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Creating a color palette: The model, the concept, and the mathematics

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Based on a brief episode of a modeling activity involving experimentation, performed by 9th graders, the goal is to reflect on criteria of authenticity for school modeling tasks. The study shows that students adhered to the experimental work and materials; still their understanding of the real event and their proposed solution to the problem seems to have missed the real problem setting.

Keywords: Concept development, experiential learning, mathematical models, proportion.

Introduction

Worldwide school mathematics curricula embrace the goal of ensuring the knowledge, skills, and competences that will enable individuals to understand today's highly mathematized and technological world and to allow them to critically and consciously engage in processes that involve mathematics and its applications. This strategic goal of mathematics education is clearly echoed by the rhetoric of international tests such as PISA and TIMSS. However, mathematical modeling is admittedly a demanding activity for students, not only because each step of the modeling cycle can become a potential blockage (Blum, 2015) but also because, for example, the focus for many students seems to be to arrive at a single solution instead of developing a suitable mathematical structure for a real world problem situation.

Recently, Blum (2015) has reaffirmed that the aims of teaching applications and modelling can be seen as having a dual function. On the one hand, the knowledge of mathematics and its use is vital to the real world and its progress, mainly in solving real problems and carrying out complex projects; on the other hand, the real world and the way it incorporates mathematical knowledge is extraordinarily important as a vehicle to provide meaning to the learning of mathematical concepts and generally to mathematics as a discipline.

Bonotto and Basso (2001) also discussed another dual nature: that of cultural artifacts, which besides belonging to the world of everyday life are also part of the world of symbols. If artifacts can be the basis for understanding the mathematical structures underlying reality, they can also be used for vertical mathematization, where the concepts embedded in real-world objects and processes become material for reflection and problem solving.

In this article we want to reflect on this duality of goals of modelling real situations and, in parallel, on the duality of roles that a particular type of task, here labeled as an experience-based task, can play in the application/understanding of the concept of direct proportionality. Experiential mathematics is a way of students dealing with living and practical knowledge of a problem or situation in the context of mathematical modeling at school. It intends to develop students' ability to collect and interpret data in addressing real world situations as well as the ability to develop mathematical thinking and communication, while reporting their ideas and findings to others. The experiential mathematics puts

the real situation as an integral part of the students' work, allowing them to understand how it works in reality and how they can "handle" it mathematically (Palm, 2008, 2009; Vos, 2011, Galbraith, 2006).

Theoretical perspective

Authenticity of a modelling situation

The question of the authenticity of the examples proposed for the implementation of mathematical modeling has been dissected by several researchers and gains particular emphasis on educational environments informed by a socio-critical view of mathematical modeling. For example, Rosa and Orey (2016) advocate the use of ethno-modeling as a tool for teaching purposes where rather than distorting the cultural and ethnic reality, the aim is to help students learn how to find and work on authentic situations and real-life problems emerging from cultural and historical artifacts, embracing mathematical ideas, thoughts and practices as developed by cultures across time and space. In any case, the perspective of the actors who actually participate in the actual settings has long been emphasized by Niss (1992), in pointing out that genuine problems, issues and situations must be recognized as such by the people who actually work in them. Despite such concerns, there is still considerable lack of authenticity in many situations that school mathematics presents to the students, which often end up just mimicking real-world scenarios, prioritizing educational goals and curriculum alignment over facing the ill-defined reality of many problems (Vos, 2011; Palm 2008, 2009; Eames, Brady & Lesh, 2015).

Blum (2015) explicitly notices that the motivating or the marketing role often attributed to mathematical modeling tasks in textbooks and curriculum materials results in the creation of problems that are only a disguise of a real situation. But other authors have strongly criticized the pseudo authenticity of modeling problems, particularly in the PISA tests. This does not invalidate the accepted notion that an out-of-school scenario cannot match completely a proposed scenario in a school mathematics assignment. And that seems to be even more unquestionable when it comes to formulating problems in assessment tests.

A school task can, of course, never completely simulate an out-of-school task situation. Nevertheless, sometimes the school situation can be organized and the assignment formulated in such a way that many of the aspects of a real life task situation may be simulated fairly well following that the students' task solving can take place under conditions fairly close to those in the simulated situation. Other times, for example in largescale high stakes testing, the conditions under which the task solving takes place put severe restrictions on the possibilities to simulate many of the aspects with high fidelity (Palm, 2008, p. 40).

