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Modelling problems in German grammar school leaving examinations (Abitur) – Theory and practice

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Examples of mathematical problems in German school leaving examinations are examined in detail as to the extent to which they verify the competence of mathematical modelling. For this purpose, the relevance, the authenticity of the context, the openness of the task and the partial competencies as criteria are examined in more detail. It is shown that authentic contexts but no relevant contexts are contained in the problems. As a rule, the use of mathematics is not authentic and problems are not open. The partial competencies in modelling are quite unequally divided and are not covered completely. It is recommended to use all the criteria for the design of future examination questions.

Key words: Modelling, examinations, Abitur, authenticity.

Introduction

In Germany, an overall policy of the Conference of Education Ministers has been in place with respect to “Educational Monitoring“ since 2006. The intent is to enhance competence orientation within the educational system. In mathematics, the focus also lies on the general competence in the field of modelling. From the year 2017 onwards, a task pool of Abitur examinations will be available out of which all Federal States can take problems for Abitur examinations. This is an important step towards improving the quality of examination problems and to align levels of requirements between the Federal States step by step. Such examination problems have been developed on the basis of educational standards and also contain, in particular, elements examining mathematical modelling. Studies regarding problems in written Abitur examinations are rare. Still today, it is obvious that there are very few theoretically and methodically profound and empirically substantiated approaches in respect of examination problems in the framework of the written Abitur examinations, which at times just focus on individual standard-related aspects (Kühn, 2016, p. 75). Therefore, this applies in particular to problems regarding mathematical modelling. At the core of mathematical modelling lie the translation of a problem from reality into mathematics, the work with mathematical methods and the application of mathematical solutions to the real problem. Usually, the entire modelling process is represented as a cycle. Idealisation here means that the representation itself is, again, a model. In Borromeo Ferri et al. (2006) the creation of a mathematical model is contemplated in detail, representing the process of the individual, creating the model, from the starting situation to the mental representation as a first step.

Modelling competence

Blum et al. (2007) describe modelling competence as the capacity to execute the relevant process steps required when switching between reality and mathematical problem adequately as well as to analyse models or comparatively evaluate such models. This can be discussed in more detail on the basis of modelling cycles by Blum and Leiss (2005, p. 19). Thus, it is possible to analyse the various partial processes of modelling activities considering various details and other aspects. It is

also possible to consider the capacity to implement such a partial process as a *partial competence* of modelling (Kaiser, 2007; Maaß, 2004). In consideration of the modelling cycle one could characterise such partial competences as shown by Greefrath and Vorhölter (2016, p. 19). We do not mention inner-mathematical working between the mathematical model and the mathematical results as a partial modelling competence since this is not specific for modelling processes.

Characteristics of modelling problems

It is not always easy to select or develop the right modelling task. As an indication, characteristics may be specified that a modelling task should fulfil. In her comprehensive classification scheme for modelling problems, Maaß (2010) focuses in particular on the type of relation to reality, to the openness and the modelling activity as criteria for modelling problems. With respect to the focus on the modelling activity, partial competences of modelling will be observed more closely. As regards the relation to reality, the relevance and authenticity of the context will be examined more closely (cf. Greefrath et al. 2013).

Relevance

Initially, we deal with the classification of reality-related problems handed down in German discussions. In connection with the factual context of problems, we traditionally talk about “*embedded problems*“, *problems formulated in text form* and “*factual issues*“. These types of problems provide facts about the relevance of the factual context used for students.

