_{\Omega_{\mathbf{x}}}$
- Signed distance to $\partial\Omega_{\mathbf{x}}$
- Discretization of $\partial\Omega_{\mathbf{x}}$



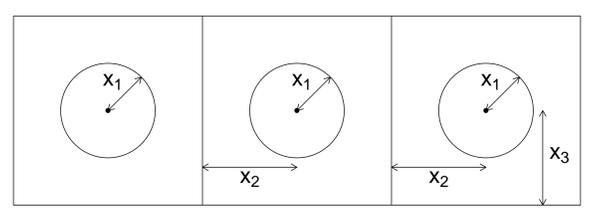
Choice of mapping ϕ is critical for identifying Ξ

Reduction by PCA in shape space

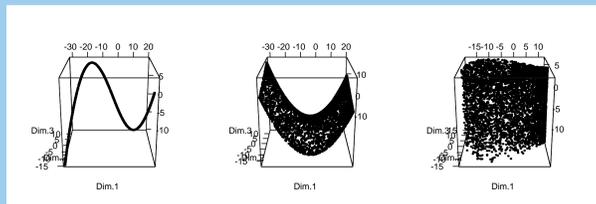
Following the work of [2, 3, 4]

- Draw N designs $\mathbf{x}^{(i)} \in X$ and compute $\Phi := (\phi(\mathbf{x}^{(i)}))_{i=1, \dots, N}^T$
- Analyze the variety of shapes by applying a PCA on Φ : the eigenvectors of $\Phi^T \Phi$, \mathbf{v}^j form a shape basis, $\Phi^{(i)} = \bar{\Phi} + \sum_{j=1}^D \alpha_j^{(i)} \mathbf{v}^j$
- Instead of $\mathbf{x}^{(i)}$, work with its eigenbasis coordinates: $\alpha^{(i)} := (\alpha_1^{(i)}, \dots, \alpha_D^{(i)})^T$
- $\alpha^{(i)}$, $i = 1, \dots, N$ form a δ -dimensional manifold

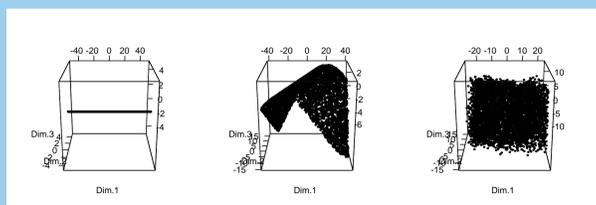
Tests cases



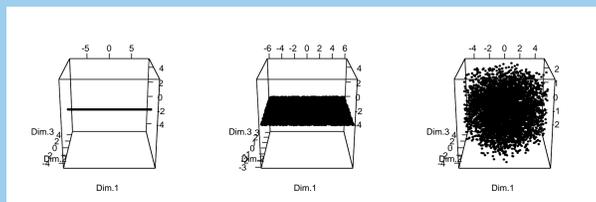
Characteristic function



Signed distance

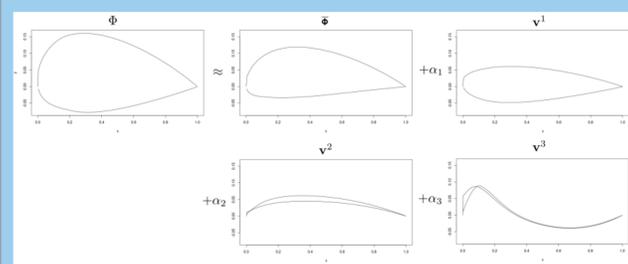


Discretization



- Dimension reduction: $\Phi^{(i)}$ approximated by retaining δ principal components from their eigenvalue contribution

$$\Phi^{(i)} \approx \bar{\Phi} + \sum_{j=1}^{\delta} \alpha_j^{(i)} \mathbf{v}^j$$