According to Palm (2008), students have been predominantly faced with unrealistic mathematical tasks and have given in many cases unrealistic answers to them. At the same time, it is not totally obvious to what extent the realism of the tasks would produce greater caution and attention from students to the realism of their answers. Furthermore, Palm notes that there is a lack of consensus in the mathematics education community on the concept of realism of a task in terms of adjustment between a school assignment and a real life situation. And the evidence of that may be found in the variety of different terms for tasks that somehow try to emulate real situations (authentic tasks,

realistic tasks, real life tasks), together with the many different meanings that have been attached to each of them.

Vos (2011, 2015) has been systematically addressing the issue of authenticity and suggesting criteria that allow us to judge the more or less authentic trend of a task. Her first concern is the use of the term authentic tasks because the educational environment will require adaptations in the same way as with a flight simulator: the criteria of the *effectiveness* of the learning environment and the *realism* of the proposed situations should therefore prevail. The leading proposal is that the concept of authenticity of a task is first and foremost a social construct, so never universally definable. Therefore, a task may have some authentic aspects but it may also have others that are deliberately introduced and designed to respond to educational purposes. In short, it must be acknowledged that not all aspects of a task need to be authentic while accepting that the tasks are more engaging if a number of issues that have their origins in the situation are respected.

Once put into perspective the question of authenticity of modeling tasks, the consequent challenge concerns the search for principles to guide the construction of interesting modeling tasks, that are both effective and intellectually honest (Blum, 2015; Galbraith, 2006; Palm, 2009). To a great extent the challenge is to convert a fruitful idea rooted in a real-world situation in a problem that is fairly well framed by the details of the situation and that is accessible to a resolution that also respects the authentic school mathematics (Vos, 2011).

There are at present some proposed formulations of basic principles to inform the authentic character of modeling tasks in school mathematics. Here we will consider the work of Palm (2009), Galbraith (2006) and Vos (2015).

Palm (2009) considers a number of variables: Event; Question; Information; Presentation; Solution Strategies; Circumstances; Solution Requirements; and Purpose. Galbraith (2006) proposes a number of principles: Link to the real world; Tractable question; Feasible solution process; Existing knowledge; Evaluation available; Structure. Vos (2015) offers a set of criteria within a pragmatic definition of authenticity where the purposes and methods of the modelling researchers and task designers converge: authentic aspects in the field of mathematics (symbols, research questions, research experience) and authentic non-mathematical aspects (apparatus, professionals, mathematics applicability, problem settings).

In this study, we will concentrate on the aspects which directly relate to experiencing the real-world situation and developing a mathematical solution. We will thus select the following parameters: the *event* and the *question*, the *link to the real world*, the *apparatus* and the *problem setting*.

Experiential learning

The use of cultural artefacts and, in particular, objects that are part of children and adults' everyday experience is suggested by Bonotto and Basso (2001) as a strategy to make connections between the mathematics involved in real-world situations and the mathematics that is targeted in the classroom. They found that for students to bring mathematics into reality, it is helpful to introduce mathematical facts that are embedded and encoded in artefacts. According to their reports, in carrying out mathematical experiences from the interpretation of artefacts, the students make the transition from the real world to the world of symbols (horizontal mathematization) but in addition the use of artefacts also give them the opportunity to advance the construction of mathematical concepts (vertical

mathematization). The artefacts (concrete materials) may also be used as tools for the application of previous knowledge in new contexts and for consolidating the existing mathematical knowledge, pushing it to a higher level (Bonotto & Basso, 2001).

Bonotto (2007) advocates the need for change if we want to create realistic situations in mathematical modeling activities, i.e., recommends less stereotypical and more realistic situations, namely with the use of concrete materials; those are relevant to the students as part of their life experience, offering meaningful references related to concrete situations.

Other theorists also argue that fundamental knowledge and skills may be more easily accessible if students are directly involved in practical and experimental activities, as those can promote not only the perception of the usefulness of the materials in question but also a better understanding of the concepts explored. The discovery learning and the learning in practice are ideas inherent to the experiential education model. The model of learning by doing, developed by Kolb (1984), consists of a four-process cycle. The four processes are: experience (perform an activity); reflect (ask questions and talk about what happened in the experiment, analyzing possible inconsistencies between the observed and the predicted); abstract (generate a new idea or modify a previous idea); and apply (use what was learned in a similar or different situation, which can in turn create the need for new experiences). Experimental activities lead students to interact, analyze, question, reflect and transfer. The activity comes first; learning comes from the thoughts and ideas that arise as a result of the process of learning by doing. Accordingly, the drive for the development of new concepts is provided by new experiences. “Learning is the process whereby knowledge is created through the transformation of experience” (Kolb, 1984, p. 38). “Learning by doing” therefore appears as a natural learning perspective if modeling is seen as an activity that is similar to the methods in the experimental sciences or to the applied mathematics research.