Embedded problems have no real relation to reality. The factual context is of no importance regarding the solution of problems and can be exchanged arbitrarily. The intention of embedded problems is the application and practice of numeracy skills. An embedded task as a modelling task is suitable only to a limited extent since the mathematical model is already implicitly contained in the problem. *Word problems* are problems formulated in text form – sometimes complemented by figures. Similar to the embedded problems, the issue is basically exchangeable as reality is often represented in a very simplified manner. The intention is to promote mathematical skills. It is not really about an autonomous development of a mathematical model because of the lack of reality and the given simplifications. Nevertheless, one major problem for students is the translation of the text into the corresponding mathematical symbolism such as terms or equations. For this reason, the known term “*mathematisation*“ in the field of modelling is normally used in this context (Schütte, 1994, p. 79). In the case of problems formulated in text form the mathematical problem dominates the embedding of it. Another focus then lies - depending on the specific problem - on the interpretation of the mathematical results in the factual situation and in the wording of a corresponding answer. The substantial and - particularly in connection with factual problems - frequently exclusive treatment of problems formulated in text form in mathematics classes has been strongly criticised. One reason is the lack of genuine reference to reality. Another reason is the process of practising similar mathematical issues, formulated in text form, as a result of which a real reflection about the context used becomes redundant. In the case of *factual issues* a factual environmental problem is of primary importance. In this context, the function of “*Factual Calculation*“ is described by Winter (2003) as “*Environmental Development*“. In this context, real data are frequently given in respect to which authentic questions are then asked. Since the processed issue plays a real role, information about the respective matters must be gathered and processed.

Therefore, the processing of factual issues is also to be considered as interdisciplinary or - ideally - even as discipline-linking. In this context, the factual issues can be viewed as modelling problems (Franke & Ruwisch, 2010).

Authenticity

Authenticity is one central characteristic of modelling problems. Authenticity means both the authenticity of the extra-mathematical context and of the application of mathematics in this particular situation. The extra-mathematical context must be real and not be specially designed for a certain arithmetical problem. The application of mathematics in this situation must also be reasonable and realistic and should not just be used in mathematics lessons. Authentic modelling problems are problems that genuinely belong to an existing subject or problem area where they are accepted by people working in those areas (Niss, 1992). Thus, an authentic task becomes credible and at the same time a realistic task for students from an environmental point of view. Authenticity helps students take problems seriously and avoid superficial substitute processing strategies as in the case of embedded problems (Palm, 2007). In the case of authentic problems, students can assume that the things they are dealing with really exist and that the task they are presented with is a real task that finds its justification outside mathematics lessons as well (the task have an out-of-school origin and a certification (Vos, 2015)). Authenticity of problems helps students take such problems seriously. However, authenticity of problems does not mean that the problems are actually important for the students' present or future lives.

Openness

Open problems are those problems that - for instance - allow for more than one approach or solution. Openness enables students to choose their own approaches or solutions regarding the problems. There are various classifications of open problems. We limit ourselves here to the examination of openness by initial state, transformation and target state (cf. e.g. Wiegand & Blum 1999). Said classifications use the known problem-solving psychological description of a problem through its initial state, target state and a transformation that transfers the initial state into the target state and are not limited to modelling problems (Klix, 1971). Open problems are divided by the clarity of their initial and target statuses as well as by transformation. Maaß (2010) suggests another classification of open modelling problems based on Bruder (2003) which includes seven different types and which distinguishes between overdetermined and underdetermined problems.

Criticism concerning Abitur examination problems with modelling elements

The use of modelling in examination problems, however, is not unreservedly viewed positively. The fact that in many cases the relevance of the factual context used is not at the focus of examination problems, gave rise to (fundamental) criticism on the part of some expert representatives with regard to modelling in examinations. On the one hand, the criticism is directed against the fact that "modelling competence" is not at all examined by the problems (Kühnel 2015, p. 76). On the other hand other authors show the categorical refusal of modelling problems (cf. Bandelt & Weidl, 2015, p. 4). Strong criticism is also directed against the greater part of texts in examination problems (cf. (Jahnke et al. 2014, p. 120): "Instead of dealing with mathematical problems, A-level students have to tackle wording problems".

