



The investigation of mathematical problem-solving processes of fifth grade students in small groups

Firdevs İclal Karatas, Öznur Soyak, Ayşenur Alp

► To cite this version:

Firdevs İclal Karatas, Öznur Soyak, Ayşenur Alp. The investigation of mathematical problem-solving processes of fifth grade students in small groups. CERME 10, Feb 2017, Dublin, Ireland. <hal-01933481>

HAL Id: hal-01933481

<https://hal.science/hal-01933481v1>

Submitted on 23 Nov 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

The investigation of mathematical problem-solving processes of fifth grade students in small groups

Firdevs İclal Karatas, Öznur Soyak and Ayşenur Alp

Middle East Technical University, Faculty of Education, Ankara, Turkey;

iclal.karatas@metu.edu.tr; osoyak@metu.edu.tr; alp.aysenur@gmail.com

The purpose of the study is to investigate mathematical non-routine problem solving processes of students in small groups. The study was conducted with nine fifth grade students in three small groups. A framework developed by Artzt and Armour-Thomas (1992) for protocol analysis of problem solving in mathematics is used for this study to determine problem-solving behaviors observed within different episodes of problem solving. The findings revealed that although the understanding episode was coded as the greatest percentage, the analyzing episode was coded as the lowest percentage within the three groups.

Keywords: Problem-solving process, small groups, fifth grade students.

Introduction

Problem solving has become an integral part of learning mathematics since it helps students to understand mathematical contents. It also leads students to understand how to apply their knowledge into their daily lives when solving problems. The NCTM (2000) also emphasizes the mathematical problem solving activities from pre-kindergarten to grade 12 in all mathematics classrooms. Moreover, problem solving is strongly emphasized in recent Turkish elementary mathematics curriculum. It is considered as a basic skill that should be developed in each content area (MoNE, 2005). Besides, students should be able to develop their own strategies and apply them to solve their real-life problems when solving problems (MoNE, 2005).

Moreover, problem solving entails engaging in a task for which the solution process is not identified beforehand (NCTM, 2000). Mayer (1992) defines problem solving as a cognitive process in which one figures out how to solve a problem of which the solution is not already known. Most definitions of problem solving emphasize problems that require problem solvers to use information and procedures in unfamiliar ways. Problem solving is an extremely complicated human endeavor. It is considerably more than the implementation of well-learned procedures or the simple recall of facts. Problem solving involves the construction of sequential procedures that build strategies in addition to the application of the structure (Hammouri, 2003). Problem solving also entails arranging several cognitive and metacognitive processes, deciding and performing suitable methods, and regulating behavior for varying demands of problems (Montague, 1991).

A variety of models are proposed that describe the processes that problem solvers use from the beginning until they finish their tasks (Garofalo & Lester, 1985; Mayer, 2002; Montague & Applegate, 1993; Polya, 1957). For example, Polya's model comprises of four stages; namely, "understand the problem, make a plan, carry out the plan, and look backwards" (Polya, 1957). Later, Garofalo and Lester (1985) revised the model proposed by Polya and include cognitive and metacognitive components. Their model is described in four stages as orientation, organization, execution, and verification. Montague and Applegate (1993) presented a model focused on seven

cognitive processes “reading, paraphrasing, visualizing, hypothesizing, estimating, computing, and checking” and three metacognitive processes “self-instruction, self-questioning, and self-monitoring”. Mayer (2003) proposed another cognitive process model that included translating, integrating, planning, and executing processes. In particular, Artzt and Armour-Thomas (1992) developed a framework to examine the problem-solving processes of individuals as they work in small groups.

Furthermore, non-routine problems are problems where how to solve the problem is not obvious immediately, or they have not been encountered before in the curriculum. Non-routine problems require critical thinking and an extension of prior knowledge that may include concepts and techniques which will be explicitly taught at a later stage, and may include finding connections among mathematical concepts (Schoenfeld et al., 1999). The findings of this study support that challenging problems are likely to enable metacognitive process so that students consciously adjust and regulate their cognitive processes (Montague & Applegate, 1993).

