



**HAL**  
open science

# Mathematical modelling as a prototype for interdisciplinary mathematics education?-Theoretical reflections

Rita Borromeo Ferri, Nicholas Mousoulides

► **To cite this version:**

Rita Borromeo Ferri, Nicholas Mousoulides. Mathematical modelling as a prototype for interdisciplinary mathematics education?-Theoretical reflections. CERME 10, Feb 2017, Dublin, Ireland. hal-01933490

**HAL Id: hal-01933490**

**<https://hal.science/hal-01933490v1>**

Submitted on 23 Nov 2018

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Mathematical modelling as a prototype for interdisciplinary mathematics education? – Theoretical reflections

Rita Borromeo Ferri <sup>1</sup> and Nicholas Mousoulides <sup>2</sup>

<sup>1</sup>University of Kassel, Germany; [borromeo@mathematik.uni-kassel.de](mailto:borromeo@mathematik.uni-kassel.de)

<sup>2</sup>University of Nicosia, Cyprus; [n.mousoulides@ucy.ac.cy](mailto:n.mousoulides@ucy.ac.cy)

*In the last years the discussion for promoting Science, Technology, Engineering, and Mathematics (STEM) education became a central goal of educational policy in many countries worldwide, in an attempt to prepare students for a scientific and technological society. However, interdisciplinary mathematics teaching and learning is not limited to the “STE” and should include other disciplines across the curriculum. Mathematical modelling, as a mathematical practice and key competence within mathematics education standards could be interpreted as an excellent example for promoting not only modelling competencies, but also interdisciplinary mathematics education (IdME) in school. In this paper we focus theoretically on the question, ‘Which core similarities and differences can be stated between the two fields along three perspectives?’, by presenting a piece of theory describing the interplay between IdME and mathematical modelling.*

*Keywords: Mathematical modelling, interdisciplinary mathematics education, theory development.*

## Introduction

The purpose of the present study is to examine, from a theoretical point of view, the interplay between mathematical modelling and interdisciplinary mathematics teaching and learning, and to propose how mathematical modelling can promote interdisciplinary mathematics education. Following our theoretical approach, we present an example of an activity, based on our previous work, which exemplifies the key features and components of such a modelling activity that can promote interdisciplinary mathematics teaching and learning.

In the following sections, we present mathematical modelling as a means for teaching and learning mathematics through an interdisciplinary lens, by referring to the characteristics of modelling that make this approach feasible. We later present the teacher perspective on using modelling in promoting an interdisciplinary approach, and an example of an interdisciplinary modelling activity. We finally present and discuss a model of the interplay between mathematical modelling and Interdisciplinary Mathematics Education (IdME).

## Mathematical modelling as means for interdisciplinary teaching and learning

In this section we firstly give a brief general view on mathematical modelling and interdisciplinary mathematics teaching and learning by defining the two fields and by posing some initial thoughts for consideration. Although these definitions show strong overlaps, one has to look deeper concerning their differences, to understand both fields as exclusive as well. To make this more transparent we discuss mathematical modelling as a means for interdisciplinary teaching and learning, along the following perspectives: (a) the modelling cycle perspective and the individual modelling routes, (b) the teachers’ perspective and the cross-link approach, and (c) the

interdisciplinary activities and the students' work. These perspectives are presented in the following sections.

