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# Didactical activities to engineering training: Methodological proposal

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*In this paper we present a methodology to design didactical activities for training engineers. One phase of this methodology is selecting an extra-mathematical context, allowing identification and analysis of mathematical models used by engineers. We selected an industry beer context and we identified the Pareto chart as a tool to solve different problems, for example faults in production lines. This work uses elements from Anthropological Theory of Didactics. We present a praxeological analysis as basis of didactical activities.*

*Keywords: Beer industry, Pareto chart, training engineer, didactical activity.*

## Introduction

Over the past 30 years, mathematical modelling and applications have been a subject of study and development in the field of Educational Mathematics, as in the ICMI 14 study, edited by Blum, Galbraith, Henn and Niss (2007). Mathematical modelling and applications are activities in which math is used to solve problems in various contexts (math, engineering, economy, medicine, etc.). They are based on developing competencies associated with the application of mathematics and constructing mathematical models in extra-mathematical contexts (Niss, Werner and Galbraith, 2007). However, mathematical modelling and applications are not widely used in primary and secondary education, which give more importance to concepts and procedures. At the university level, modelling has emerged as a new educational paradigm (Bissell & Dillon, 2000). In the future, teaching professionals will be required not only to create but also adapt mathematical models to solve practical problems (Bissell & Dillon, 2000) related to interpreting solutions proposed by other professionals and to employ technology to perform mathematical tasks. This demands teaching math in such a way that it includes the use, adaptation, handling, and interpretation of mathematical models in order to adequately deal with tasks in extra-mathematical contexts. One of perspectives that comes closest to this demand is known as the realistic perspective, which is described in the proceedings of the Topic Study Group 21 (TSG21) of the 11<sup>th</sup> International Congress on Mathematical Education in 2008, as follows: “In this perspective, mathematical modelling is viewed as applied problem solving and a strong emphasis is put on the real life situation to be modelled and on the interdisciplinary approaches” (Blomhoj & Carreira, 2009). In this work, we consider the work of Kadijevich (2009), which illustrates this perspective, is a very interesting approach to the mathematical modelling due to the type of problems chosen to address it and to the use of information technology that is proposed. However, we believe that is not clear, from a theoretical point of view: how do you choose the actual context? How is a modelling activity generated in the context chosen? How is analyzed their relevance to the classroom? In general, it seems that the question of choosing the extra- mathematical context in order to propose the modelling activity has been theoretically little worked, it appears as a little cloudy element, leaving the emphasis on the characteristics of didactic activities (Galbraith & Stillman 2006) and on the modelling cycles that

allow to describe and analyse it (Blum & Borromeo Ferri, 2009). In an effort to attend to this lack of theoretical framing of the choice of additional mathematical context for the design of didactic activities, we have proposed a methodology based on the extended praxeological model (Castela & Romo, 2011) for designing didactic activities based on modelling that incorporate elements of the use of models in real contexts.

## Elements of the Anthropological Theory of Didactic

Since the extended praxeological model (Castela & Romo, 2011) is based on the Anthropological Theory of Didactics (ATD), we present in this section some of its elements. The ATD is an epistemological model that allows the study of human activity in its institutional dimension. An institution is a stable social organization that defines the human activities generating resources that make them possible. These materials or intellectual resources, which are made available to the subjects, have been produced by communities along the confrontation of problematic situations with the objective of solve them in a regularly and effectively way (Castela & Romo, 2011). The classic praxeological model, proposed by Chevallard (1999), recognizes the praxeology  $[T, \tau, \theta, \Theta]$  as a minimal unit of analysis of human activity. Its four components are: the task type ( $T$ ), the technique ( $\tau$ ), the technology ( $\theta$ ); and the theory ( $\Theta$ ). The ‘task’ refers to what is to be done; the ‘technique’ is how it is to be done; the ‘technology’ is a discourse that produces, justifies and explains the ‘technique’; while the ‘theory’ produces, justifies and explains the ‘technology’. Mathematical praxeologies or mathematical organizations can be of different level and they serve to a hierarchy of levels of determination proposed by (Chevallard, 2002). Mathematical institution imposes a model of subjection to the mathematical praxeologies: it rests on a structure that organizes praxeologies in different interlocked levels that are in increasing order of size as follows: specific, local, regional and global. The most basic level of a mathematical organization is the punctual  $[T, \tau, \theta, \Theta]$  and it has only one technique for performing such tasks. The next level is the local, which groups all punctual mathematical organizations associated with the same  $\theta$  technology. The regional level regroups all punctual organizations associated with same theory  $\Theta$ , global or domain regroups certain regional mathematical organizations; discipline is the top level and combines all domains.

