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Geometric modelling inspired by Da Vinci: Shaping and adding movement using technology and physical resources

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We report on the experience gathered in a study using a geometric modelling approach based on dynamic geometry systems complemented by physical modelling. Our strategy intends to strengthen the interconnections between the current trends in Science, Technology, Engineering, Art and Mathematics Education. To help students to better understand how certain physical mechanisms work, some da Vinci machine prototypes were reconstructed and used as a starting point for this study. Building upon previous experience, our work currently concentrates on the analysis of connecting physical and digital resources and on how they contribute to students' creative thinking and problem-solving. We discuss the concept of geometric modelling, focusing on spatial thinking, joints and their movements. Further, we present some new manipulatives that are being tested together with digital applets and discussions from this practice.

Keywords: geometric modelling, GeoGebra, problem solving, creative thinking.

Introduction

Geometric modelling has been used in a wide range of contexts, but continues to deserve more attention in mathematics education. The term “geometric modelling” usually refers to different digital techniques for representing specific objects or surfaces. This concept seems to be associated mostly with computer-aided design, the tasks of which are usually related to shape and to improving models in order to approximate real structures by means of sophisticated algorithms and software. This work intends to bring geometric modelling, supported by dynamic geometry systems (DGSs) and combined with physical resources, to the classroom. While an analogue model provides “hands-on experience” to students so they can comprehend certain mechanical movements (and their restrictions), a digital model forces them to develop suitable strategies for transcribing such actions. Students must therefore think about how to apply mathematical concepts properly in order to use them successfully in their digital models. In our case, creating a digital representation is not only a mathematical exercise, but also an opportunity for the students to refine and review their comprehension processes and to improve their physical models. In this modelling approach, students are faced with problem-solving in Science, Technology, Engineering, Art and Mathematics (STEAM) contexts.

Modelling is characterized as the branch of mathematics that deals with the translation of a real-world situation into mathematical language. In publications discussing mathematical modelling (e.g., Brinkmann & Brinkmann, 2008; Lingefjärd & Holmquist, 2001), data are used to optimize particular processes or to develop algorithms for analysis or prediction. Geometric modelling approaches also concentrate predominantly on processes rather than on actual goals and concepts (e.g., Mason, 2001; Henning & Keune, 2008; Siller, 2008). We use geometric modelling for both the virtual (on the computer screen) and physical (concrete manipulative) representation of objects, which allows both the objects' functionalities and the interactions between them to be analyzed. Exploring linked

mechanisms by using either concrete manipulative or digital tools such as GeoGebra, students can examine mathematical ideas in order to improve their constructions. We focus on the process of using modelling to support geometrical relationships and vice versa. Our goal is that students gain a better understanding of how certain mechanisms work, beyond connect such STEAM areas. As a starting point, we used some mechanisms introduced by Leonardo da Vinci and shared them in an interactive GeoGebra book (see <https://ggbm.at/AnHK7nCX>). Although da Vinci invented them more than 500 years ago, their basic physical principles are still used in engineering and in education. Reconstructing these models does not only mean redoing what has already been done; it means connecting ideas and strategies for learning and using new resources based on some classical ideas, and maybe even improving them along the way.

In this paper, we present an activity developed within a vocational course in Brazil. Building upon this preliminary experience, we intend to investigate (in the form of a PhD project) how the combined use of physical and digital tools is of mutual benefit and promotes learning in science and mathematics.

Theoretical framework

Enhancing geometric modelling through Dynamic Geometry Systems

More than a “mathematical playground”, dynamic geometry systems (e.g., GeoGebra) must be considered as a proper space for transforming students into explorers – a platform where mathematical understanding takes on another dimension and goes beyond merely applying formulas. Numerous studies have advocated this (e.g. Schumann, 2004; Gawlick, 2005; Bu & Hohenwarter, 2015) and stress the importance of transcending traditional geometry courses before the increasingly sophisticated and widespread application of geometry in science and daily life. In particular, they highlight how DGSs have supported changes in mathematics teaching and learning as well as in professional mathematical practice. Among the perspectives associated with the DGS approach, we outline those that are closer to geometric modelling: problem-solving, concept formation, construction, measurement, visualization, exploring, and variation/animation. DGSs often grant access to information that would otherwise be inaccessible. For instance, in spatial geometry, GeoGebra enables users to see an object from different points of view and its cross sections. Such features provide an important link between 2D and 3D representation. Investigating whether some ideas established in 2D also work in 3D or how they should be adapted is a promising strategy for promoting spatial thinking. Consider an example where the reasoning based on circles in a plane can be extended to a sphere in space: We have a point given in a plane and a line at a certain distance d from the point. In order to mark those points on the line that are located at the same distance $e > d$ from the point, one could use a circle in 2D, but this would not suffice in 3D space (a more appropriate approach would be to use a sphere in this case). The attempts to find a solution using DGS software enhance the students’ conjecturing processes – another benefit of DGSs.

