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Mathematical modelling in an archaeological context: Their complementarity as essential tool for inquiry

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This paper focuses on analyzing the potentialities of using a codisciplinary inquiry context to enhance the appearance of mathematical modelling in teaching and learning practices. We focus on an archaeological context where modelling becomes an essential tool to enquire into and progress on the study of questions emerged from the interplay between two main disciplines: Mathematics and History. We present the design, implementation and analysis of a teaching sequence based upon the ruins of a Roman theatre discovered in Badalona (Catalonia) to promote inquiry and modelling students' competences. The sequence was implemented in 2015 with 12 and 14-year-old students. The central question introduced (what kind of building could have been the discovered ruins) involved students in a reflective inquiry process that facilitate them to progress on modelling, at the time modelling facilitate that the inquiry process could follow.

Keywords: Inquiry competence, mathematical modelling, co-disciplinary context, geometrical model, task design.

Introduction

The main aims of our research are to (1) design codisciplinary teaching sequences to promote inquiry students' competence, (2) implement them and (3) analyze their affordances to enhance inquiry and modelling students' competences. In particular, this paper focuses on the design, implementation and analysis of a teaching sequence based upon an archaeological context—the ruins of a Roman theatre discovered in Badalona (Catalonia, Spain) and how modelling became an essential tool to enquire into several questions that emerged from this extra-mathematical context, in order to progressively build up the necessary subject knowledge to provide and validate answers.

As aforementioned, the sequence started with an initial situation that leads to the formulation of the specific initial question and became the core of the students' research. The starting situation which is presented to students is the discovery of some Roman ruins in a Badalona's suburb by the archaeologists' team of the city Museum. The archaeological report that is provided explained that these ruins could belong to an ancient Roman building, but *what kind of building could it have been? Could be a theatre? A circus? An amphitheatre?* In order to face these questions, it is essential to make students unfold their inquiry and modelling competences and help teachers to guide student's reflections and to organize inquiry activities so that (mathematical and historical) knowledge could emerge and be used.

In the research we present, the codisciplinary context here considered for the design of the sequence—an archaeological context adapted from a real research—presents several advantages, which will be further developed in the following sections. On the one hand, requires that Mathematics and History work together to face most of the questions that appear in the teaching and learning

sequence. This no so common complementarity between both disciplines allows us to provide authenticity (Vos, 2011) to the questions faced and to promote the blend of inquiry and modelling in different steps of the study, as also some previous investigations have shown (Sala, Giménez & Font, 2013; Sala, Barquero, Font & Giménez, 2015). On the other hand, this codisciplinary context is enough close to students (as it is located in the same city that the school is) that facilitates that students can have access to the real ruins and to experts' answers about their real investigations, providing legitimacy to their inquiry process and responses. After describing the main theoretical frameworks upon which the design of the teaching sequence was proposed and analysed, we present the main research questions about how the use of codisciplinary inquiry contexts can help to break disciplinary boundaries (in particular between Mathematics and History) and about how their interaction leads to a rich complementarity between inquiry and modelling.

Theoretical framework and research questions

The research we present in this paper considers different theoretical aspects. First, it is assumed the notion of *basic competence* considered in Catalan curriculum. More concretely, we are interested in the notion of *inquiry competence* that, following Sala (2016) and framed according to the Catalan curricula of Primary and Secondary school education, it is defined as:

The ability to mobilise the suitable knowledge and the appropriate resources that facilitate the development and application of a logical and critical methods —under the teachers' guidance— in order to look for and find answers to problematic questions or situations in some school and/or out-school context. (Op. Cit. 2016, p. 64)

In the same line than Artigue & Blomhoj (2013) underlines, an important step in legitimising inquiry-based approaches was the publication of the National Science Education Standards (NSES; NRC, 1996), which called for students to do and know about scientific inquiry, and that teachers should foster the development of inquiry skills. When the inquiry-based approaches migrate towards mathematics, it seems important to consider approaches paying attention to establishing connections among mathematical and extra-mathematical world, such as modelling approaches. In this sense, some of the inquiry requirements (such as making observations, posing questions, examining sources of information, identification of assumptions, considering alternative explanations; NRC 1996, p.23) are closely related to some essential steps of the cycling process through which mathematical modelling is developed. So that, sometimes, it is difficult to find the differences between both processes (see for instance, the modelling and inquiry cycles presented in Blomhoj, 2004) and the linked competences. In particular, as pointed in Niss, Blum & Galbraith. (2007, pp.3-8), modelling constitutes a competence in its own right, which needs to be developed through appropriate modelling activities. From our viewpoint, to promote inquiry mathematical modelling should be placed at the core of the mathematical (and scientific) teaching and learning practices to ensure the right and rich development of important enquire abilities. Also, in the other way round, to promote and ensure mathematical modelling practices, inquiry should nourish some essential steps that may make modelling successfully progress.

