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Levels of analysis of a mathematical modelling activity: Beyond the questions-answers dialectic

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This paper presents the a posteriori analysis of an study and research path (SRP) on comparing reality versus forecasts of Facebook users, which appears as a teaching and learning proposal for mathematical modelling. We present the main elements of the SRP that have been designed through a virtual platform developed in the frame of the European project MCSquared and experienced in a first-year course at university in management sciences degrees. The three-layer analysis we present is based on three main study dialectics: the questions-answers, media-milieu and individual-collective dialectics, which are central for an SRP and for mathematical modelling. In particular, we focus our a posteriori analysis on how these three dialectics were fostered and which are the main weaknesses as well as strengths of the SRP experienced.

Keywords: Mathematical modelling, study and research paths, dialectics, university level.

Introduction: The SRP as teaching proposal for mathematical modelling

During the last decades, researchers and practitioners agree that teaching should not be focused only on the formal transmission of knowledge, but also should provide students of the tools for enquiring into the study of real phenomena and integrate mathematics as an essential modelling tool. It is thus important to foster a change of the school paradigm, with new functionalities to mathematical knowledge, novel responsibilities to teachers and students, different ways of questioning mathematical knowledge, that is, moving from a school paradigm which most of the time focused on introducing students to already built mathematical knowledge devoid of its rationale to a paradigm of questioning it, ‘questioning the world’ in the words of Chevallard (2015). In the particular case of the research on modelling and their applications and on inquiry-based approaches some big steps have been made showing how, under certain suitable conditions in different educational levels and curricular frames, modelling activities may be successfully put into practice (Artigue & Blomhøj, 2013; Burkhardt, 2008; among others). Hence, the dissemination and long-term survival of activities based on modelling, enquiring and other innovating proposals are one of the main challenges for Mathematical Education. Therefore, to support and analyse any kind of alternative teaching proposal, researchers need reference models that allow them to analyse and evaluate the impact that these innovative teaching practices have on the way mathematics is conceived, on the nature of the didactic systems and milieus emerged through these practices, and on the conditions and constraints that help or hinder the viability of these practices.

In our research, developed in the framework of the anthropological theory of the didactic, we bet on the use of the study and research paths (SRP) as epistemological and didactic model (Chevallard, 2015; Winslow et al., 2013) to face the problem of moving towards a functional teaching of mathematics and, particularly, where mathematics are conceived as a modelling tool for the study of problematic questions.

Traits and levels of analysis of an SRP

According to Barquero and Bosch (2015), the starting point of an SRP should be a ‘lively’ question of real interest for the community of study (students and teacher/s). The study of Q_0 , called the *generating question*, evolves and opens many other *derived questions* Q_1, Q_2, \dots, Q_n . The continuous looking for answers to Q_0 (and to its derivative questions) is the main purpose of the study and an end in itself. As a result, the study of Q_0 and its derived questions Q_i leads to successive temporary answers A_i that can be helpful in elaborating a final response R^\heartsuit to Q_0 . These first characteristics can be associated to the first level of analysis of the SRP that we here consider, it consists in the dialectic establishing between the questions posed and the likely answers appearing (*questions-answers dialectic*) which also provide the basic structure of an SRP to be implemented and to be enriched after each implementation. This first layer can be linked to the *cronogenesis* dimension of the teaching and learning practices, referring to the evolution of questions to be faced and the necessary knowledge to be used.

Another central dimension for an SRP is the *media-milieu dialectic*, which constitutes the second level of analysis. As described in the aforementioned investigations, the implementation of an SRP can only be carried out if the students have some pre-established responses accessible through the different means of communication and diffusion (that is, the *media*), to elaborate the successive provisional answers A_i . These *media* are any source of information, such as: textbooks, treatises, research articles, class notes, or the teacher acting as main media. However, the answers provided are constructions that have been elaborated to provide answers to questions that are different to the ones that may be put forward throughout the mathematical modelling process. Thus they have to be de- and re-constructed according to the new needs. Other types of *milieus* will therefore be necessary to test the validity and appropriateness of these answers. This second level of analysis put attention to the *mesogenesis*, that is, the evolution of the experimental milieu. Finally, we may consider the collective dimension of the research and study of questions in an SRP. This third dimension focuses on the roles and responsibilities that, far from the traditional didactic contract, students and teachers may assume in experiencing an SRP and how the individual work is shared, transferred and agreed with the wider community, and vice versa. We will denote this third level as the *individual-collective dimension*, which refers to the *topogenesis*.