Teaching approach using analogues and DGS

In the didactic domain, Alsina (2007) proposed that students gain insights from the functionality of objects and thus engage themselves in a creative process because they can identify the potential or restrictions of a particular phenomenon. Gravina (1996) suggested that dynamic geometry can foster an approach to geometric learning in which assumptions are made from experimentation and the

creation of geometric objects. In the same vein, Swan et al. (2007) stated that students can refine their own thinking by interacting with different representations of problem situations. In our case, the aligned and parallel modelling process led students to a practice where they had to hone their ideas in every construction step. Facing the need to convey their ideas, students tapped into their previously acquired understanding (an example is discussed in more detail further below).

Recent studies (e.g., Sinclair, Bussi, de Villiers, et al., 2016; Camou, 2012; Lesh and Sriraman, 2010) support the positive effect of the design and implementation of a multi-representational approach to exploring 3D objects using crafts, computer technology, and paper-and-pencil methods. In this context, we seek to provide an integration of geometry with algebra and trigonometry (using the example of joints with circular movements) that goes beyond technical instrumentation.

In our case in particular, the use of mechanical principles provides the background for the modelling process, as can be seen from the diagram in Figure 1, which was adapted from De Sapio and De Sapio (2010): they considered the relevance of applying an approach to problem-solving at an elementary stage through constructing mechanical analogues to geometric problems. In this case, mechanical reasoning supports geometric reasoning. Note that we added the arrow in the opposite direction, since the reverse case (geometric reasoning supports mechanical reasoning) is equally possible, as shown in our study: On the one hand, mechanical reasoning was essential to discussing the proper ratio for a pulley system in one case. On the other hand, with the help of rotational simulations (i.e., geometric reasoning) by means of digital modelling, the students figured out how to build a functioning physical prototype in another case.

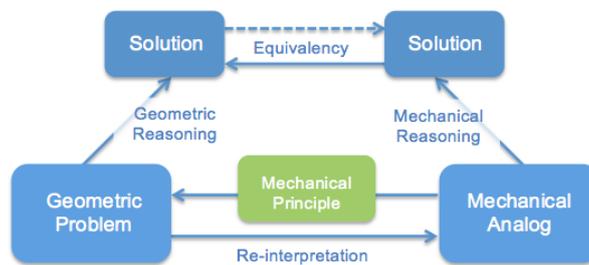


Figure 1: the solution's correspondence come out in both directions

Concerning the activity's driving, the activity is consistent with DeHaan (2009), who stated that some strategies can transform the lecture hall into a workshop or studio classroom (even partially), and stressed the use of computer-based interactive simulations as a promoter of creativity instruction.

In fact, researchers investigating creativity generally argue that projects tend to be more creative when the solution is redefined, revisited, and questioned numerous times during the process (Lee & Carpenter, 2015). Furthermore, there are many different ways of developing prototypes. The process of refining ideas and designs puts students on an unrestricted path. Siswono & Novibasari suggested that problem-posing activities using the "What's another way?" strategy could improve students' abilities in creative thinking (as cited in Siswono, 2010). In our study, the students discussed their different ideas, especially in digital modelling. In this phase, they considered various points of view and also tried to gain some insights to check whether their ideas were feasible or not in order to do make the model as simple as possible.

Methodology

Our experiment started in September 2015 at the Federal Institute of Education, Science and Technology in Brazil. It took approximately four months and had the form of a partial extra-class activity with two weekly meetings to follow the progress. The students (most of them were 16 years old) participating in the vocational (informatics) course were supported by two additional teachers (physics and mathematics). Although supervised by teachers, they chose their own topics to investigate. We first present the Da Vinci Rotatory Bridge Project (Figure 2) developed by 4 students. It was agreed that they should develop both physical and digital models in order to try to improve the joints of the existing mechanisms. Our intention was that, by comparing similarities and differences between the models, the students should be able to use one to support the other. No particular order was prescribed, but parallel development was suggested. Use of GeoGebra was also optional but recommended, since we were exploring it during class. In fact, da Vinci projects have been promoted since 2011, but this was the first time GeoGebra materials and GeoGebra 3D features were integrated. Particularly in the digital modelling process, the students concentrated on principles of rotation, translation, and spatial geometry.