### **General view and definitions**

Whereas there is a strong consensus in the international discussion that mathematical modelling can be described as an activity that involves transitioning back and forth between reality and mathematics, the definition of interdisciplinary mathematics education is very vague. Recently several researchers from different disciplines, including both authors of this paper, published a monograph entitled “Interdisciplinary Mathematics Education – State of the Art” (Williams et al., 2016). Without going into detail here, it became clear that describing a ‘discipline’ is much easier than to think about, if more disciplines could be multi-, inter-, trans- or meta –disciplinary. So what does interdisciplinary mean? An interesting paper by Nikitina (2006) described three core approaches to the teaching of science and mathematics in integrative ways, that differ from one another in form and purpose. These three strategies, namely conceptualizing, contextualizing, and problem-centering, ask different questions of mathematics and science, and serve different learning goals. The authors discussed these strategies based on their empirical study and furthermore they claim that understanding the strengths and weaknesses of each strategy can help educators choose the optimal way to present their interdisciplinary material. In contrast, a brief pragmatic definition is formulated by Roth (2014), who stated that: “Interdisciplinarity denotes the fact, quality, or condition of two or more academic fields or branches of learning. Interdisciplinary projects tend to cross the traditional boundaries between academic disciplines” (p. 317). In the following we use this definition of IdME as a basis for our theoretical reflections in this paper. Following Roth’s (2014) definition, some thoughts arise by contrasting it with mathematical modelling: Without having a real life problem, mathematical modelling activities are not possible. At first, real life questions come out of another ‘(scientific) discipline’ than mathematics. This makes sense and so we have the solution and found the overlap between these fields by arguing that mathematical modelling is the same as interdisciplinary mathematics and vice versa! – Stop, this would be too easy! Can we easily speak about ‘disciplines’ or is it better to say that real life questions of modelling problems come out of other ‘realities’? Do modelling problems always include or promote further disciplines/realities and what is the importance when having one or more of them? If there are other disciplines implicitly and explicitly distinguishable in a modelling problem, is it the teacher’s goal to connect them and make them understandable for the students? Are modelling problems per se a prototype for interdisciplinary mathematics education? In the next section the cycle perspective gives some answers to the raised questions and again new thoughts are presented.

### ***Cycle perspective and individual modelling routes***

Recently the importance of modelling cycles, independent of type (see Borromeo Ferri, 2006), became clear for the teaching and learning of mathematical modelling in the classroom. In addition to promoting general modelling sub-competencies, fostering the meta-cognitive modelling competency can be seen as a strong goal in the learning process as well. Research on students’ cognitive processes while modelling also showed that the individual’s process of modelling is far from linear. So, individual modelling routes (Borromeo Ferri, 2007) better describe students’ jumps backward and forward within the cycle. These jumps could be empirically reconstructed between phases, not only inside “reality” or “mathematics”, but mostly between “reality” and “mathematics”.

Looking from the cycle perspective on modelling and interdisciplinary mathematics we would like to formulate two main thoughts/ideas, where (b) is dependent on (a).

(a) The modelling cycles which were developed in the field of applied mathematics and mathematics education during the last decades have a strong focus on the mathematics itself of course (Pollak, 1979; Blum, 1985). The interdisciplinary view is not explicitly visible. The often used terms in the modelling cycles, like “simplifying” or “working mathematically” do not imply that other disciplines are involved. This shows exclusiveness and no overlap when just focusing on the cycles. The same phenomenon can be observed when looking at modelling cycles in physics or chemistry (e.g. Goldhausen & DiFuccia, 2014), because they indeed focus on their discipline, but applying mathematics, if necessary, is only a side product.

(b) Considering that the interdisciplinary view is not explicitly visible in the known cycles, it is clear that it can only happen in connection with appropriate modelling problems. The individual has to interpret by reading the problem the other disciplines/realities that are included in the problem. This means that the extra-mathematical knowledge is not only limited to one’s own experiences, but to the knowledge of other disciplines, like physics or ICT. One could argue that the stronger the “discipline knowledge” and the “mathematical knowledge” the better a student’s modelling process will be. Speaking on an abstract level, the individual modelling routes are on a multi-dimensional level, when the other discipline(s) included in the modelling problems is understood by the students. From this point of view, we see strong overlaps between mathematical modelling and interdisciplinary education.

