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# TWG06: Applications and modelling

# Introduction to the papers of TWG06: Applications and modelling

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## Introduction

Mathematical modelling and its teaching at various educational levels are widely accepted all over the world. There is consensus of the need to integrate mathematical modelling and applications in curricula and this has already taken place in many European countries. However, there is no unanimity on how to integrate mathematical modelling and applications into the processes of teaching-and-learning mathematics. In addition, there is little secure empirical knowledge available on how to implement efficiently the necessary new learning environments. In its discussions at CERME 10, the thematic working group TWG 6 on modelling and applications aimed to contribute answers to these open questions and hence further develop the work from previous ERME conferences. The contributions discussed at the congress are characterized by a strong and fruitful diversity in the research questions considered, the school levels addressed and the theoretical approaches taken. On the whole, the papers address theoretical, methodological, empirical or developmental research on the teaching and learning of applications and modelling. The group involved 35 participants from 16 countries – participants were from Europe, and also from South and Central America. A total of 20 papers and four posters were presented and discussed during the working group sessions.

In the following, we describe all presented papers, although not all have been submitted for the proceedings, grouped around eight comprehensive themes, which refer to current issues within the teaching and learning of mathematical modelling.

As a first important theme, we identify the *interplay between disciplines into modelling activities* referring to the specificities of interdisciplinary modelling activities, especially in engineering teaching. The second theme relates to the *connection of the problem solving perspective and mathematical modelling* and to the development of problem solving strategies and competences, when students work in groups and develop individual competences. The third theme covers *developing modelling strategies and competences*, for example theoretical and empirical work focused on the analysis and use of heuristic strategies adopted by teachers and/or students which illuminates strategies used to foster students' performance when solving modelling problems. The fourth theme refers to the *tools and methodologies used to analyse modelling processes*, namely

studies focusing on elaborating specific methodologies for analysing and evaluating modelling practices. The fifth theme focuses on *teachers' beliefs in relation to the teaching of mathematical modelling*, including, for example, research on the role of teachers to foster modelling practices. The sixth theme covers *teachers' interventions in mathematical modelling*. The seventh theme refers to *experimental materials and technology in modelling*, which covers two topics, a first primarily focused on the role of the auxiliary material and its impact on modelling and a second concerned with how to combine different resources with technology in concept development by means of real word contexts. The eighth theme refers to the *assessment of modelling practices*.

## Overarching themes

### Modelling and interdisciplinary teaching

This theme focuses on the interplay between disciplines in modelling activities. The first study by *Borromeo* and *Mousoulides* describes theoretical reflections about the differences and similarities between mathematical modelling and interdisciplinary mathematics education. Besides its focus on underlying similarities and differences, this paper provides some examples of projects that make connections explicit and how these may be useful for teachers while discussing modelling as a means for solving interdisciplinary problems. Within this thematic strand, other empirical works were presented based on particular case studies involving interdisciplinary projects with mathematical modelling as a central issue. In particular, *Brake* and *Lantau* described a pilot study of an interdisciplinary project used with experienced students of grade 12 based on modelling segways which supported them to build models based on linear systems of differential equations. Furthermore, *Sala*, *Font*, *Barquero* and *Giménez* reported on the design and analysis of an implemented interdisciplinary project, where mathematical modelling was embedded into an archaeological context. The study showed how the complementarity between two subjects (history and mathematics) can be an important tool in supporting modelling and inquiry. Besides the potential of designing interdisciplinary modelling situations, such as those presented in these papers, a couple of important question remain unresolved:

- What are the specificities of interdisciplinary mathematical modelling in relation to mathematical modelling more generally?
- How is it possible to manage the interplay between mathematics, mathematical modelling and non-mathematical knowledge to enrich teaching practices in the learning of mathematics?
- Can some analytical tools widespread in the research frame of modelling, such as the modelling cycle, be adapted to analyse mathematical modelling in interdisciplinary contexts? And, if yes, how?
- What relationships exist between the mathematical modelling cycle and the inquiry process?
- How do we best integrate different disciplines in developing modelling tasks?

Another important topic referred to modelling in the particular case of teaching of engineering. The papers from *Romo*, *Tolentino* and *Romo-Vázquez*, and *Siero*, *Romo* and *Abundez* reported intentions to design and analyse modelling activities for the mathematical education of teachers. Both studies were based on the Anthropological Theory of Didactics. They focused on analysing the roles and interplay of institutions in the educational programmes of engineers and on the institutional

conditions that the design of the study and the research activities expose in the teaching of modelling in such contexts. Some of the questions discussed were:

- How can different institutions become involved in mathematical modelling in engineering education?
- How can practical knowledge be grounded on mathematical knowledge?
- How can the steps of an engineering project (design, mathematical model, prototype) be described? How are these steps interrelated? How are they connected to the modelling cycle?