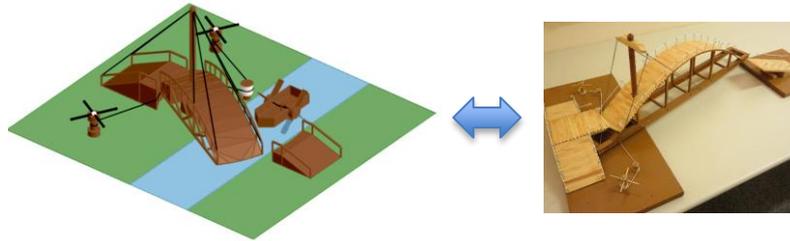


Figure 2: The digital prototype developed using the GeoGebra 3D feature (left) and the physical model made of wood (right) were developed in parallel

As a further development we are combining this activity with a new resource, 4Dframe¹, which is a flexible material and easy to manipulate. In the next example (Figure 3), we follow the development of a digital catapult in two versions. The second one is based upon the 4Dframe model.

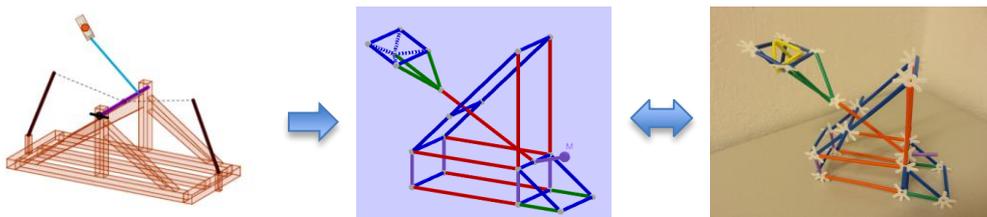


Figure 3: Catapult evolution and becoming easy to represent

Since mechanical principles are highlighted in a simple way, it becomes easier for students to represent them in GeoGebra. Furthermore, elementary models with straws and connectors become part of the digital modelling more easily when only segments and points are used to represent the structures. The functional principle of the joints, however, is preserved. In addition, various colours are used to represent the corresponding elements and to contribute to the visualization and facilitate spatial comprehension.

¹ For some examples, see <https://www.geogebra.org/m/xCxJUyyx>.

Finally, another important benefit is that the digital model allows a wide range of representations to be created by simple cursor movements (in the example in Figure 4 by dragging the blue points). Using 4Dframe, the students can think freely about different possible solutions and the constraints arising for an eventual construction, and are engaged in a learning process involving critical and creative thinking.

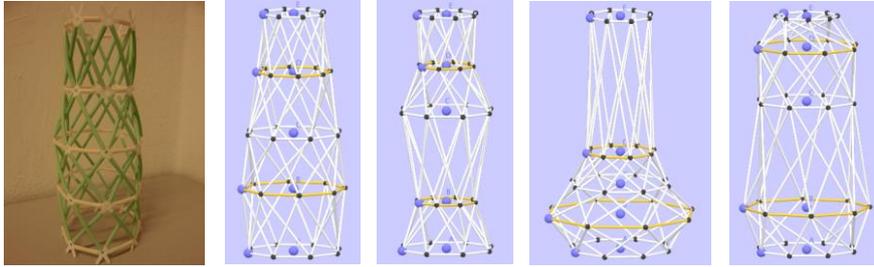


Figure 4: Multiple representations promote creative thinking. By dragging the blue points in the digital model, students can create a range of possible solutions.

Construction's ideas and discussion

In order to illustrate some basis used by students to support such models, we share some parts of them. The students were free to construct according to their own previous knowledge and were not required to use any specific content. However, if the need arose in a particular task, the students received proper support. In such cases we enjoyed discussing the problem at hand with them and, together, introduced new concepts or strategies, as presented below.

Given a circle with its centre in A (located on the x-axis) and an arbitrary radius² we started the task (Figure 5). First students investigated the relative position of a point B on the circle in relation to its center while sliding along a line. Since the goal here was to simulate the movement of a wheel, they needed to implement rotational movement. To figure this out, they changed the definition of point B to $(x(A) + \sin(x(A)), y(A) + \cos(x(A)))$. Naturally, this result was obtained after several attempts and discussions. Note that, in this case, the point B is a function of point A.