The perspective of experimental modeling environments described by Halverscheid (2008) focuses on activities supported by experiments that give the opportunity to build mathematical models and produce mathematical knowledge around the questions investigated during experimentation. The role of experience is to lead the search for a suitable mathematical model that can explain the real data and results. The practical experience, as advocated by the author, becomes the rest of the world inside the classroom or school laboratory. The models are therefore produced to explain and interpret that intended authentic world.

Our theoretical approach, intends to combine the matter of authenticity with the idea of experimental modeling environments. We therefore consider that the authenticity of the school modelling task requires a clear account of an *event* that takes place in an out-of-school situation and the formulation of a *question* that is relevant and pertinent in that real world. But we add that the search for a solution entails a *link to the real world* that is made by an experimental *apparatus*, and a *problem setting* which is experientially emulated in the classroom.

Methodological approach

This study adopts a qualitative methodology of action research as the pillar of the methodological approach is a teaching experiment on the implementation of a new pedagogical approach involving modeling and applications in mathematics classes (Loughran, 2007; Latorre, 2003). The pedagogical motives of the teaching experiment corresponded to the need of providing meaning and practical

sense to mathematics, to contribute to change the negative view that many students held about mathematics, and to develop their appreciation for mathematics in their daily lives.

Two classes of 9th graders participated in this study. The ages of the students ranged between 14 and 17 years old, with an average of 14 years old. They were usually collaborative and committed students. Most of the lower achievers worked hard to improve their performance but showed many difficulties in applying mathematical concepts and in problem solving. None of the students had come across modeling activities in previous school years.

The modelling activities included 4 sections. The first is the introduction to the topic under study. The second consists of a practical activity, using manipulatives and everyday objects. The third comprises the analysis of the data obtained in the experimental stage, with the purpose of creating a model that might be used in similar situations. Finally, a written report should entail the following points: explanation of the experimental situation, assumptions made, strategies used, results, evaluation of the work and the difficulties found.

To solve the tasks students worked in groups of three or four elements organized at their will. The time allocated to each task was between 90 minutes (1 lesson) and 180 minutes (2 lessons). During the activities, the students had the opportunity to move around the room and discussing with colleagues the ideas from another group.

The data collection included participant observation and field notes from the teacher. The classes were recorded on video and audio with a mobile camera and the activity of a target group was also recorded on video and audio with a still camera. For each activity, a target group was randomly chosen. The written reports delivered by all the groups at the end of each activity were also collected. Here we will focus on the activity and work of a single group of four students. We will consider in particular what they developed in the experimentation phase and on the model proposed.

The task involved color dispensing and mixing paint solutions. It aimed to simulate the procedure used in industrial machinery called tintometric systems. In a tintometric system once the white base has been dispensed the container automatically positions itself under the volumetric tinting system, which dispenses all the coloring pastes and then mix them. The task was presented as follows:

Introduction. An indoor and outdoor paint shop uses a mixing paint machine in which it is only necessary to place the can with the neutral paint and introduce the code of the chosen color in the catalog. But a problem arose in the machine and it partially crashed. It no longer dispenses the required pigments, it can only mix them. Now the shop assistants must manually place the pigments to obtain the color chosen by the client. This means a new problem. They do not have information about the quantities of pigment to be used for each color of the catalog. So, your mission is to create a palette of colors and provide the amount of pigments to be used for each color.

Practical activity. You have at your disposal white liquid (milk) and colored liquid pigments (food coloring), a gauge and 1 ml syringes. Your mission is to produce a color palette that involves two primary colors and a reference table with the precise amounts to be used by the assistants.

Analysis. Choose two primary colors and prepare a palette of colors with various shades and make a table with the amounts of pigments for tins of 1L, 5L, 10L and 20L.

Report. Record all your procedures and results and submit your final report.