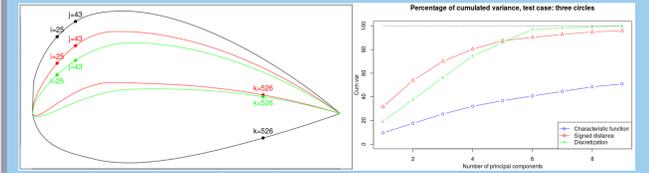


References

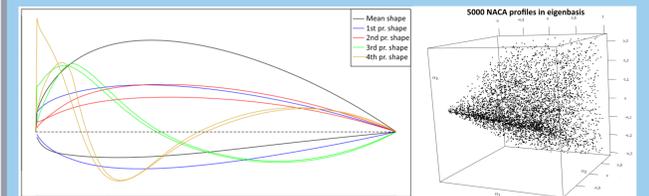
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Kriging in reduced basis

- ϕ = shape discretization: most efficient mapping to retrieve δ

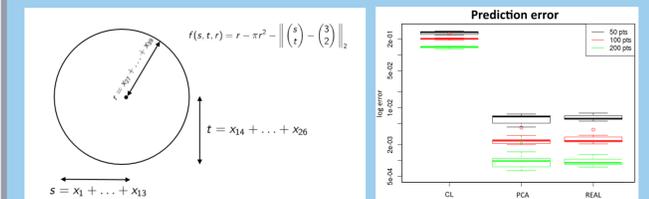


- Principal axis: eigenshapes

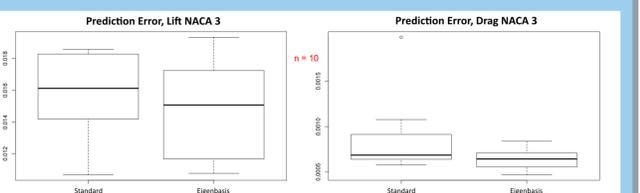
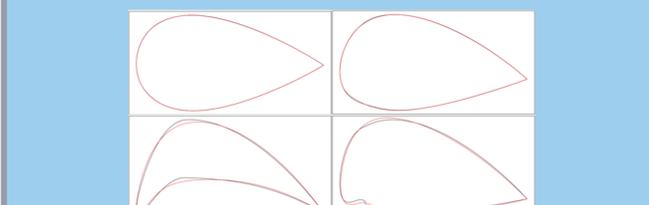


Kriging on principal components: test cases

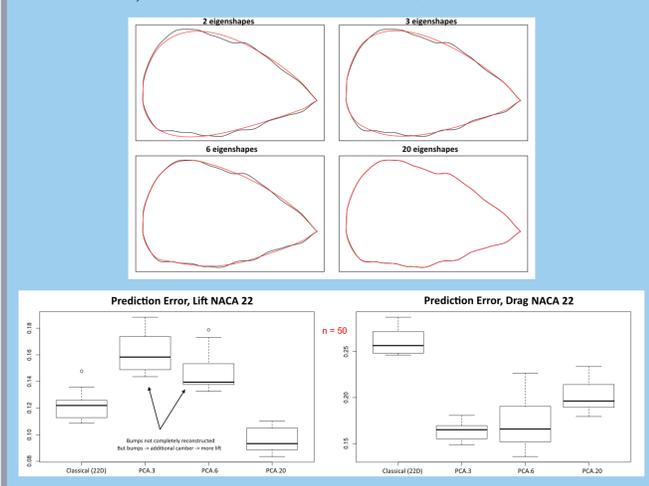
(a) Over-parameterized circle, $\mathbf{x} \in \mathbb{R}^{39}$
 $\rightarrow \delta = 3$



(b) Meta NACA 3 dimensions, $\mathbf{x} \in \mathbb{R}^3$
 $\rightarrow \delta = 3$



(c) Meta NACA 22 dimensions (bumpy airfoil), $\mathbf{x} \in \mathbb{R}^{22} \rightarrow \delta = 3, 6, 20$



Conclusions

- Lower/true dimension retrieved
- Enhanced predictability in the reduced eigenbasis
- Shape discretization: better than characteristic function and signed distance

Further work

- Invariance of shape discretization under some permutations (points indexing, multiple shapes)
- Use this framework to perform (multi-objective) optimization