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Fifth-grade students construct decimal number through measurement activities

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In this study, a teaching experiment was used in which fifth grade students developed their own knowledge about decimal number based on their prior knowledge and real life actual experiments. A Realistic Mathematics Education approach was used for designing this research. Participants in this study were 27 students from a primary school and they did not have formal instruction at school on decimal number until participating in the current study. These students engaged in some measuring activities during four sessions in two weekends. They discovered the idea of calibration of a measurement unit and tried to use this idea for measuring the length of objects. This study shows how real life experiments of students help them to calibration a measurement unit.

Keywords: Decimal number, measurement activities, realistic mathematics education, primary students.

Introduction

Understanding of decimal numbers is very important for people who live in twenty first century because they use computer, calculator, digital monitor and other measurement activities in their real life experiments. Usually, people encounter with decimal number in their real life. For example nutrition facts written on foods, factors for buying products, vaccination card of children and ... have a lot of things related to decimal number. All children encounter with decimal number before formal education at school, although its meaning is not understood. According to Bonotto (2009) connection between outside of school experiments of children and formal education can support students' conceptual understanding.

In traditional teaching, decimal numbers developed through place value. Usually students use tens and hundreds blocks for consolidating decimal numbers concept. Operations and procedures explain by teachers abstractly and then ask students to do some similar exercises. Indeed, students hadn't active role in developing their own knowledge and there is no connection between out of school students experiment and their math classroom activities.

It seems to we need to develop new approaches for teaching decimal number. One of the useful ideas in this regard is realistic mathematics education approach. In current study, we try to use this approach for finding some real contexts which are authentic for students and these contexts used for starting point for constructing mathematics by students themselves. Indeed, this paper will be introducing a teaching experiment in decimal number domain. During this teaching experiment, students do sequence of measuring activities to develop a measurement unit and calibrate it for measuring different lengths. Students also discover different representation for decimal number through these activities. The main purpose of this research was that show how students develop their

own knowledge about decimal number in the context of real world with using their common sense and prior knowledge.

Literature review and theoretical framework

Decimal number is one of important topics in school mathematics which has plenty of application in students' real world experiments. But several studies show that students and even adults haven't good understanding about decimal numbers (Moloney & Stacey 1997; Steinle, 2004; Lai & Tsang, 2009; Sengul & Guldbagci, 2012). Lai and Tsang (2009) show that procedural knowledge of students in decimal number was very good, but their conceptual knowledge in decimal number and decimal notation were so weak.

One of the important concerns of Lai and Tsang (2009) is that do the mathematics teachers know how to deliver decimal conceptual knowledge to the students? Bonotto (2001) believe that students' difficulties in decimal number rooted in teachers teaching strategies which have no connection to real life of students. Indeed, students usually encounter with decimal number in format of some stereotype word problems. Niss, Blum, and Galbraith (2007) said word problems exist several centuries in school math curricula and used as application of mathematics, but these types of problems in fact are a pure math problem in cover of words. Greer (1997) believes that word problems are artificial. Verschaffel (2002) states that emphasis on word problem cause to suspend common sense during mathematical problem solving.

Bonotto (2004) in the line with realistic mathematics education, believe that engaging students into the contextual activities that related to their own personal life, help them to enhance their conceptual knowledge in mathematics and having positive attitude toward math. She mentions two below factors that separation between school mathematics and real life facts: Stereotype problems of mathematics textbooks and Classroom environment.

If teachers of mathematics wish to establish situations of realistic mathematical modeling in problem-solving activities, Bonotto (2005) proposes below suggestions:

- The type of activity to which teachers delegate the process of creating interplay between math classroom activities and everyday-life experience must be replaced with more realistic and less stereotyped problem situations;
- Teachers must endeavour to change students' conceptions, beliefs, and attitudes toward mathematics;
- A sustained effort to change classroom culture is needed.