All over the world the importance of exploring elementary school students’ problem solving abilities is highlighted. There are some international examinations such as Program for International Student Assessment (PISA) and National Assessment of Education Progress (NAEP) to determine the performance on non-routine problems and in problem solving. In the 2012 version, students’ PISA ranking scores in problem solving show that Turkey is among one of the worst in the world (42nd out of 65 countries, 2012 PISA). Therefore, researchers need to find out why this is so. As a country students’ problem solving abilities can be improved if how students think, and their awareness of their actions while solving non-routine problems are determined. In addition, exploring cognitive and metacognitive abilities is difficult in problem solving. Especially, elementary school students may not be aware of what, and why, they are doing. As a result, the purpose of this study is to investigate problem solving processes of fifth grade students when they solve non-routine mathematical problems.

Significance of the study and research question

Many research studies and projects have pointed out the importance of learning problem solving in school mathematics courses (Higgins, 1997; NCTM, 2000; Verschaffel et al., 1999). One of the major goals of mathematics education is the acquisition of the skill of learning how to solve problems. However, there are conflicting views about the attainability of these goals (Verschaffel et al., 1999). Despite long years of instruction many research studies show that children are insufficient and not confident in having the aptitudes required for approaching mathematical problems in a successful way (Higgins, 1997; Doorman et al., 2007). The reasons for these deficiencies, particularly in elementary students, can be attributed to two factors. The first of them is the lack of specific domain knowledge and skills (concepts, formulas, algorithms, problem solving). The second factor is shortcomings in the heuristic, metacognitive and affective aspects of mathematical competence. When confronted with unfamiliar complex problem situations, students mostly do not spontaneously apply heuristic strategies such as drawing a suitable schema or making a table. They usually only glance at the problem and try to decide what calculations to perform with the numbers. In addition to this, many students have inadequate beliefs and attitudes towards mathematics itself, learning mathematics, and problem solving. These beliefs exert a strong negative influence on their willingness to engage in a mathematical problem. Some examples of such beliefs and attitudes are

that there is only one correct way to solve a problem; that a mathematical problem has only one right answer; and that ordinary students cannot solve problems which requires higher order thinking. These insufficiencies in students' beliefs are related to the nature of the problems given in the lessons and the classroom culture. Hence, problem solving activities should give opportunities to students for investigation, reasoning and deciding on the solution process and improve their problem-solving skills. Small groups in problem solving may provide natural setting for interpersonal monitoring and regulating of students' goal directed behaviors. In this study, the problem-solving processes that occur as individuals engage in mathematical problem solving in small-group settings are examined.

The findings of the study may contribute the studies on the process of fifth grade students' thinking. The determination of these students' thought processes will helps teachers to design and adjust problem solving instruction and better support the development of students. The findings of this study may be applicable for developing teaching methods and materials to enable the development of fifth grade students' problem solving skills in future non-routine problem solving classrooms. In addition, this study can be significant for the design of curriculum in that the results support the design of educational or special programs that can be more effective and supportive of elementary students. Thus, the research question of the study could be stated as follows:

Which are the most dominant problem solving processes of fifth grade students when solving non-routine problems?

Theoretical framework

It is necessary to appraise the information about problem solving processes to develop a framework that can explain how students figure out mathematical problems. Mathematics educators and psychologists have suggested various problem solving process models. Polya (1957) proposed four phases called "*heuristics*" to understand problem solving processes. The phases are known as "understanding the problem, devising a plan, carrying out the plan and looking back". Polya also proposes several strategies that can be used when students solve problems. His strategies include using diagrams, looking for patterns, trying special cases, working backward, intelligent guessing and checking, creating an equivalent problem and creating a simpler problem. Considering the problem-solving processes, an appropriate strategy can be essential to reach the solution of the problem.

After that, Schoenfeld (1982) developed a model for mathematical problem solving based on the Polya's model. The model includes five episodes; namely, "reading, analysis, exploration, planning/implementation and verification". Adding cognitive and metacognitive aspects of problem solving to Polya's and Schoenfeld's model, Garofalo and Lester (1985) proposed a framework with orientation, organization, execution and verification phases. Montague and Applegate (1993) also proposed cognitive-metacognitive aspects of mathematical problem solving. This model focused on seven cognitive processes "reading, paraphrasing, visualizing, hypothesizing, estimating, computing, and checking" and three metacognitive processes "self-instruction, self-questioning, and self-monitoring". These various models have been used to investigate problem solving processes, but only two models by Garofalo and Lester's model as well as Montague and Applegate's model have been used with gifted students as a framework to describe problem solving processes in the literature (Garofalo, 1993; Montague, 1991; Montague & Applegate, 1993; Sriraman, 2003). Several metacognitive actions during problem solving were described in each phase of those models by