### ***Teachers' perspective and the cross-link approach***

With respect to the previous section we focus now on the teachers’ perspective. If we want teachers to be qualified in interdisciplinary mathematics, is it expected from them to become experts, for example, in all of the four STEM (Science, Technology, Engineering, and Mathematics)-fields? This question was the starting point for our theoretical conceptualization of STEM from a teaching and educational perspective, which is also based on the existing debate in STEM-education. Borromeo Ferri and her colleagues (2016) distinguish between the single-field teaching approach and the cross-link field teaching approach. The first approach describes promoting a single discipline in school very deeply, like for example an engineering learning environment (e.g. English & King, 2015). The other disciplines are not fundamentally included within this environment. The second approach means to promote multiple disciplines; at least two disciplines are promoted in *one* learning environment in order to cross-link these disciplines (see Star et al., 2014 for an example). Within the empirical classroom study of the “Leonardo-da-Vinci Project” (Borromeo Ferri et al., 2016) mathematics, physics, engineering and art were explicitly included in one learning environment. Grade 9 students (14 year olds) built and modelled the Leonardo bridge in an Inquiry-based Learning environment. The main goal of the lesson-unit was the permanent reflection of cross-linking the disciplines. On the basis of the empirical data and theoretical thoughts “cross-link” could be characterized as follows: One can speak from cross-linking, if at least two (scientific) disciplines are combined during one lesson or within the whole lesson-unit and are reflected with students on a metacognitive level (Borromeo Ferri et al., 2016). Again, the main aspect of making several disciplines explicit is at the foreground. If the teacher decided to look at the Leonardo bridge from only a strong mathematical perspective by neglecting the other disciplines, it is also possible.

The students had the opportunity to understand and to model the bridge by also using and naming the other disciplines.

### ***Interdisciplinary activities and students' work***

A great number of research studies has focused on the development of activities and learning materials, following an interdisciplinary approach. In this paper, we focus on the development (and the characteristics) of learning activities that have adopted a modelling perspective (e.g. English & King, 2015; English & Mousoulides, 2015; Mousoulides, 2016). Such activities are set within authentic contexts, and allow for students' multiple interpretations. With regards to mathematics, such activities provide students with opportunities to be engaged in important mathematical processes, such as describing, analysing, constructing, and reasoning (Lesh & Doerr, 2003).

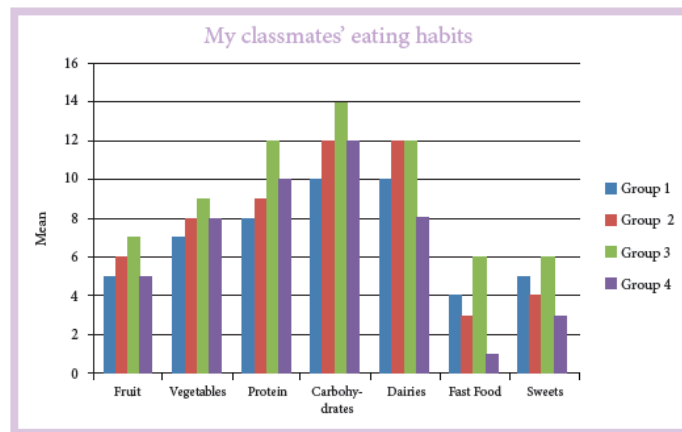
Research in the field listed six design principles for developing such learning activities, following a modelling perspective. These design principles are based on the work of teachers and researchers and that have subsequently been refined by Lesh and Doerr (2003). The 'Model Construction Principle' ensures that the solution requires the construction of an explicit description, explanation, procedure, or justified prediction for a given mathematically significant situation. The 'Reality Principle' requires that students can interpret the activity meaningfully from their different levels of mathematical ability and prior knowledge. The 'Self-Assessment Principle' ensures the inclusion of criteria that the students themselves can identify, and use to test and revise their ways of thinking. Specifically, the modelling activity should include information that students can use for assessing the usefulness of their solutions, for judging when and how their solutions need to be improved, and for knowing when they are finished. The 'Model Documentation Principle' ensures that while completing the modelling activity, the students are required to create some form of documentation that will reveal explicitly how they are thinking about the problem situation and their solutions. The fifth principle is the 'Construct Share-Ability and Re-Usability Principle', which requires students to produce share-able and re-usable solutions that can be used by others, beyond the immediate situation. The 'Effective Prototype Principle' ensures that the modelling activity is as simple as possible yet still mathematically significant. The goal is for students to develop solutions that will provide useful prototypes for interpreting other structurally similar situations.