Bonotto (2001) maintains that children's understanding of decimal numbers can be fostered by classroom activities where learners can transfer their out-of-school knowledge and utilize familiar tools, such as the ruler, that they also use in out-of-school contexts. She believes it is possible to attempt an innovative teaching trajectory in which decimal numbers are introduced through contextualized measuring activities. Indeed, measuring activities requiring vast use of a ruler can offer children good opportunities to move toward the construction of a comprehensive understanding of decimal number and notation. For example, a study of Astuti (2014) shows that, if students use paper strip and calibrate it by themselves, then their understanding about decimal numbers and notation will be more developed.

Method

In this study, a teaching experiment about decimal number will be introduced. Based on the research aim, the type of the research is categorized as design research. This research contributes to develop a Local Instruction Theory (LIT) to support students develop the understanding of decimal number and notation. LIT has cyclic spirit (Gravemeijer, 2004) that in this study is prototyped by a Hypothetical Learning Trajectory (HLT) (Simon & Tzur, 2004) which is elaborated and refined when conducting the design. The initial step of HLT in this study is developed based on the analysis of key areas of decimals from literature review, the analysis of Iran mathematics curriculum, and the analysis of the potential use of contexts and model based on the framework of RME.

Current study is part of larger study that investigates conceptual understanding of fifth grade students in several aspects such as constructing a measure, calibration a measure unite, familiarize with decimal and notation, comparison of decimal number, density of decimal number, and submission and multiplication of decimal number. This study concentrates only on constructing a measure, calibration a measure unite, familiarize with decimal and notation. Main purpose of this study was familiarizing students with concept of decimal number. In this regard five activities designed which related to students real life facts (see figure 1).

1. Select a measuring unit arbitrary and measure length and width of classroom board, approximately. Represent length and width of classroom board with mathematical symbol.
2. How we can get better approximation? Write your proposed method completely, then record measured length in a mathematical form.
3. Select an object that smaller than your measuring unit. How you measure the length of this object? Explain your method and write measured length.
4. In this week a one meter non-graded tape give to each group of students and ask them where do you hear about “half” concept? What is the meaning of “half”? What is mathematical symbol for that?
5. Divide non-graded tape in 10 parts. Then try to measure a selective object approximately, and then show it with new mathematical symbol (decimal number).

Figure 1: Activities of first week

These activities implemented in four 80 minutes sessions in two continuous weekends' day. First three activities implemented in session 1 and 2 in the first weekend. Fourth and fifth activities implemented in session 3 and 4 in the second weekend.

This study conducted in a primary school in the beginning of school year (Fall 2014). Participants of this study were 27 fifth grade (10-11 years old) female students. These students had no formal program in these two days and all of them participate in this extracurricular class voluntarily. All students work on activities in group. Each group contains three students. Two types of complementary communications occur in this study: group discussion and whole class discussion.

During these teaching experiment sessions, second author and two other math educators record all communication of students in group. In the end of each session, students' group works collected also. Below considerations navigate activities designing process, data collection and data analysis.

- Using of non standard measuring units for measuring objects in the classroom;
- Measuring with high accuracy;
- Calibrating a measuring unit;
- Numerical representation of length of object with using calibrated unit;
- Importance of decimal division and decimal representation of length of object.

After each session, video record of session and students' group works and researchers note analyzed and use them for leading teaching experiment in next sessions.

Results

Results of first day (Sessions one and two)

The students used different tools for measuring the length and width of classroom board, such as notebook, math book and A4 paper size. As it was asked them to measure the length and width approximately, so they write these sizes as follow and in term of a complete unit. In fact, the extra parts were neglected. A group of students who chose their notebook size as measuring unit (module) had stated that the classroom is 23 and a half notebook size length. Using the term "half" showed that they know the decimals informally.

In the second activity, it was asked them to measure the length more carefully. In all the groups it was seen that they divided the measuring unit (module) to small sections. The difference between group operations is in the numbers of divisions and choosing denominator. The mathematical symbols which were used in this activity are as the length of a natural number plus a fraction or a Mixed number.