cognitive theorists, in mathematical problem solving. To examine the problem-solving behaviors and cognitive processes of individuals as they work in small groups, Artzt and Armour-Thomas (1992) developed a framework based on Schoenfeld's (1982) framework. Schoenfeld (1985) defined an episode as "a period of time during which an individual or a problem-solving group is engaged in one large task" (p.292). The framework for the protocol analysis of problem solving in mathematics is used for this study to differentiate between cognitive and metacognitive problem-solving behaviors observed within the eight episodes (read, understand, analyze, plan, explore, implement, verify, and watch and listen) of problem solving. The framework synthesizes the problem-solving phases identified in mathematical research by Garofalo and Lester, Polya and Schoenfeld, and of cognitive and metacognitive levels of problem solving behaviors studied within cognitive psychology, in particular, by Flavell (1981). This framework used in this study is to examine the interactions between two levels of cognitive processes (cognitive and metacognitive) observed in the problem-solving behaviors of students working in small groups on mathematics problems.

Methodology

This project consists of qualitative research in which case studies are employed. A qualitative design is appropriate for this study because the study focuses on gaining in-depth information about what actually occurs during the problem-solving process. The study conducted with nine fifth-grade students in a private school in the capital of Turkey. Purposeful sampling was used to select the participants as the researcher wanted to obtain more knowledgeable information about the problem-solving processes within the groups. Voluntary participants were involved in the study. The students who have high self-expression skills were selected by two mathematics teachers.

This study used multiple methods including a think aloud procedure when the students are engaged in solving problems, researcher's field notes of observation, and analysis of students' solution papers to collect data. Prior to the data collection, the participants practiced the think aloud technique with a sample problem. The procedure provides participants with important practice for understanding and developing confidence prior to utilizing the technique with the research problems. Over the one day 3-hours period of data collection, three mathematical problems which were selected from PISA problem solving sample questions (going to the cinema, transportations system and holiday) from decision making units (OECD, 2005), were given to participants to solve in small groups by using think aloud method. All three students in small groups had their own paper and problem sheet to follow the process. They continuously spoke aloud while they work on the problems explaining their thoughts. Also, they had unlimited time to solve each problem. Since misinterpretations of the data might have resulted with only a single researcher as the data collector, the researcher maintains a record of field notes explaining her reflections about the activities. The field notes included the explanations of questions, reactions, and behaviors that occurred during data collection.

In the study, group members were chosen from different classes, and they had never studied together before. Data collection was in a one-to-one setting between the participants and the researcher to have some field notes. One researcher observed exactly one group and took field notes. The researcher videotaped all the processes to record the participants' behaviors, how they responded to the problems, and what mathematical language they used. All data from the think aloud session, participants' solution papers, and researchers' field notes were transcribed for analysis by the

researchers. To generate the categories, the researchers read through all transcribed data sentence-by-sentence and identify words or phrases that described the participants' responses. For example, in group 1, student 1 says for cinema question: "First we will read the question then we will discuss". This sentence is coded as *Read*. Again in group 1, student 1 says for cinema question: "Until now, what did you understand?" and student1 suggests: "Let's underline the important sentences" These two sentences are coded as *Understand*. Also, student 2 says: "They cannot go to that film because it is for above 18 years old". This sentence is coded as *Analyze*. For the second question, student 1 says: "Let's try the other way" and coded as *Explore*. Student 2 says: "They cannot go to Children in Deep, Carnaval and Pokemon" coded as *Implement*. Student 2 says: "Let's look at it carefully. They can all go to Mystery. Let's check" coded as *Verify*.

After each interview is transcribed, participants check the accuracy of the described experiences and themes. Then, the codes were applied based on a review of the data and the concepts emerging from the data. The responses of one student were compared with those of other students in the same problem, as well as the same student across other problems. Multiple data sources were used to triangulate and confirm patterns that emerged. Each response was compared with other responses with the same idea, regarding the source of the responses. The codes were grouped into categories. At this point, preliminary categories were developed. Responses were compared across categories in terms of similarities and differences. Next, the researchers revised categories with transcribed data again and again until the final categories are confirmed. The final categories were also reviewed against the transcribed data for the last time.