Didactic analysis of a mathematical modelling process: The case of an SRP about comparing reality against forecast

We focus on analysing the case of an SRP on *comparing forecasts against reality in the case of Facebook users’ evolution*. This SRP was designed in the frame of the European project MCSquared (<http://www.mc2-project.eu>), the goal of which is the design of innovating teaching proposals (the so-called c-book units) to foster creative mathematical thinking. This c-unit in particular has been produced by a group of five multiple background designers: 2 Maths Education researchers, 2 university lecturers, who were then in charge of its implementation, and an expert on modelling in the field of Operations Research, which enrich a lot the way to structure the teaching sequence to prompt mathematical modelling. The teaching proposal was made in the virtual socio-technical environment developed along the project, the c-book that integrates some narratives with several applets of different factories of different educational technologies. Next we present the details about the design of the SRP experienced, combining the virtual environment offered by the c-unit with face-

to-face sessions during the winter term of the academic year 2015-16 with first-year students of Business Administration Degree and of Innovation Management (BAIM), all from the ‘Escola Superior de Ciències Socials i de l’Empresa-Tecnocampus’, Pompeu Fabra University.

The initial situation starts from a real news about a research developed by Princeton University in 2014, in which it was predicted that Facebook would lose the 80% of its users before 2017. Hence, the initial question Q_0 presented to students is about: *Can these forecasts be true? How can we model and fit real data about Facebook users’ evolution to provide our forecast the short- and long-term evolution of the social network? How can we validate Princeton conclusions?* The experimentation was structured in three interconnected phases linked to the generating question Q_0 , building up the a priori design of the SRP, then reflected in the design of the c-book unit. A first phase that focuses on the open research of real data about Facebook users, a second one focused on which mathematical models (mainly based on elementary functions) can provide a good fitting to real data, and a third one about the use of these models to provide the short-, medium- and long-term forecasts of Facebook users and about how to decide about best and most reliable model. The students, working in ‘consultant teams’ of 3-4 people, got the order from MS2 Consulting (‘Mathematical Solutions Squared’) previously described as Q_0 and they were asked to deliver a final report by the end of their work as an oral presentation as response to the order. The implementation combined face-to-face sessions in the teaching device called ‘Math modelling workshop’ (in a total of six 90-minuts weekly sessions) for the miss-in-common of the junior consultant teams’ partial reports, with work out of the classroom. The guiding c-unit that gives the workshop support was ready before starting in order to let the different teams attach their answers, pose new questions by providing an interacting device (as chats or shared spread-sheets) and use some applets designed especially to get through the different phases of the SRP. Next we sketch how different dialectics were prompted by both: (a) the c-unit design (by its initial design but also by the different changes that were introduced according to students’ requirements: new questions and answers not envisioned, new media required, etc.) and (b) the didactic gestures and devices to manage its implementation.

The questions-answers dialectics

We can visualize the first level of description of the SRP designed and experienced as an arborescence of the *questions* that were proposed and faced and the *answers* foreseen and appeared. This questions-answers structure constitutes a first layer of analysis of the process designed and of the trajectories followed in the implementation of the SRP. As introduced above the generating questions Q_0 of the SRP on comparing forecasts against reality in the case of Facebook users’ evolution was broken into three main derived questions (see Figure 1 for the SRP questions’ organization), which guided the successive phases of its implementation. A first phase that focuses on the open research of real data about Facebook users, a second one focused on which mathematical models (mainly based on elementary functions) can provide a good fitting to real data, and a third and last one about the use of these models to provide the short-, medium- and long-term forecasts of Facebook users and about how to decide about best and most reliable model.

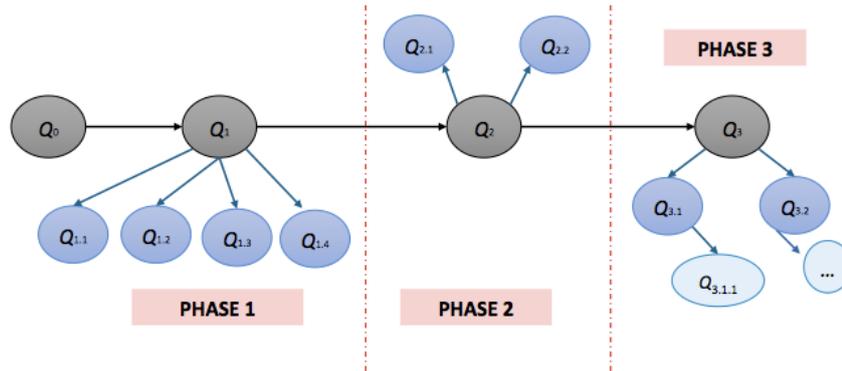


Fig 1: Tree of questions and answers of the different phases of the SRP

Q_1 : Which data sets about Facebook users are better to consider in our research? $\rightarrow A_1$: Each group look for the data set to be used and shared; the whole community agree on the terminology (year, period, units, etc.) and on the dependent and independent variables to consider.