## Codetermination of the mathematical and didactic

Chevallard (2002) develops the model presented below, in order to take into account the subjections that weigh on the didactic organization of the study of praxeologies. In this work, the author notes that didactic organizations cannot be developed if they are found far from higher levels, domain and discipline; reciprocally these levels cannot be imposed without considering the conditions of the educational institution. In that sense yields a co-determination of mathematical and didactic organizations.

[...] each level imposes, at a given moment during life of educational system a set of constraints and support points: ecology that results is determined both by what restrictions prohibit or drive, and the exploitation that actors make to the support points that different levels offers. (Chevallard, 2002, p.49)

As you can see the fact that Chevallard be interested in teaching leads to extend the range of levels. He introduces three higher levels: society, school and pedagogy, noting that levels of domain and discipline are also subject to restrictions imposed by these three levels that complement the scale

upward: Society → School → Pedagogy → Discipline → Domain → Sector → Local → Specific. This hierarchy makes us consider that the study of mathematical praxeology or modelling praxeology in an institution that is subject to various restrictions imposed by institutions of higher levels.

### **Moments of the study**

In the frame of ATD, the study is seen as the construction or reconstruction of elements of a mathematical praxeology, in order to perform a troublesome task (a task type for which a mathematical praxeology does not exist or is not available). In order to finely describe this process of construction or reconstruction, the ATD proposes a model of study of a punctual mathematical praxeology. This model distinguishes six moments, which are also associated with groups of activities. A moment is a dimension of the activity, a phase in the process of study, which may appear several times but following an internal global dynamics. Chevallard (2002) presents the model as follows:

#### **Group I (Study and Research Activities [SRA])**

1. Moment of *the (first) meeting with T*;
2. Moment of *the exploration of T and technical emergency  $\tau$* ;
3. Moment of construction of *the technological-theoretical block*.

#### **Group II (Synthesis)**

4. Moment of *institutionalization*.

#### **Group III (Exercises and problems)**

5. Moment of *work of the mathematical organization (specifically of the technique)*.

#### **Group IV (Controls)**

6. Moment of *the evaluation*.

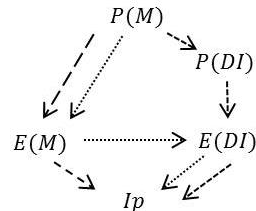
These moments are not detailed in this paper, but they are presented with the aim of showing how the ATD in the process of construction and reconstruction of a praxeology is conceived.

In particular, we are interested in considering group I and generating an SRA: a didactic device for students to construct, in this case, a modeling praxeology involving the Pareto Diagram. One of our questions is, what legitimizes the chosen mathematical modeling praxeology? Why is it important for students to build this praxeology? And in particular, future engineers. For us it is important that the mathematical modeling of the classroom is related to the mathematical modeling of its professional practice, which is seen as a relationship between institutions as shown below.

### **Training in mathematics and the professional world seen as institutions**

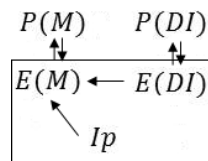
In the framework of the ATD, analysis of mathematical activity is considered in its institutional dimension. Given that our proposal is to generate a methodology for designing activities based on mathematical modelling that links mathematical knowledge teach at the training institution and the one used in the professional field, we must identify institutions that are adequate to participate in this process, and their interrelations. Romo-Vázquez (2009) argued that training engineers involves three types of institutions: Production (*P*), where praxeologies are produced, Teaching (*E*) responsible for transmitting the praxeologies. Use or users *Ip*, where the praxeologies are employed. By producing institutions we refer to mathematics *P(M)* seen as a discipline, together with its

intermediate disciplines  $P(DI)$ , which we will also call Engineering Sciences (e.g. signal processing, control theory, electrical circuits, etc.). Teaching institutions are represented by mathematics  $E(M)$ , and the intermediate disciplines  $E(DI)$ ; while the practical institution is  $Ip$ . The latter is examined at two levels: the professional practice of engineers, and the devices that, upon approaching practice, are developed in schools; for example, a project to innovate a product or service. Also taken into account were three inter-institutional tours that can be followed by a mathematical praxeology by going from  $P(M)$  to  $Ip$ . These can be represented schematically as follows (Figure 2):



**Figure 1: Institutional tours of a mathematical praxeology to go from  $P(M)$  to  $Ip$**

The transpositive effects (changes that occur upon moving from one institution to another) can be so large that a mathematical praxeology in  $Ip$  may not be recognized as mathematical. Research by Hoyles, Noss and Pozzi (2000) shows that some professionals utilize techniques and strategies in their practice that are based on mathematical models, but that when automated are no longer recognized as such. This begs the question: what sorts of didactic activities can be generated so that the mathematical models used in  $Ip$  practice or  $E(DI)$  find a place in methods of teaching mathematics  $E(M)$  (see Figure 2)? To answer these questions we propose the following methodology.