By adopting the principles mentioned above, Mousoulides and colleagues (e.g. English & Mousoulides, 2015; Mousoulides, 2016; Williams et al., 2016) have developed a number of interdisciplinary modelling activities for students. These activities have been piloted and mainstream tested in various schools in a number of countries. Such an example, the '*How can I lose weight*' activity is presented in the monograph by Williams and colleagues (2016). The activity, which targeted 11-12 year olds, focused on the balance between nutrition and physical activity for a healthy life. The activity required students to actively participate in the collection, presentation and interpretation of data regarding their nutrition and exercise habits. Based on an analysis of their own data, students had opportunities to explore the variables (and their dependencies) that may affect the amount of energy intake on a daily basis (e.g. height, mass, age) and suggested specific diet and exercise plans, always taking into consideration the need of balancing the two.

The activity consisted of three parts. In the first part, the case of Mary, a 14-year-old girl who cannot fit into her favourite clothes, was presented. The students then considered the general question, "Is

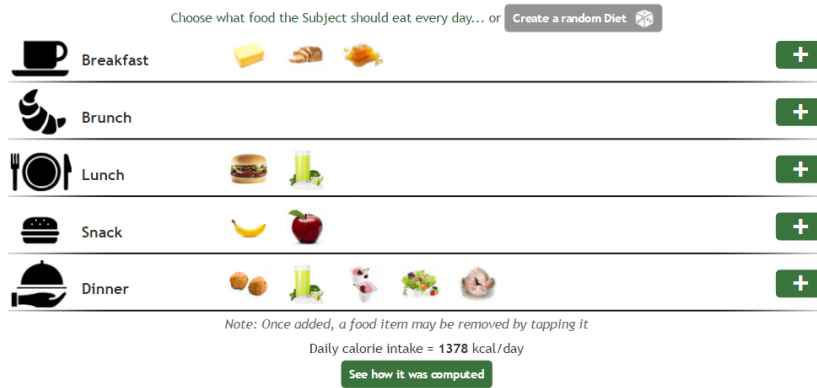
not eating the best approach to losing weight?” Students, with teachers’ support, quickly realised that the question needed to be refined in order to be answered meaningfully. On refining the question in their own ways, students acknowledged that real (actual) data on nutrition, and also on physical activity are needed. Students were then encouraged (by teachers) to work with their parents to collect the required data, through an anonymously completed questionnaire. Using their own data, students worked in groups to summarise their results, by categorising data into the different food categories (e.g. protein, carbohydrates, dairy products, fruits, vegetables, sweets, etc.), and by discussing the advantages and disadvantages of each category (Reality Principle). Students also explored trends and relationships in their data, by using a spreadsheet software (Self-Assessment Principle). An example of their work is presented in Figure 1.

During the second part of the activity teachers guided a student-centred exploration for identifying the factors that determine a person’s daily calorie intake (age, gender, height and body mass). Students worked on analysing tables and graphs by using an applet software, designed to support the interdisciplinary activity.