$Q_{1.1}$: Which time intervals may be considered? $Q_{1.2}$: How can data be well-organized? $Q_{1.3}$: How to organise and visualise data? $Q_{1.4}$: What can we say about the growth tendency of the data analysed?

Q_2 : Which mathematical models provide the best fitting of real data about FB users? $\rightarrow A_2$: Each consultant group is asked to propose and justify three mathematical models fitting real data.

$Q_{2.1}$: Which models (based on elementary functions: linear, parabolic, exponential, etc.) may fit the data?

$Q_{2.2}$: How can the coefficients of the model be determined?

Q_3 : How can we decide about the ‘best’ models fitting data? Can we use this model to predict the future evolution of FB users? $\rightarrow A_3$: Need to create tools to justify why a mathematical model/s is/are the ‘best’ with respect to: (a) fitting data and (b) forecasting the evolution of FB users.

$Q_{3.1}$: How can we compare the error committed between reality and forecasts provided by models?

$Q_{3.2}$: Can be the same model used for the short- and long-term forecasts?

Let us comment the main aspects of the *a posteriori* analysis of the experimentation referring, in particular, to the questions-answers dialectic level. About the first phase, we should remark the ease with which the students found real data about the evolution of the social net. The most format they found the information was by means of a graphical representation (for example, a bar chart). This fact strongly determined their analysis, since they focused mainly in the graphical analysis of the data growth tendency, but not in their numerical versant (variation tax, that appeared in a tangential way). Besides, the fact that many groups found the same data triggered an intense debate and interchange of ideas among them, which took us to consider a brainstorming session about the previous hypothesis in the classroom, and as a consequence, the duration of the first phase was extended from 3 to 4 sessions. Due to the wealth of answers collected by the teams during the brainstorming session we decided to ask the students to deliver a first report in a poster format, so that each team could synthesize their findings and share their conclusions at that moment. About the second phase of the SRP, since many groups worked finally with very similar data on the worldwide evolution of FB users, we made two new decisions: (a) give each team a second set of different data, corresponding to different geographical areas, in order to contrast their hypothesis and extend their study; and (b) ask for more than one fitting model for each data set. The analysis of the teams proposals made arise

a non-expected aspect: many of them proposed using piecewise functions, so that the expected answers to Q_2 about the consideration of models based elementary functions (linear, quadratic, exponential, etc.) was extended. Otherwise, during the brainstorming in the first phase the teams started enquiring in the history of FB and about the possible reasons of the changes in the tendency of the data or number of users (IPO, new rival social nets, purchases of the company, new developments, etc.), and also the moments of change of tendency. New questions and answers appeared at this stage about changing the fitting model in accordance to a particular action or decisions of FB. Concerning the third and last phase, we only dedicated two face-to-face sessions of the workshop (one with the whole group and the other for the consultant teams' doubts) and were not enough for a rich development of Q_3 . Although this time constraint, there were some applets designed and integrated in the c-unit to help on the simulation of models and its contrast to real data, as we explain in the following section about the media-milieu.

The media-milieu dialectic

Since we have the first layer of analysis of the SRP in terms of the arborescence of the questions-answers, it is important to ask when, where and how questions can arise and answers can be developed. It is at this new level when there may appear the different elements taking part of the milieu, composed of varied elements: questions, temporary answers, pre-existing answers in or out school, means to validate answers, experimental data, etc., accessible through different kind of media (textbook, lectures, website resources, etc.). The relation among these elements can be analysed through the *media-milieu dialectic*. In our SRP it has been central the constant dialectic between the search for data (for instance, real data about Facebook users, or about the company changes) and of pre-existing answers (ways to organise data, common models to fit population evaluation, elementary functions, tools to control error, etc.) that exist in different media that were available to students, such as web resources, contents of Mathematics course, answers from lecturers from other courses; and the creation of the appropriate means (milieu) to integrate (or refuse) them in their SRP path study. Let us stress the importance of some of them.