**Figure 2: News institutional relations between  $Ip$ -  $E(M)$  and  $E(DI)$ - $E(M)$**

## Methodology for designing didactic activities (SRA) based on mathematical modelling

This methodology for designing didactic activities based on mathematical modelling emerged from research by Macias (2012). Here, modelling activities are seen as praxeologies (mathematical and/or modelling) to be performed in  $E(M)$ , but in relation to praxeologies of  $E(DI)$  and/or  $Ip$ . It consists of four phases: 1) Selecting an extra-mathematical context, 2) Praxeological analysis and identification of a mathematical model. 3) Analysis of the mathematical model identified and its relation to  $E(M)$  and 4) Design of the didactic activity (SRA) for  $E(M)$ .

**1) Selecting an extra-mathematical context.** First, we must consider the educational level at which teaching will take place, then the contexts where the mathematical applications will be put to use. For example, if we consider teaching programs for engineers, the natural contexts of use are specialty training  $E(DI)$  and professional practice  $Ip$ . After that, one must identify some of the elements (resistance of materials, control theory, data structure, among others) that are of macro scale and may include various sub-institutions for the analysis of the modelling activity that occurs there. Selecting this context must be based on an approach to the institution or sub-institution

through interviews with one or more of the subjects involved (e.g. professors, expert users, researchers), a review of relevant documentation (suggested by the aforementioned subjects, and/or one's own search), and visits aimed at identifying the type of mathematical and modelling activity that is used. Specifically, it is important to analyse whether the mathematical models identified as being in use correspond to those that are actually taught  $E(M)$ ; examples could include functions, vectors, matrixes, mathematical optimization, or differential equations, among others. In this way, one can determine whether the context chosen provides a suitable analytical basis for designing the didactic activity.

**2) Praxeological analysis and identification of a mathematical model.** In this phase mathematical modelling activity is analysed through praxeologies. Modelling activities in an extra-mathematical context may consist of mathematical praxeologies and/or mixed praxeologies.

**3) Analysis of the mathematical model identified and its relation to  $E(M)$ .** A mathematical model that is in use but that is also taught in  $E(M)$  is identified and then analysed through the functions of the technology practice; i.e., describe, validate, explain, facilitate, motivate and evaluate. Describing the model in use allows us to elucidate the reasons relative to context on the basis of which that particular model was chosen to resolve tasks in the extra-mathematical context. Identifying the elements that validate the use of the model, and under what conditions, makes it possible to understand what contextual elements must be considered in designing didactic activities. For example, many mathematical models are used in “ideal” conditions such that they make it possible to resolve certain tasks more easily, though the solutions obtained will later need to be adapted to reality. This adaptation is conducted on the basis of certain elements that *validate* it. Recognizing the *explanations* of use allows us to understand what each element of the model represents and to what degree the model used allows us to model the context (or part of it). Analyzing the elements that *facilitate* the use of the model reveals the process of mathematical modelling, which entails not only assuring that the mathematical model chosen will make it possible to resolve a problem in an extra-mathematical context but also that the resolution reached will be the least complex one. Identifying precisely what it is that motivates the use of the chosen model is a medullar phase in designing didactic activities, but this analysis of use must be complemented by a didactic analysis of the model in the context in which it is taught.

**4) Designing didactic activities (SRA).** Designing a didactic activity must be based on both the praxeological analysis of use (praxeologies present in an  $Iu$ ) and the mathematical model identified; i.e., one must recognize the praxeologies of both use and teaching in order to perceive the relations between them. One must choose the types of tasks that, because they emerge from use, can be adapted for a type of school task; for example, studying the behavior of a continuous signal, determining the total cost of an inventory, or calculating the inverse of a mixed matrix, etc. These types of tasks require mathematical techniques that may be school-related (being part of curriculum) but are also used in the professional field, so mathematical and non-mathematical technologies will have to be built by the students (3<sup>rd</sup> moment) in order to validate, explain and justify techniques that emerges when confronting the problematic tasks. Specifically, technologies of use (part of practical praxeologies) must be adapted in such way the students can build them in the first college courses. The objective of the didactic activities must be oriented towards the type of praxeologies that figure

in the activity, whether this be constructing, mobilizing, or searching, for knowledge. It is intended that this activity may be an SRA which allows building a praxeology of modelling.