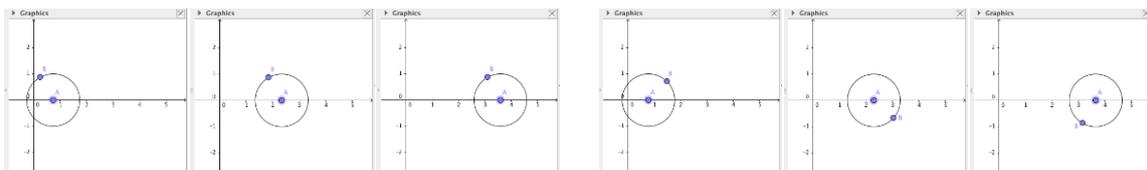


Figure 5: Three-frame cycles represent the transition from a "dragging circle" to a "rolling circle"

In a 3D representation, the same logic is preserved, but this concept was totally new for the students: Each point then has three inputs. If a circle is perpendicular to one of the standard axes, then all points belonging to this circle have a constant input regarding to such axis. The other two coordinates repeat the idea from the previous 2D example. For instance, in the case below (Figure 6), B is given by $(x(A) + \sin(x(A)), 0, z(A) + \cos(x(A)))$. The connected elements can be completed by rotation or symmetry. Additionally, some principles applying to spatial coordinates were used to define proper points as a

² The radius could also be controlled by a slider. In this case, the scale had to be in accordance with the remaining construction.

basis for such constructions. In the boat example (Figure 6), a first reference segment was built to guide the following marks through translations and reflections.

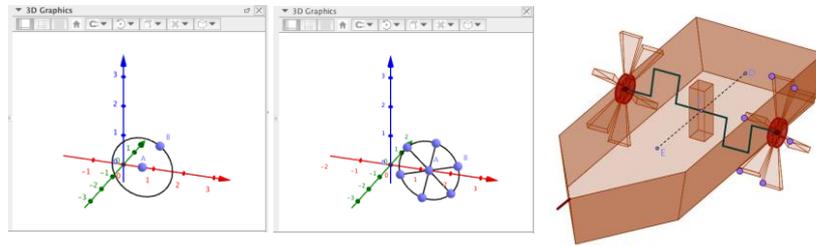


Figure 6: Developing a 3D representation in 3 steps

In this activity, the students should preserve the coherence between analogue and digital model as much as possible. To illustrate this, we refer to the bridge example, where the relation between the turns of the driver pulley and the bridge was to be determined; this is an issue easily identified in the physical prototype but not as readily in the digital model. When the students realized that the models were not in agreement, they concentrated on the geometrical problem and concluded which ratio between the number of turns of the bridge and of the pulley should be appropriate. In contrast, when students modelled only the digital boat (in a previous experiment), they recognized the following misconception before building the physical model: if the paddle wheels spun together at the same speed, the boat would move only forward and backward. They then fixed this problem in the physical model. This suggests that it does not matter which model they build first as long as the whole model contributes to their experience and improves their learning process. When digital geometric modelling was the goal, students needed to use their knowledge of trigonometry and parameters (functional thinking) in order to establish links between elements to obtain the desired representation. Questions such as “*If you want to change the direction of the rotation, what do you have to do?*” often initiated their investigative process and sometimes became a challenge.

Conclusion

“Inviting” students to reconstruct historical models is one of many possible ways to teach mathematical concepts and to promote students’ creative thinking processes. Students must decide how they can use their previous knowledge in order to solve a given task. In this way, students increase their autonomy and become more involved in their own learning processes. An important step of Pólya’s heuristic strategies can be outlined: “If you cannot solve the proposed problem, try to solve first some related problem” (as cited in Schoenfeld, 2016).

While we promote STEAM on the one hand, we introduce different dimensions of learning to students and enable deep learning driven by their own interests on the other. Problem-solving and geometric modelling can also become a basis for the integration of mathematical learning into trans-disciplinary educational frameworks, currently referred to as STEAM.

In the course of this study, students left some testimonials on Moodle. One such testimonial referred to some physics aspects: “*I enjoyed two videos posted by the teacher that showed how da Vinci’s bridge worked. They enabled a better understanding of the functioning of the rotatory bridge. I understood better how the system of ropes and pulleys works in order to reduce the workforce.*”

Feedback such as this and examples such as those reported above illustrate the students' interaction with different resources that supported them. They indicate that the geometric modelling approach can motivate and contribute to their learning process. We therefore seek to evaluate and promote this activity among teachers as part of our current research. We are now working on developing new resources that connect the physical and the digital world and will report on these at a further occasion.

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