In the first phase of the SRP, it was important to count on the help of the teacher of another course called 'Introduction to digital communities' (running in parallel to the workshop) who helped on providing a general sense and functionality to Q_0 and to show how the students could look for real data about FB and some techniques to organise them. Also, having open access to the news and papers published by Princeton University about FB, as well as FB answer to Princeton or their monthly report about users growth was very useful for students. All these elements took part of the *media* accessible to students, at the time it enriches students' *milieu* mainly composed at this stage of the data sets that each team chose to work with, shared and debated with the whole class sessions. All these elements helped them to prepare a first report with the first temporary answer A_1 , in a poster format given in the c-book platform, and then shared and debated in the face-to-face session (see Figure 1, left side). With respect to the second phase, the a priori design of the c-unit contained some applets (designed with Geogebra) proposed to help students to explore different models based on elementary functions (Q_2). These applets provided the main *media* for students to visualize data jointly with model simulation, and also took part of their *milieu* as main tools for contrasting, comparing and deciding on the 'best' models to choose. As aforementioned, in the SRP experienced students suggested using piecewise functions, which pushed designers to make changes in the

questions and applets as these necessities were coming up. Moreover, some groups started to present new hypothesis about model to use with non-elementary functions (such as Gaussian function), most of which were part of their milieu because they had been introduced in previous courses. So that, designers had to quickly cover this demand by designing a new applet with Geogebra to let them manipulate also these types of mathematical models.

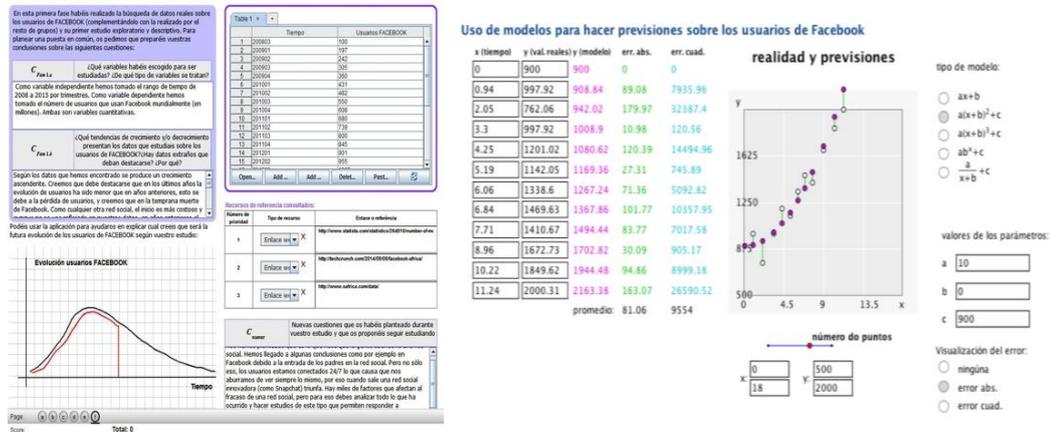


Fig 2: Example of page format and applets of the c-unit about Facebook users

With respect to the last phase, despite of the lack of time, the study of Q_3 that the consultant teams developed was in general very rich. Concerning $Q_{3.1}$ about comparing the error committed by models' forecasts against reality, we decided to integrate a new applet from Cinderella (see Figure 2, on the right) to provide students with the main media, also milieu, to simulate models they had bet for and to be able to compare graphically and numerically the error between data and forecasts coming from models' simulation. Through this tool, one could obtain the numerical calculation and graphical representation of the punctual and the averaged error (absolute and quadratic error) at the time one can changed the parameters that define the model to obtain a better fitting. Although the several advantages that this applet provided to students, students assumed and used uncritically the tools proposed by the applet. The lack of time and these designers' decisions made that the media-milieu dialectic at this stage was not so rich as it could be.

The individual-collective dialectic

In this level of analysis we focus on the relation between the teaching devices habilitated for the implementation of the SRP and the changes on the traditional didactic contract that are necessary, that is, changes on the roles and responsibilities that both students and lecturers involved in this experience had to assume. This layer, closely related to the two previously introduced, provides a finer detail about how individuals and the group developed their work in the SRP. And, how all the actions and objects of study and research (looking for questions and answers, proposing new media, adopting external answers, enlarging the milieu, etc.) are shared, agreed and transformed from the individuals to the community. Although we will be not able to go very deep in this description, due to space restriction, let us stress some important features of the SRP about its collective dimension.