### The context proposed: Pareto chart in beer industry

To make a praxeological analysis we are chosen a Beer industry that is constituted by ten production lines and produce about one million hectoliters of beer per month. Apart from domestic beer this industry produce lots of beer for export to countries in all continents. To meet domestic demand and shipments abroad, it requires each of the ten production lines, meet high levels of efficiency. However, in each production line problems requiring immediate attention in order to achieve the planned goals they are presented. Thus various problems for the maintenance of thousands of machines, components and parts that make up each of the production lines are also presented. And logistics to control the flow of materials required in the production and control of shipments. In Tolentino (2015), these questions are studied: What Mathematics used in this industrial environment? Are there some common mathematical tools to manage the wide range of problems in the brewing? Tolentino, was both a master's student of mathematics education program and he was working in the industry as an engineer, he found that a Pareto chart is used to solve problems in the industry. This chart is based on the principle that if 20% of the causes of problems is attended a solution of 80% is obtained in effect.

### Praxeology faults problems

**Type of task.** Identify the most important causes of faults in production lines of an industry. **Task.** To solve faults (time) that arise in different production lines of beer production. **Technique. Step 1. Data collection.** It is recorded in a table stop time of production lines due to operational faults, faults in machinery or material defects. **Step 2.** Is ordered from highest to lowest the time column of this table, the percentages of stop time are obtained for each line, in relation to the accumulated from 157.62 hours. A column for cumulative percentage is added.

Line	Wasting time (h)	Percentage	Cumulative Percentage
Line 2	41.02	26.02	26.02
Line 8	25.65	16.27	42.30
Line 12	24.34	15.44	57.74
Line 6	14.73	9.35	67.09
Line 10	14.17	8.99	76.08
Line 3	13.8	8.76	84.83
Line 11	11.03	7.00	91.83
Line 5	8.33	5.28	97.11
Line 4	4.55	2.89	100.00
cumulative	157.62		

**Table 1: Wasting time of production lines**

**Step 3.** Two graphs are performed in Excel: in the bar graph the columns for each line indicate stop time. Graph of the Lorenz curve or cumulative percentage. On the right vertical axis measures the time from 0 to 41.02 h. For the Lorenz curve on the right vertical axis the percentage of 0 is measured at 100 percent (See figure 3). **Step 4.** Identify the diagram. Finally add a line from the 80 percent that intersects the graph of the cumulative percentage, and descends to the x axis, to separate lines that are to the left of this line it is the line 2, 8, 12, 6 and 10 (see figure 4). These are the lines that generate 80 percent of the total time of overall production stoppages during this time period.

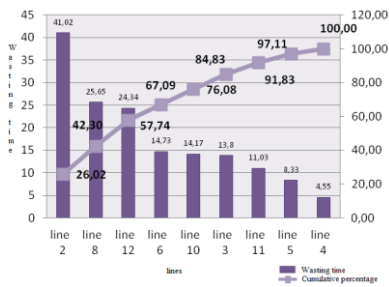


Figure 3: Bars and Lorenz curve

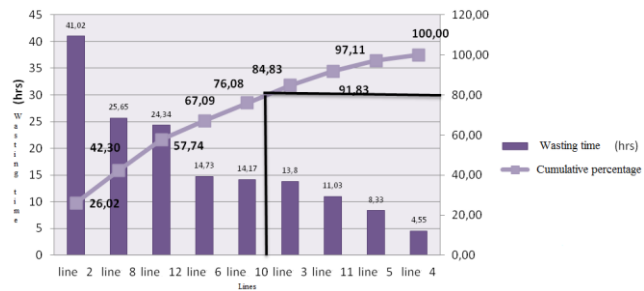


Figure 4: Analysis of the Pareto Diagram

**Step 5. Analyze the Pareto chart.** As a result of the above analysis, attention is directed to lines that generate 80 percent of stop time, *ie* lines 2, 8, 12, 6 and 10. Therefore proceeds to Pareto analysis of each one of these 5 lines, starting at line 2. Leaving as trivial the 4 lines: 3, 4, 5 and 11. That is although the line 11 was stopped 11 hours during this week, is considered out of importance according to the analysis Pareto realized. In this case, the Pareto Diagram is used again to analyze lines 2, 8, 12 and 6, to make decisions about the elements that must be repaired urgently, in each of the lines. The Pareto Diagram is made by engineers and allows them to identify the main causes of the problem, however a deeper analysis of the industry is necessary to recognize how the practical and theoretical knowledge allows solving the identified causes. This analysis is repeated in the brewing industry several times to have elements to act and address problems optimally, using fewer resources and obtaining the greatest benefits.