A case of proportional reasoning

The process held with the experimental apparatus was the same in most of the groups. It was clear that the students knew the idea of a paint catalog and knew that colors are labeled with names. Therefore, in general, they were creating a sequence of shades and were making labels for each shade. They used a small amount of pigment at a time and always added that amount to the previous mixture. At the beginning they only had milk (equivalent to the neutral paint) and then added color by the addition of two pigments. Throughout the experiments all groups recorded the values in the tables and subsequently calculated the amounts of pigment for different paint tins. Some of the groups did not present a formula to generate any amount of paint (unknown value) and only made the calculation of the amounts necessary to make the quantities asked in the problem.

The target group used yellow and green pigments. In their report the group described how they created a palette of four shades:

Using the syringe we started with 0.1 mL of yellow pigment in the cup with milk (60 mL) and this color was **Vanilla**. Then we added another 0.2 mL of yellow pigment to the glass that already had 0.1 mL of yellow and the color was **Cream**. After we added another 0.2 mL of yellow pigment and 0.1 mL of blue pigment to the glass which already had some amount of pigment and the color was **Mint**. Lastly we added to the glass another 0.4 mL of yellow pigment and another 0.5 mL of blue pigment and the color was **Amazon Green**. Now, using the results of our table, we will determine the amount of pigment to be used with 1L, 2L, 5L and 10L of paint. To obtain those values, we use the rule of three. For the case of Vanilla we get:

$$\begin{array}{l} 60 \text{ -----} 1000 \\ 0.1 \text{ -----} x \end{array} \quad x = \frac{1000 \times 0.1}{60} = 1.6 \text{ mL}$$

To complete our description of the results of the group we note that they created a table for each tin size and the four shades, showing the quantities of *paint*, yellow pigment and blue pigment. For the Amazon Green, the table indicated: 20000 mL of paint, 300 mL of yellow, and 200 mL of blue.

Discussion and conclusion

In this teaching episode and from the empirical data presented we want to emphasize that the students apparently attributed significant authenticity to the task, namely: to the *event* of producing a color catalog of paint; to the *question* of finding the quantities of pigments for manual mixing; to the *link with the real situation*, that is, helping the assistant to retrieve the quantities of pigments for each shade; and to the use of the experimental *apparatus* which was essential for the ongoing mathematization and emulated the real process.

However it seems that some aspect of authenticity has lacked. At a certain point the mathematical model is clearly formulated by the students and it shows their awareness of the fact that there is direct proportionality between the quantities of white paint in the experimental model and in the real sized tin, as well as direct proportionality between the quantities of a pigment in the model and in the real sized tin. That is obviously a sound conclusion. However, they went on assuming that the amount of white paint in the real tin was equal to the capacity of the tin, which is equivalent to say that the tin would be completely full with white paint and thus with no capacity available for an additional amount of color paste.

We are suggesting that the missing aspect of the modelling situation concerns the authenticity of the *problem setting*. The students had actually visited a paint supplying store in the neighborhood and could learn about a machine used to automatically dispense and mix the paints and produce the desired color. But one detail is relevant on the machine operation: it begins by asking the operator the size of the tin and then retrieves the amounts of neutral and of colored pastes. The total of the mixture has to be equal to that volume (or slightly below). Instead, the setting in which they worked was a scenario involving much lower quantities than the actual tins. Therefore, no absolute urge for correctness of the calculations was at stake, as Vos (2011) points out. Indeed, the initial volume of milk was little and imperceptibly changed by adding small doses of pigments. In other words, there was almost no distinction between the volume of white and the volume of the mixture in the glass of the experiment (Figure 1). But that is no longer valid when we want to produce a can of 20 liters of Amazon Green color. In this case, the value obtained for the combined quantities of yellow and blue pigments was 500 mL and that would be a quantity which, if added to the neutral paint, greatly exceeds the capacity of the tin.



Figure 1: Two shades and not distinguishable volumes

Surely, other factors can be considered as responsible for the inaccurate mathematisation carried out. One of them refers to the difficulty students have with the notion of ratio, which is well documented in research. In fact, the rule of three was a ready strategy for students. And the ratio was never really considered. In any case, what happened is that students did not differentiate the amount of colored paint from the amount of white paint, as it is evident from their table that shows 20000 mL of “paint” instead of “white paint”. Still, that value was the correspondent to the 60 mL of milk in their rule of three. In conclusion, the concept of ratio was never activated and instead proportionality was used as a process of enlarging the “size” of each ingredient in the mixture (enlarge a small cup to a large tin). Therefore, questions of authenticity that are recognized by those who work with them in practice (Niss, 1992) seemed to have been absent from the process of translation between students’ real model of the mixture produced in the classroom and the actual process of paint production.

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