First, the *devolution of Q_0* was presented as an external order coming from MS2 Consulting ('Mathematical Solutions Squared') and students were asked to spend more than a month to prepare and deliver a final report by the end of their work as an oral presentation. The lecturer of the (official)

Maths course now changed their role to become only a guide of the study and research process, transferring most of the responsibilities to students on elaborating their answer. Moreover at the end of the implementation, two external persons carried out the *validation* and *evaluation* of the final consultant team's answers, in coherence with what was asked at the beginning and trying to make students assume these responsibilities. Second, students were organised all the time in *consultant teams* with an autonomous functioning, who were asked since the beginning to jointly deliver a collective report, although in its evaluation each of them had to be responsible of explaining responses to one of the phases and all could be asked about any phase. To help on assuming this autonomy, lecturers took several decisions: (a) distribute different data sets to help them not to be very influenced by other teams work and rhythms; (b) use the workshop sessions for the common debate of groups work and to share the main advances in finding answers (although different) and new questions emerged, and (c) constantly introduce changes in the design of the c-unit design, which work as the shared support, with the new questions that had emerged and with the media that could help them at each step. For instance, at the last of phase 1, designers decided to integrate a poster format to fix the things students had to share with the rest of the groups and dedicate a workshop session when consultant's teams could explain their answers to Q_1 ; or, as explained in the previous section, in phase 2 and 3 designers decided to create new applets to deal with new models proposal and to facilitate them the media and milieu to make the SRP progress.

Final remarks and conclusions

In this paper we focus on the case of an SRP on comparing forecasts against reality in the case of Facebook users' evolution to show the use of three dialectics: the one of the questions-answers, of the media-milieu and of the individual-collective, corresponding to the three complementary level of didactic analysis of teaching and learning processes (Chevallard, 2008). Besides their analytic use, they suppose a productive framework to enrich teaching and learning practices, in particular, on modelling.

In what concerns to the *questions-answers dialectic*, since the beginning of the workshop, the generating question Q_0 about the controversy of the article by Princeton was adopted by the students with a great interest and, up to the end of the process, was kept alive. From its implementation we can underline very important conditions that were created. First, the flexibility of the lecturers and designers team that were opened to readjust the schedule according to students' team work, that is why we devoted more sessions to the first phase and consequently reducing the ones to the last phase. Furthermore, they were very attentive to integrate in the c-book unit all new questions and means that students asked for. Second, students were very active on workshop session to share their proposals from which many derived questions appeared, some of them planned in the a priori design, some others that extended the initial proposal. About the *media-milieu dialectic*, in the case of this SRP and with the support of the c-unit infrastructure, we took several decisions along the implementation of transforming the media offered to students to help them in the modelling process and also to observe the impact new media had on students' milieu. We may again insist on the importance of very important contributions, such as: collaboration with other subjects (as the one of 'Introduction to digital communities'), focusing some workshop sessions on the discussion external answers that students brought, or by the creation of widgets to foster students' experimental work, etc. Last but not least, about the *individual-collective dimension*, we may underline that students easily accept the

request of presenting their final response in front of an external committee. The last session that we dedicated to these presentations brought to the light the richness and variety of answers given to the stated initial question, as well as the complementarities of consultant teams' answers. But, we are aware of the weakness and insufficiency of mechanism to collect individual and teams internal work, as most of the workshop sessions were dedicated to the common debate. This is one aspect to be improved in the following experimentations and to integrate means to get access to the dynamics established between the individual and collective work.

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References

- Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM Mathematics Education*, 45, 797–810.
- Barquero, B., & Bosch, M. (2015). Didactic engineering as a research methodology: From fundamental situations to study and research paths. In A. Watson & M. Ohtani (Eds.), *Task design in mathematics education. An ICMI study 22* (pp. 249–272). Dordrecht: Springer.
- Burkhardt, H. (2008). Making mathematical literacy a reality in classrooms. In D. Pitta-Pantazi & G. Pilippou (Eds.), *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 2090–2100). Larnaca: University of Cyprus
- Chevallard, Y. (2008). Un concept en émergence : La dialectique des médias et des milieux. In G. Gueudet & Y. Matheron (Eds.), *Actes du séminaire national de didactique des mathématiques 2007* (pp. 344–366). Paris: IREM de Paris 7 et ARDM. Brest, France.
- Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Dordrecht: Springer.
- Winsløw, C., Matheron, Y., & Mercier, A. (2013). Study and research courses as an epistemological model for didactics. *Educational Studies in Mathematics*, 83(2) 267–284.