Criticism regarding examination problems can be absolutely justified, in particular if it is about embedded problems and not about factual problems in examinations (cf. Fig. 1). Henn and Müller (2013, p. 205) comment: “Unfortunately, most of the so-called “modelling problems“ at school - and in particular in the Abitur exam are not at all modellings according to our way of thinking. Almost always one starts out from a more or less complex function equation, allegedly describing a ski jump, a tower, a playground or another construction. Now, with this function, a common functional examination is to be made. However, the whole thing is not a modelling task, but entirely a mathematical problem.“

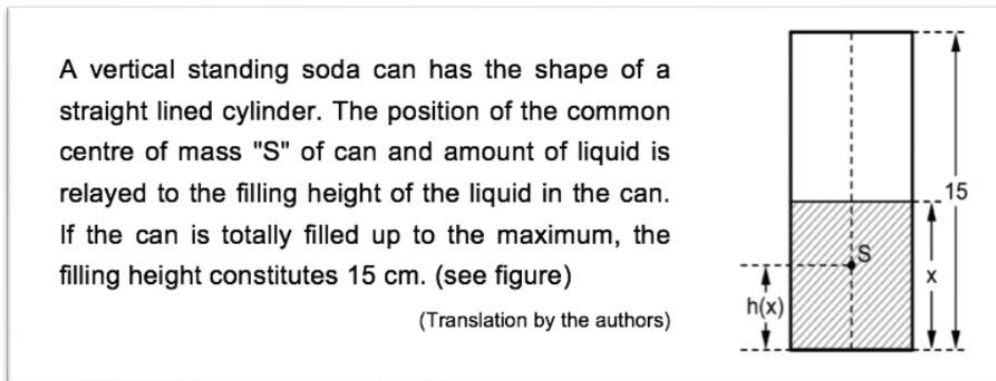


Figure 1: Stimulus text of an examination task (www.iqb.hu-berlin.de)

This brief insight shows that the use of the term “modelling problem“ - in particular in examinations - does not at all guarantee the clear characterisation of a certain type of problem. It should be considered, however, that the conditions given in examination or in normal lesson situations are different from each other (cf. Siller et al., 2016, p. 381/382). Ultimately, in examination situations the focus is on the term of “measuring“ the students’ performance as outlined by Siller et al. (2016, p. 384). Thus, less attention may be paid to a creative phase in a test situation. In addition, psychometric findings (cf. Rost, 2004) reveal that - for the measurement of performance - such performance must be addressed explicitly and structured into small units in order to obtain a valid statement.

Research issue

Against the background of criticism directed towards current Abitur examination tasks and in consideration of the importance of competence of mathematical modelling for German levels of education and of international discussions as well as of the theoretically clarified criteria for modelling problems, we have to ask ourselves the question, how to evaluate the quality of existing Abitur examination problems with modelling elements in Germany. We limit ourselves here to the calculus subject area, which accounts for the largest part of the Abitur examination in mathematics in Germany. With regard to the criteria discussed above, the question is: How can German Abitur examination problems in calculus be assessed with regard to the partial competences of modelling, to relevance, to authenticity and to openness?

Research method

For preparing for the pool of Abitur examination problems which will be available from the year 2017 and from which all Federal States can easily draw Abitur examination problems, the Institute for Educational Quality Improvement in Berlin will provide a collection of examples (cf.

www.iqb.hu-berlin.de/bista/abi). Such samples were chosen as a basis for the examination because they best fit with the Abitur examination problems of 2017. Among the sample problems there is a total of four examination problems for calculus for general grammar schools. Two of the problems are of an increased level and a further two are intended to be used in Computer-Algebra-Systems (CAS). All in all, the four examination problems include 50 items.

.... b) Initially the empty can gets filled up slowly with liquid until the maximum filling height of 15 cm is reached. Describe the movement of the center of mass „S“ during the filling cycle. Constitute schematic the can, filling height and center of mass at that point the center of mass is conditioned to the minimal height and describe the characteristic of this situation and describe the feature of this situation. (Translation by the authors)

Figure 2: Part b) of an examination task (www.iqb.hu-berlin.de)

Problems were evaluated by items according to different criteria. The selection of criteria follows the frequently quoted central characteristics of modelling problems (cf. Bruder 1988, Greefrath & Vorhölter 2016, p. 17, Maaß 2004, p. 22). Problems were discussed and evaluated per item and on the basis of Tab. 1 in the framework of a qualitative research process including three evaluators. Finally, all items could clearly be allocated the corresponding characteristics (cf. Bortz & Döring 2006) – by using 0 (does not match – e.g. Relevance 0 in Fig. 2) or 1 (matches the criteria – e.g. Authenticity in Fig. 2). In this way, a detailed compilation of the examined items was created and an allocation to the aforementioned criteria became possible.