## Conclusion

We consider that this praxeological analysis (phase 2 of methodology), briefest presented, allow us to see the importance of Pareto chart on the beer industry. To design the SRA, it is necessary to analyze university courses likely to identify a local Praxeology the Pareto chart. However, analysis of the Pareto chart of the beer industry gives elements for SRA: from data of different problems ask propose a model that allows the company to identify the major causes of the problems. Considering the problem of faults, you can ask students a model to identify the line that causes the greatest wasting time or the lines that cause 50% of the strikes, then 75% and then 80%. The interesting thing about this proposal may lie not only that students can work with data from businesses, but they reach handle the Pareto Principle, the 80-20 ratio. The Pareto Diagram also involves mathematical elements that support them as the Lorenz Curve, which has hardly been mentioned here. In Tolentino (2015) the mathematical analysis of the origin and evolution of the Pareto Diagram is presented and this must also be considered in the design of the SRA, involving three institutions  $P(M)$ ,  $E(M)$  and  $Ip$ .

## References

- Bissell, C. & Dillon, C. (2000). Telling tales: Models, stories and meanings. *For the Learning of Mathematics*, 20, 3-11.
- Blomhoj, M. & Carreira, S. (Eds.). (2009). *Mathematical applications and modelling in the teaching and learning of mathematics*. Roskilde, Denmark: Roskilde University.
- Blum, W., Galbraith, P., Henn, H.W. & Niss, M. (Eds.). (2007). *Modelling and applications in mathematics education: The 14th ICMI Study ICMI Study*. New York, NY: Springer.



- Blum, W. & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application*, 1, 45–58.
- Castela, C. & Romo-Vázquez, A. (2011). Des mathématiques à l'automatique: Étude des effets de transposition sur la transformée de Laplace dans la formation des ingénieurs, *Recherches en Didactique des Mathématiques*, 31, 79–130.
- Chevallard, Y. (1999). La recherche en didactique et la formation des professeurs: Problématiques, concepts, problèmes. In M. Bailleul (Ed.), *Actes de la Xème Ecole d'Eté de Didactique des Mathématiques* (pp.98–112). Caen: IUFM de Caen et A.R.D.M.
- Chevallard, Y. (2002). Organiser l'étude. 3. Écologie et régulation. In J. L. Dorier, M. Artaud, M. Artigue, R. Berthelot, & R. Floris (Eds.), *Actes de la XIe Ecole d'Eté de Didactique des Mathématiques* (pp.41–56). Grenoble: La Pensée Sauvage.
- Galbraith, P. & Stillman, G. (2006). Framework for identifying student blockages during transitions in the modelling process. *Zentralblatt für Didaktik der Mathematik*, 38, 143–162. doi:10.1007/BF02655886A
- Kadijevich, D. (2009). Simple spreadsheet modelling by first-year business undergraduate students: Difficulties in the transition from real world problem statement to mathematical model. In M. Blomhoj and S. Carreira (Eds), *Mathematical applications and modelling in the teaching and learning of mathematics: Proceedings from Topic Study Group 21 at the 11th International Congress on Mathematical education in Monterrey, Mexico, July 6-13, 2008*. Roskilde: Roskilde Universitet. (IMFUFA-tekst: i, om og med matematik og fysik; No. 461).
- Niss, M., Werner, B. & Galbraith, P. (2007). Introduction Part I. In Blum, W., Galbraith, P., Henn, H.W. & Niss, M. (Eds.). *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 3-32). New York, NY: Springer.
- Romo-Vázquez, A. (2009). *Les mathématiques dans la formation d'ingénieurs*. Paris: Irem de Paris.
- Tolentino, S. (2015). *Uso del principio de Pareto en la industria cervecera y su posible vinculación con la enseñanza de las matemáticas*. (Unpublished master's thesis). CICATA-IPN, México.