Findings

The coding for each of the three groups was done and the behavior of each group was categorized by episode. As it was suggested by the study, the three groups' episodes or problem solving behaviors were recorded and ranged. The audio records of the groups were coded in 1-min intervals based on the emergent behaviors through sentence-by-sentence and identify words or phrases that described the participants' responses. Groups were differentiated from one to another by giving numbers such as Group 1, Group 2 and Group 3.

Table 1 lists the number and percentage of behaviors across all groups, of the 519 behaviors that were coded, 25% belonged to Group 1, 21.65% belonged to Group 2 and 53.4% were demonstrated by Group 3.

According to the results students in Group 1, out of 130 items (sentences, phrases, words), show "understanding" behavior (e.g. Student 1 says: "Stanley cannot come to the cinema on Sunday also he cannot watch Pokemon" which represents rephrasing the questions in different ways) 44 times, 33.8%, which is the most frequent behavior observed, "exploring" behavior 25 times, 19.2% which is the second most frequent behavior observed and "implementing" behavior, 22 times, 16.9% which is the third most frequent behavior. As it was mentioned, the greatest percentage of existing behaviors was in understanding followed by exploring.

For Group 2, out of 112 items, students show "understanding" behavior 53 times, 47.3%, which is the most frequent behavior observed, "exploring" behavior 19 times, 16.9%, and "implementing" behavior, 15 times, 13.3%, which is the third most frequent behavior observed.

For Group 3, out of 178 items, students show “understanding” behavior 75 times, 42.1%, which is the most frequent behavior again, “exploring” behavior 27 times, 15.1%, which is the second most frequent behavior observed and “planning” behavior 19 times, 10.7%, which is the third most frequent behavior observed among the other behaviors. Differently from Group 1 and Group 2, this group shows more “planning” behavior than other groups.

Table 1: Percent distribution of behavior categories (episodes) by problem solving group

Behavior Category	Groups					
	Group 1		Group 2		Group 3	
Read	12	(9.2%)	6	(5.3%)	17	(9.5%)
Understand	44	(33.8%)	53	(47.3%)	75	(42.1%)
Analyze	1	(0.7%)	2	(1.7%)	1	(0.6%)
Explore	25	(19.2%)	19	(16.9%)	27	(15.1%)
Plan	7	(5.3%)	9	(8.0%)	19	(10.7%)
Implement	22	(16.9%)	15	(13.3%)	15	(8.4%)
Verify	11	(8.4%)	4	(3.5%)	10	(5.6%)
Watch & Listen	8	(6.1%)	4	(3.5%)	14	(7.8%)

Of all the episodes coded, the understanding episode was the coded as the greatest percentage within three groups while analyzing behavior was very rare. Among 419 items, 172 items represent “understanding” episode, 71 items represent “exploring” episode, 52 items represent “implementing” episode. The percentages of each episode are given in Figure 1.

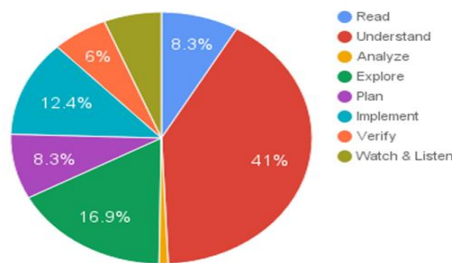


Figure 1: The percentage of the problem-solving behaviors for each episode in all groups

Discussion

According to Artzt and Armour-Thomas (1992) the greatest percentage of time was exploring (60.4%). In contrast to their study, we found that understanding was the most frequent observed behavior (33.1%). Watching and listening can play an important role in addressing the issue of communication between group individuals. These low and high percentages of watching and listening behaviors may be as a result of different reasons. For example, in Group 1, one student interrupted the other two students and also this student was writing the majority of the solution. The records and observations indicated that in each group some students assumed a leadership role. Therefore, it was possible that this kind of act would discourage the other two students in the group. However, during the problem-solving procedures, some productive interactions occurred while Student 1 (in Group 2) was not only supporting and guiding others but also got benefits from group members' ideas. These

results are in agreement with Artzt and Armour-Thomas' (1992) findings which showed different patterns interactions between group members and show the significance of intergroup relations for active and productive contribution.