Reference to reality	Does the problem refer to an extra-mathematical factual context?
Relevance	Is the factual context relevant to the students (factual problem)?
Authenticity	Is the factual context authentically related to the actual situation?
Authenticity	Is the factual context authentic with regard to the use of mathematics?
Openness	Is there more than one possibility to solve the problem (solution variety)?
Partial competences of modelling	Which partial competences of modelling are required for dealing with the problem (simplifying / structuring, mathematising, interpreting, validating)?

Table 1: Criteria used for the Assessment of Items

Results

In an initial assessment of the existing data material, we focus on the criteria reference to reality, authenticity, openness, relevance and modelling competence. Due to the small data basis, it was not possible to make valid quantitative statements. However, certain trends are visible. 20 items out of a total of 50 contain a reference to real life situations. The problems, though, included in the basic level more items with reference to reality (52 %) vs. items without reference to reality than in the higher level group (31 %). Correspondingly, on average, problems with the use of CAS contain more items with reference to reality (46 %) than in the group without the use of CAS (33 %).

Authenticity: If one looks at the problems considering the requirement that the respective items should contain realistic situations, our analysis reveals that such items appear both at the increased

level and at the level where CAS are used. More than one half of the items with reference to reality also contain realistic situation at both levels. Regarding a realistic use of mathematics which from our point of view represents a further aspect of authenticity (cf. table 1), this has been identified in just three items. Students were required to explain in the factual context of a parabolic modelled trajectory that only certain parameter values are possible. It should be noted here that quadratic functions are used even by experts for modelling trajectories.

Openness and Relevance: Open problems have not been identified in the items examined. Also with respect to the relevance of the problem, no item can be identified in the present examination problems that would fulfil this requirement.

Modelling competences: The analysis concerning the partial modelling competences show that the items normally address one of the partial competences. However, the partial problems are not designed in such a way that the sequence of an idealised modelling cycle reflects the appearance of the partial competences of simplifying, mathematising, interpreting and validating one after the other. Most items focus on mathematising, followed by required interpretations. Simplifying or structuring or the validation within one item is not required by the current examination problems. One item requires both mathematising and interpreting. This item requires the determination of an intersection point on the basis of a model given by a function equation with an unknown parameter with self-determined conditions interpreted in the framework of the factual context.

Discussion

Generally, it can be said that around 40% of the items examined include reference to reality. However, this does not mean that such items also contain modelling problems. The application of common criteria for modelling problems shows that we can talk about modelling problems only to a limited extent. Not a few items include an authentic factual situation, but a realistic use of mathematics is given just in exceptional cases. None of the problems is open or relevant. This is why criticism towards Abitur examination problems must be taken seriously if one must conclude – as already concluded by Henn and Müller (2013).

It is comprehensible that in individual items only a partial modelling competence is dealt with due to the examination situation or to performance measurement, respectively. This can also be observed essentially in respect to the problems examined. It would be desirable though, that all partial competences of modelling were contained in a test problem and that this problem was tested in the typical sequence as in the modelling cycle. This is not the case with regard to the problems examined. There are even partial competences, which are not tested at all, such as simplification.

Numerous general conditions must be observed when preparing examination problems. In order to actually test the modelling competence, it is necessary not just to use problems with reference to reality which describe an authentic situation only partially and to test just a few competences of partial modelling competencies, but to start with an intensive and criteria-led development of modelling problems for an examination on the basis of the current educational standards.

In order to improve the competency modeling and the development of modelling problems for examinations, the criteria used here should also be used for the development of examination problems and for training problem developers. In order to increase the share of suitable modellings

in examinations, it seems that a substantially greater proportion of reality-related problems or a substantially targeted development of suitable items is needed.

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