### **Connection of the problem-solving perspective and mathematical modelling**

The second theme refers to the connection between problem-solving activities and mathematical modelling. First, *Clohessy* and *Johnson* examined the relationships between the problem-solving performance of small groups with that of individual students in order to identify the influence of group work as an effective instructional strategy when teaching problem solving. Second, *Karatas*, *Soyak* and *Alp* presented an investigation about mathematical non-routine problem solving processes of fifth grade students in small groups. Their study aimed to determine problem solving behaviours within different episodes of problem solving. As both papers focused on the description and measurement of problem solving competences in small groups and individually, some common questions appeared:

- How might individual competences improve when students are participating in a group?
- What instruments can we use to measure the improvement of individual competences before, during and after mathematical modelling processes? Which are the most valuable indicators we might use to measure these changes?
- What are the cultural aspects that have most impact on planning the implementation of problem-solving activities?

### **Developing modelling strategies and competences**

This third theme concerns the use of heuristic strategies to support modelling practices. The paper from *Stender* and *Kaiser* presented a study on the usage of heuristic strategies by students in school within modelling activities and the promotion of strategic help provided by academic tutors, who guide the modelling activity of the school students. The paper of *Schmelzer* and *Schukajlow* focused on the relationship between reading comprehension and mathematical modelling. The study identified strategies to help learners comprehend a modelling problem and described ways these strategies might be implemented in the classroom. The following important questions emerged from discussions of the working group:

- How far can heuristic strategies developed in the frame of problem solving be transferred to mathematical modelling?
- Can their identification and characterisation be helpful in supporting teachers' strategic interventions?
- How, in different teacher education programmes, may these heuristics be made explicit? If we make them explicit, do we risk narrowing the radius of action of teachers when guiding modelling activities?

- What are the effects of text length and superfluous elements on reading comprehension in modelling problems?
- How can we prepare teachers to foster students' reading comprehension in modelling?

A closely related topic was that focused on the metacognition of modelling competencies as an essential part of developing competence in modelling. *Vorhölter, Krüger* and *Wendt* presented their results from a pilot study about the identification and measurement of metacognitive modelling competencies in small groups when working on modelling activities. The following relevant questions were discussed:

- How might we best define metacognitive competencies? How can their characterisation be used to evaluate students' development of competencies?
- How is it possible to separate the individual progress of metacognitive competences from that of the collective group?
- How might a detailed evaluation of students' teamwork enrich the understanding of metacognitive competences?

### **Analysis of modelling processes**

This fourth theme addressed tools and methodologies used to analyse modelling processes. On the one hand, *Barquero, Monreal* and *Ruíz-Munzón* presented a study, proposed within the frame of the Anthropological Theory of Didactic, about how to forecast the increasing number of Facebook users. The analysis of the implemented research path was based on three dialectics essential for mathematical modelling: the questions-answers dialectic, that of the media-milieu and that of individual-collective dynamics. On the other hand, the paper from *Delgadillo, Viola* and *Vivier* presented an analysis of a modelling task in the context of pre-service teacher education based on the theory of the Mathematical Working Space. In this latter study, the modelling cycle was used as an essential tool to analyse the personal Mathematical Working Space of students solving a modelling task. Some questions appeared in the discussion of both papers:

- Which different dimensions, or levels, of analysis may be taken into account when analysing mathematical modelling practices?
- Which are the most valuable observables (depending on the focus of study)?
- Up to what point can tools for analysis be used as tools to help with designing mathematical activities? In which more general approaches to task-design do they appear?
- How do we carry out analysis that takes into account both individual activity and collective interactions in modelling processes?

### **Teachers' beliefs on teaching modelling**

This theme refers to a study on teachers' beliefs on modelling tasks. The paper from *Ramirez* explored mathematics teachers' beliefs about teaching and learning mathematical modelling and about modelling itself. It presented an exploratory study of responses from teachers collected in an online questionnaire related to the characteristics of modelling practices. Several questions came up about the relation of teachers' beliefs with their experience and knowledge about modelling:

- While analysing beliefs many aspects have to be taken into account, which are difficult to separate. Thus, what is the relation between ideas, beliefs, previous experiences, and modelling competences?
- The role of teachers' beliefs in connection with teachers' knowledge is examined. Therefore, the question arises: How do we take into account teachers' knowledge about modelling?
- The role of teachers as individuals within institutions is studied. Thus, which cultural and school conditions influence their opinions?

### **Teachers' interventions in teaching modelling**

This section refers to teachers' interventions in teaching mathematical modelling. *Ferrando, Donat, Diago* and *Puig* presented an analysis of the different kinds of interventions teachers made during a project in which students worked on a modelling task about the intensity of sound distribution throughout a classroom. The study aimed to identify the influence of such interventions on students' learning opportunities. The following questions arose:

- How do available resources and means influence the openness of the task and students' possible responses?
- Who validates the final answer in a project?