To guide students to a more accurate calibration, they were asked to measure the length of an object that is smaller than the length of their module and write its mathematical symbol in the third activity. In this activity, student should choose smaller objects than unit. They chose the length of pencil lead packet, pen, notebook and etc to measure. The students' performance was divided in three classifications.

- Four groups of students neglect the previous division and creating the new one for their measuring. They changed the number of their part and explained that the length of chosen object is smaller than their unit, so they change the numbers of divisions. Indeed, they divided the length of measuring unit into larger equal parts.
- Three groups of students keeping their previous division. They just divide their previous division again.
- Two groups solve this problem in different way. They divide the unit into the five parts and then divide each part into the five parts again.

Maryam: we divided our set unit into 2 parts. We divided each into 3

Researcher: can you represent the length of measurement in math symbols?

Aida: one second (one a half) and three of this part.
(i.e. $1/2$ and $2/3$ of this part).

Researcher: would you please represent it in math symbols?

Aida: we should calculate it.

Maryam: we should add $1/2$ and $2/3$.

Marjan: no, it's not the $2/3$ of the notebook length. It is $2/3$ of one half of its length... It means its $5/6$. (She shows it on the picture to her partners).

In fact, this group is faced with a challenge in calculating the length with mathematical symbols. They needed to be able to add the fractions in unequal denominator. Dividing the previous parts into same part numbers, the operation of one group was different. First, they divided the unit into 5 divisions and do it again for each part. They stated:

"Although we can re-divide each part into 5 sections and we repeat it again and again for smaller parts. But if we want to notate the length, it will be difficult. For example, we have to write one fifth plus two twenty fifth plus ... so, probably the next denominator is 125. Again 125 times 5 ... then set the common denominator..."

The fourth group operation was, writing the numbers in base 5. They stated interestingly that they can continue this trend. They expressed that for setting the denominator, it's necessary to multiply the denominator to 5, so it was obvious that they found a regular algorithm for approximating the length of object.

In the classroom discussion, which is performed at the end of the first session, students expressed that writing the mathematical symbol for the length is very important. They have some problems in irregular dividing of parts, so they choose the third method as the best way to calibrate the units of measuring.

Results of second day (Sessions Three and Four)

In the first activity in the second session, the students were asked to express that where in real life have seen the word half and their symbol "half" and its mathematical symbol. Some of the students' expressions were as the following ones.

- We usually say in the grocery store: I need 2.5 kg lentils or in my mother's shopping list it was written 1.5 kg beans.
- The house is 15.5 m length.
- The jar contains 2.5 litter of water.
- The volume of the coke bottle was specified 1.5 liters of water.
- We need 1.5 meters fabrics for making this shirt.

All these written expressions showed that the students have seen and heard of these decimal symbols during shopping, in parents' shopping lists, food labels such as cokes etc. and they understand this concept. The following classroom discussion was conducted:

Researcher: what does one half mean?

Student: it means a half.

Researcher: what does number 5 mean based on 0.5?

Fatemeh: half a kilogram means a half of one kilogram, it is 500 grams. Half a meter means a half of a meter, so it is 50 centimetres. You can find number 5 in both of them.

Zahra: in my opinion, one second is equal to one half, so $\frac{1}{2}=0.5$.

Rezvan:so it means that there are many fractions which are equal to $\frac{1}{2}$ For example, $\frac{1}{2}=5/10=50/100$...that all of them are half or a half.

Maryam: eureka, eureka. $\frac{1}{2}$ is one out of two, so 0.5 is 5 out of 10.

Parisa: that's right teacher. 0.5 is 5 out of ten. It is the same as 50 out of 100 or 500 out of 1000. All are the same and mean a half.

Teacher: could you tell me that what 4 out of 10 looks like? Please, write down your answers on the paper (all the students wrote 0.4).