The framework contributed the observation of individuals while working in small group settings. As it can be realized from the records and observations, group composition affected the group life. As group members were chosen from different classes, it is interesting to note that in all three groups students reflected a pragmatic desire in order to achieve the common goal by working together productively. With the exception of one student in Group 3, the small group study enabled researchers to observe peer to peer communication in a small group environment.

'Understanding' was the behavior that was coded the greatest percentage by students in this study. It would be expected that after this phase, students could decompose the problem into basic components and examine the relations between given elements and common goals at the analysis level, and then explore the problem by guessing and testing. In our study, understanding led group members to the exploration without making visible analysis.

References

- Artzt, A. F., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem solving in small group. *Cognition and Instruction*, 9, 137–175.
- Doorman, M., Drijvers, P., Dekker, T., van den Heuvel-Panhuizen, M., de Lange, J., & Wijers, M. (2007). Problem solving as a challenge for mathematics education in The Netherlands. *ZDM: The international Journal on Mathematics Education*, 39, 405–418.
- Flavell, J. H. (1981). Cognitive monitoring. In W. P. Dickson (Ed.), *Children's oral communication skills* (pp. 35–60). New York: Academic.
- Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education*, 16, 163–176.
- Hammouri, H. (2003). An investigation of undergraduates' transformational problem solving strategies: Cognitive/metacognitive processes as predictors of holistic/ analytic strategies. *Assessment & Evaluation in Higher Education*, 28, 571–586.
- Higgins, K. M. (1997). The effect of long instruction in mathematical problem solving on middle school students' attitudes, beliefs and abilities. *Journal of Experimental Education*, 66, 5–24.
- Mayer, R. E. (1992). *Thinking, problem solving, cognition*. New York: Freeman.
- Mayer, R. E. (2002). Cognitive, metacognitive, and motivational aspects of problem solving. In H. J. Hartman (Ed.), *Metacognition in learning and instruction*, (pp. 87–102). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Mayer, R. E. (2003). Mathematical problem solving. In J. M. Royer (Ed.), *Mathematical cognition* (pp. 96–92). Greenwich, CT: Info age Publishing.
- Ministry of National Education (MoNE). (2005a). *İlkogretim matematik dersi 6-8. sınıflar ogretim program ve kılavuzu*. Istanbul: Milli Eğitim.
- Montague, M. (1991). Gifted and learning disabled gifted students' knowledge and use of mathematical problem-solving strategies. *Journal for the Education of the Gifted*, 14, 393–411.

- Montague, M., & Applegate, B. (1993). Middle school students' mathematical problem solving: An analysis of think-aloud protocols. *Learning Disabilities Quarterly*, 16, 19–32.
- National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.
- OECD (2003). The PISA 2003. Assessment framework- mathematics, reading, science and problem solving knowledge and skills. Paris: Author. Retrieved June 14, 2016, from <http://www.oecd.org/dataoecd/46/14/33694881.pdf>
- OECD (2005). Problem solving for tomorrow's world – first measures of cross curricular competencies from PISA 2003. Retrieved from <http://www.oecd.org/edu/school/programmeforinternationalstudentassessmentpisa/problemsolvingfortomorrowsworldfirstmeasuresofcrosscurricularcompetenciesfrompisa2003.htm>
- Polya, G. (1957). *How to solve it: A new aspect of mathematical method*. Garden City, NY: Doubleday & Company, Inc.
- Schoenfeld, A. H. (1982). Some thoughts on problem solving research and mathematics education. In F. K. Lester, & J. Garofalo (Eds.), *Mathematical problem solving: Issues in research*, (pp. 27–37). Philadelphia, PA: Franklin Institute Press.
- Schoenfeld, A. (1985). *Mathematical problem solving*. New York: Academic Press.
- Schoenfeld, A. H., Burkhardt, H., Daro, P., Ridgway, J., Schwartz, J., & Wilcox, S. (1999). *High school assessment*. White Plains, NY: Dale Seymour Publications.
- Verschaffel, L., De Corte, E., Lasure, S., Van Vaerenbergh, G., Bogaerts, H. & Ratinckx, E. (1999). Learning to solve mathematical application problems: A design experiment with fifth graders. *Mathematical Thinking & Learning*, 1, 195–229.