### **Experimental materials and technology in modelling**

This theme focused on the role and use of experimental materials and of technology. Two topics were dealt with. The first focused on the role of auxiliary material and its impact on modelling and, the second concerned the combination of different resources with technology to assist concept development by means of real-world contexts. *Guerrero-Ortiz, Mena* and *Morales* discussed how the handling of auxiliary material can favour knowledge transfer between real world situations and mathematical models. The research, which was conducted within pre-service teacher education in Chile, presented insights into the design of modelling tasks and the affordances of auxiliary materials in supporting modelling. *Carreira* and *Baioa* base their research on an episode of a modelling activity with grade 9 students which aimed to reflect on the authenticity of the modelling task and to examine how students used experimental work to help them succeed in modelling activities. Some of the matters discussed were:

- How can auxiliary material support the learning process? Is it necessary to support students in its use or is it self-explanatory?
- Does the auxiliary material simulate the real processes taking place in the real setting?
- How far are problems authentic or meaningful to students?
- To what extent can we include the way things are really done in the real world in modelling tasks for students?

Regarding the second topic, *Karimianzade* and *Rafiepour* presented a study about the introduction of decimal numbers. The study showed how different resources were introduced and how experimental work helped fifth grade students to develop their knowledge and understanding of decimal numbers in the context of measurement. *Lieban* and *Lavicza* reported about students' work in using dynamic geometry systems in a geometric modelling situation. They suggested the use of some new manipulative resources together with digital applets that progressively enrich the development of the modelling process by students. Several ideas were debated around the questions:

- Which alternative ways exist to introduce decimal or rational numbers taking into account the introduction of standard units?
- How can we produce computer simulations of physical models?
- What are the goals for the different people (students, teachers and researchers) involved in creating GeoGebra models?

### **Assessment of mathematical modelling**

This last theme refers to the assessment of mathematical modelling. The paper of *Greefrath, Siller* and *Ludwig* analysed the official school leaving examination in Germany allowing university entrance, the so-called Abitur examination, which is supposed to contain elements that examine mathematical modelling. For this purpose, they based their analysis on certain criteria: reference to reality, relevance, authenticity, openness and partial competence of modelling, to analyse the potential of problems included in the official examination. In addition to the difficulties of deciding how to evaluate modelling practices in these kinds of official examinations, several questions were discussed:

- Which criteria can be used to describe good examination questions for modelling?
- Can all the sub-competences of modelling be assessed within examination tasks?
- How can we design school examination tasks that cover various aspects and goals that include mathematical modelling?

Within this strand, *Ärlebäck* and *Albarracín* proposed an analysis of various definitions of Fermi problems from a modelling perspective. They focused on analysing how the definition and descriptions of Fermi problems in the literature align with different perspectives on modelling. They also discussed how far Fermi problems and modelling are connected and how strongly that connection is influenced by the definition of Fermi problems.

### **Concluding remarks and perspectives**

The overarching themes tackled in TWG 6 show the variety of research questions the papers dealt with, for example concerning the theoretical frameworks used or the underlying perspectives on the teaching and learning of mathematical modelling (Kaiser & Sriraman, 2006, Kaiser et al., 2007). Furthermore, the educational levels involved span from primary to tertiary education.

In their analysis of the development of the teaching and learning of mathematical modelling over the last decades, Cai and colleagues (2014) introduce five perspectives, which can be helpful in order to identify the progress made during ERME conferences and potential future development.

The first perspective, the mathematical perspective, describes the differences between modelling at practitioner level and at school level. However, industrial examples can play a powerful role in education, because they are authentic and of varying complexity. There is a long tradition of discussing examples from engineering education in TWG 6 at various ERME conferences. In addition, the relation to other disciplines has always played an important role and needs to be mentioned under this perspective.

The second perspective, the cognitive perspective, focuses on students' cognitive processes when modelling, and cognitive barriers, when students work through the modelling cycle. Cognitively oriented analyses have played a prominent role in many sessions of the modelling group at previous



conferences. At CERME 10, this aspect can be found within two thematic strands, that on developing modelling strategies and competences and also that on the analysis of modelling problems. The discussions there broadened our focus on the cognitive perspective and helped us develop further our thinking.

The third perspective, the curricular perspective, refers to the inclusion of mathematical modelling in the curricula. This question was addressed in all meetings at previous conferences and is addressed in nearly all thematic strands of TWG 6 at CERME 10 and highlights the significance and urgency of this theme.

The fourth perspective, the instructional perspective, claims the necessity of high quality modelling education in order to promote effective learning. The question on how to implement effective modelling environments is a hot topic that has persisted for decades and was addressed at CERME 10 within various strands of TWG 6, especially within the themes that addressed experimental materials and technology in modelling and teacher interventions.

As a final and fifth perspective teacher education and teachers' activities in school are addressed, because of the obvious necessity of preparing pre-service teachers for the teaching of mathematical modelling, although the importance of this topic at the various sessions of the Applications and Modelling TWG at previous ERME conferences was less prominent than at this. At CERME 10, teachers and their role in teaching as well as teacher education played a prominent role within the work of the Applications and Modelling TWG. This was integrated into various themes such as modelling strategies, teacher interventions and teachers' beliefs. This shift shows a clear further development of the discussions and the work of TWG 6 and needs to be fostered and broadened. Teachers and their education are the key for the effective and efficient integration of mathematical modelling into mathematical education at various levels.

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