In this discussion, the teacher tries to help the students using their own knowledge of "half" concept in real world to find an equivalent phrase for $\frac{1}{2}$ in mathematical world. Using the concept of one half and $\frac{1}{2}$ and in regards to their knowledge about equal fractions, they can interpret 0.5 by their own and this process shows the horizontal mathematizing. In fact, these students are moving back and forward between the real and mathematical worlds for exploring this concept.

In the second activity in which a one meter ribbon was given to all the groups, all the students divided the ribbon into 10 parts. In fact, it was asked them to specify the decimetres on the ribbon and measure the object in decimetre or one tenth accurately. In other words, they realize the relationship between number 10 in denominator and decimal numbers go out one decimal place.

According to the class discussion in the previous session, the students divided decimetres into 10 parts for more accurate measuring and made the centimetres. They made a calibrated ribbon in centimetre accuracy. In fact, it was asked them to specify the decimetres on the ribbon and measure the object in decimetre or one tenth accurately. In other words, they realize the relationship between number 10 in denominator and decimal numbers go out one decimal place (figure 2).

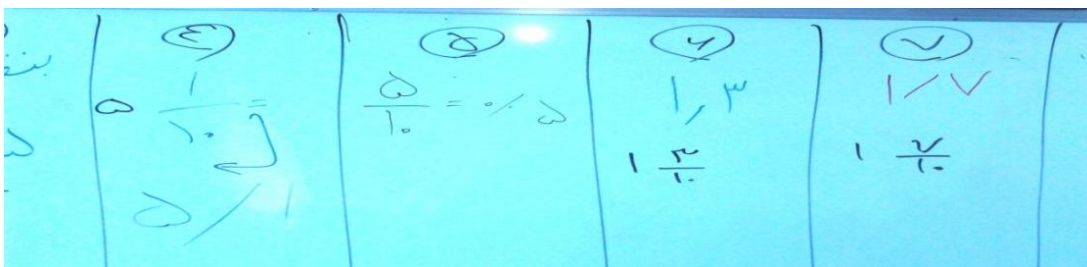


Figure 2: related to measuring the length of objects by calibrated ribbon in decimetre accuracy

When the teacher pointed that the ribbon is one meter length while one of the students said:

“That’s interesting, we divided the ribbon into 10 parts, and re-divided it into 10 so it means we have one meter into 100 divisions. These are centimetres. We knew one meter is 100 centimetres but I perceive it now”

The process of making a calibrated tape measure helps the students to realize the relationship between meter, centimetre and decimetre.

Discussion and conclusion

In the first session of teaching test, the students are allowed to calibrate a measuring unit freely but they had no idea about how to calibrate the unit. In the results of this study, it was seen that they faced a challenge with writing the mathematical symbol for the length, in the other words, these challenges caused to create a regular algorithm for calibrating the unit of measuring. The results of this study showed that choosing a division in base 10 is not natural. In the experience of Astuti (2014), the idea of direct division in based 10 was provided by researcher but here it was tried to conduct the students to dividing the unit in base 10.

This study confirmed the Van de Wall (2001) quote as said "students' mind is not like a whiteboard as entering the class". As it was mentioned, although the students had not learnt the decimal numbers formally and before the research but they used them in their real life frequently or found them on some objects covers, their parents' notes and etc. also they had seen the decimal numbers and separator mark. As Freudenthal (1991) stated, using students' background experience and information could help them in learning decimal notation and its concept. Measuring the length of an object for many times and in more accurate ways help them to realize the meaning of the digits after the decimal point. In fact, when they divided one meter into 10 sections and re-divided it again and again it means that they realize the position and concept of decimal and centesimal scales. One of them expressed, for more accurate scales; we can divide the centimetres into 10 and make it smaller and smaller. In fact, he noted the millesimal position and could guess the decimal demonstration correctly.

This study represented that if the classroom environment changes and better information in regards to the students’ real life experience are provided they can find new unknown mathematical structures by discussing and talking to each other and develop their mathematical knowledge.

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