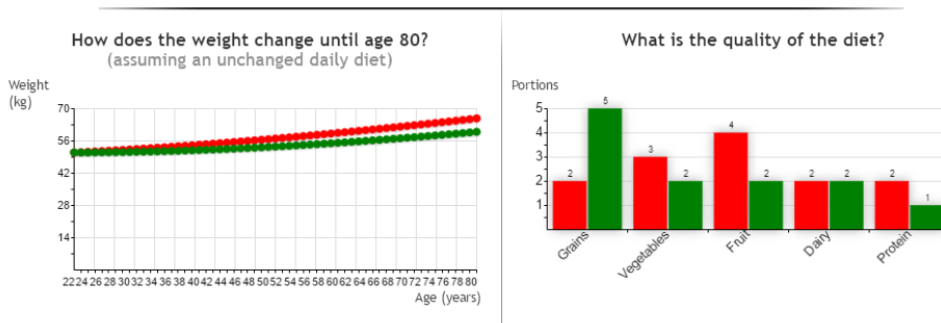
**Figure 1: Student eating habits**

In the third part of the activity students worked on suggesting a balanced diet plan (for a single day), taking into consideration the daily amount of energy a person needs (Model Construction Principle). Students could use the provided ‘food database’ for creating the person’s diet for a day (Model Documentation Principle) and then explore the appropriateness of the diet with regards to the calories taken and the food categories (Figures 2 and 3). After completing the tasks and sharing their results in a whole class discussion, students then moved to the last part of the task, in which they designed their own balanced nutrition and physical activity case (Construct Share-Ability and Re-Usability Principle).



**Figure 2: Creating a person’s diet**

- **Plan A** = the current Subject-Diet combination
- **Plan B** = a balanced Subject-Diet combination (the point at which the caloric needs are exactly met)



**Figure 3: Examining the appropriateness of a diet in terms of calories and food quality**

The interdisciplinary nature of the activity focused on the role of the bridging concept (balance between calories intake and exercise) (Effective Prototype Principle). The quite complex activity setting provided opportunities for students to explore important concepts from mathematics and biology. The implementation of the activity revealed that both teachers and parents found the interdisciplinary nature of the activity challenging; for teachers, it provided a new way of thinking and working, while for students it provided a real world problem framework, in which they could explore and connect concepts from different, yet connected, school subjects.

### Theorizing the interplay between mathematical modelling and interdisciplinary mathematics education

When summarizing the presented theoretical analysis, it becomes clear that there are strong overlaps between mathematical modelling and IdME. The real context of modelling problems, like the case presented in the previous section, in fact evokes interdisciplinary activities, but the teacher is at first the person who should make them more explicit to students, and finally actively connect the different fields, through her/his teaching. Although in some ways mathematical modelling could serve as a prototype for interdisciplinary mathematics education, mathematical modelling has its own conditions. Theorizing the interplay between mathematical modelling and IdME is a challenge, which we like to think about furthermore and also find an appropriate visualization. At this point we argue that mathematical modelling has its own “theory(-ies)”, because the characterization of mathematical modelling, and further terms/ concepts/ processes like “problem understanding”, and “validation”, are part of the theory(-ies) of modelling. This can be seen as the theoretical part of

mathematical modelling, based on the theoretical and empirical research in this field in the past decades. It is rather difficult to separate mathematical modelling from IdME, because on one hand mathematical modelling as itself is (can be) a part of IdME, but also on the other hand we can view mathematical modelling as a comprehensive research field. By adopting this modelling oriented approach in the “nutrition-exercise” case study, students could work in finding/proposing a model for balancing the intake-consumption of calories. For instance, students could be asked to propose models for different people (e.g. peers, professional athletes, teachers, parents), which balance their daily diets and their exercise habits. In doing so, the emphasis of the activity would not be within mathematics or biology, but rather on modelling.

IdME can be situated in its own field, if the (interdisciplinary) task does not fulfil the criteria of the modelling problems, e.g. when you have some kind of a “word problem”. Not every interdisciplinary task, which has (some) mathematics in it, is a modelling problem per se. IdME can be done differently and cannot be connected with a modelling problem, so one can cross the disciplines of mathematics, and for example biology in a task, but only focusing on mathematics when dealing with the problem at last. This is what we mean with pure crossing disciplines. The interplay of modelling and IdME is clearly observed, when a real life question is embedded in a real modelling problem, in which students understand the context, recognise all the disciplines involved, and use or get to know about the extra-mathematical (other disciplines) knowledge. By adopting this perspective, in the Nutrition-Exercise case study students could work in solving a problem related to finding algebraic formulae for calculating the number of calories in various types of food and/or sport activities. In doing so, students have to work with both mathematical and scientific concepts, in solving the required problem, but the emphasis would be on the mathematical concepts or the biology ones (e.g. different types of food and relation to calories per gram, etc.).

There is a number of possible implications, especially for the teachers, in promoting both modelling competencies and IdME-competencies. By using modelling problems, teachers can work with their students in crossing the boundaries between disciplines. In doing so, interdisciplinary modelling activities can provide unique opportunities for teachers to collaborate, synthesize and integrate more interdisciplinary pedagogies and teaching methods in their teaching (Berlin, & White, 1995), and for students to develop better and more coherent solutions for complex, yet interesting and real world based problems. Such approaches raise teachers and students’ expectations and confidence in working in a more interdisciplinary way, and lessened their focus on the difficulties in using and working with interdisciplinary modelling activities.

## References

- Berlin, D. F., & White, A. L. (1995). Connecting school science and mathematics. In P. A. House & A. F. Coxford (Eds.), *Connecting mathematics across the curriculum* (pp. 22–33). Reston, VA: National Council of Teachers of Mathematics.
- Blum, W. (1985). Anwendungsorientierter Mathematikunterricht in der didaktischen Diskussion. *Mathematische Semesterberichte*, 32, 195–232.
- Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM*, 38(2), 86–95.



- Borromeo Ferri, R. (2007). Modelling problems from a cognitive perspective. *Mathematical modelling (ICTMA 12): Education, engineering and economics* (pp. 260–270). Chichester: Horwood Publishing.
- Borromeo Ferri, R., Meister, A., Kuhl, D., & Hülsmann, A. (2016, July). Inspired by Leonardo da Vinci-Stem learning for primary and secondary school with the cross-link approach. *Paper presented in TSG22: Interdisciplinary Mathematics Education, 13th International Congress in Mathematics Education*. Hamburg, Germany.
- English, L. D., & King, D. T. (2015). STEM learning through engineering design: Fourth-grade students' investigations in aerospace. *International Journal of STEM Education*, 2(1), 2–18.
- English, L. D., & Mousoulides, N. (2015). Bridging STEM in a real-world problem. *Mathematics Teaching in the Middle School*, 20(9), 532–539.
- Goldhausen, I., & Di Fuccia, D. (2014). Mathematical models in chemistry lessons. *Proceedings of the International Science Education Conference* (pp. 622–642). National Institute of Education, Singapore.
- Lesh, R., & Doerr, H.M. (2003). *Beyond constructivism: A models and modelling perspective on mathematics problem solving, learning and teaching*. Hillsdale, NJ: Lawrence Erlbaum.
- Mousoulides, N. (2016, July). A modelling perspective in designing interdisciplinary professional learning communities. *Paper presented in TSG22: Interdisciplinary Mathematics Education, 13th International Congress in Mathematics Education*. Hamburg, Germany.
- Nikitina, S. (2006). Three strategies for interdisciplinary teaching: contextualizing, conceptualizing, and problem-centering. *Journal of Curriculum Studies*, 38(3), 251–271.
- Pollak, H. O. (1979). The interaction between mathematics and other school subjects. In UNESCO (Ed.), *New Trends in Mathematics Teaching IV* (pp. 232–248). Paris: UNESCO.
- Roth, W.-M. (2014). Interdisciplinary approaches in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 647–650). Berlin, Heidelberg: Springer.
- Star, J, Chen, J, Taylor, M, Durkin, K, Dede, C., & Chao, T. (2014). Studying technology-based strategies for enhancing motivation in mathematics. *The International Journal of STEM Education*, 3–19.
- Williams, J., Roth, W. M., Swanson, D., Doig, B., Groves, S., Omuvwie, M., Borromeo Ferri, R., & Mousoulides, N. (2016). *Interdisciplinary mathematics education: A state of the art*. Cham: Springer International Publishing.