Second, we considered different theoretical elements to ensure that the sequence design could offer a rich and functional teaching and learning of mathematics. With this aim, the mathematical and

didactic design quality is justified based on the three criteria of didactic ‘suitability’ —*emotional suitability*, *ecological suitability* and *epistemic suitability*— proposed by the onto-semiotic approach (Godino, Batanero & Font, 2007). Different aspects of its design considered justify the emotional suitability of the teaching sequence: the students could work with real data evidence —data from the research report of the discovery of real Roman ruins—; they could interact with the archaeologists’ team of the city Museum and could share their results with them, etc. In turn, *ecological suitability* was justified by the curricula of these secondary-school students having a competency-based approach. Likewise the implementation allowed students to unleash relevant processes of mathematical activity, in particular processes of mathematical modelling that justifies the *epistemic suitability* or mathematical quality.

Finally, we also use the notion of *research and study path* (SRP) proposed in the framework of the anthropological theory of the didactic (Chevallard, 2015) and their main characteristics to design the sequence of tasks to achieve a high epistemic suitability (see also Sala et al., 2015). The SRP proposal emphasises the necessary dialectics between ‘research’ or ‘inquiry’ (facing open-problems, problem posing, making observations, examining different media or source of information, etc.) and ‘study’ (attending teachers’ explanations to provide mathematical knowledge, building up models, testing the validity of mathematical tools, etc.). As underlined in Barquero, Serrano & Serrano (2013), mathematical modelling cannot be considered only as an aspect or modality of mathematical activity but has to be placed at the core of it. Modelling is not only as a way to make the functionality of mathematics visible, but also as a key tool for the functional construction and connection of mathematical and extra-mathematical knowledge. In this sense, inquiry and modelling have to be mutually enriched. On the one hand, modelling activity needs from more inquiry moments (posing questions, looking for external resources, looking alternative proposals, etc.). For instance, in the SRP the role of the questions and of question-posing is essential (as the starting point of an SRP with a generating question, powered enough to pose many derived questions) so that it is essential to help students develop some reflective inquiry gestures such as posing new questions along their particular study or using extra-mathematical context to validate mathematical models and make new questions emerge and thereby starting new loops in the modelling cycle. And, on the other hand, inquiry needs from modelling processes (when mathematical tools and models are build, used analysed, validate, etc.) to make the study progress.

Research questions

The aim of our research is to analyse if the implementation of the sequence we had designed, with features as the ones detailed above, could promote the inquiry students’ competences and how modelling could become an essential tool to enquire into the inquiry questions that were emerging in order to provide and to validate answers.. Therefore, our main research questions that we focus on are the following:

Can a teaching sequence based on the study of an archaeological problem in a codisciplinary context help students to face an inquiry that needs modelling processes as an instrumental tool to find, test and validate answers? Which is the relationship between modelling and inquiry in this kind of teaching sequence?

Design and implementation of the teaching sequence

As it has been introduced, the starting situation is based on the discovery of certain Roman ruins, some years ago, in the centre of Badalona (a city next to Barcelona, Catalonia) by the archaeologists' team of the Badalona's Museum. They concluded that these ruins could have been an antique public building belonging to the classical *Baetulo* (the Roman old name of Badalona), surely a theatre, and they explained their research in an article (Padrós & Moranta, 2001).

The sequence was named «*What are these ruins hiding? Investigating the Roman ruins of Baetulo*» and all the sources, devices designed, links recommended and the worksheets that they had to follow were available in the blog designed by the authors: <http://ruinesdebaetulo.blogspot.com/>

We implemented the teaching sequence with a group of a 30 students (12-13 year-old) of a Secondary school education in Badalona during two weeks of June in 2015. The selection of the participants was intentionally chosen due to the facility that the first author of the paper had to the school, and the age and syllabus adequacy of the group class. Students worked during all sessions in 'inquiry teams' of three or four members, set up in the first session. They worked as real research teams, formulating hypothesis and doing tasks to validate them, discussing the partial results obtained during the process, finding points of agreement and writing an inquiry report to gather all the ideas, proofs and work done. Each team had on member in charge of explaining and defending their temporary report. Moreover, during all the study process, students got an inquiry guide to help them to progress in such a new activity.

From the archaeological reality to the emergence of questions

The didactic sequence was inspired in a real archaeological investigation, a situation very close to a real extra-mathematical context, which was introduced to the students by the teacher and the first researcher paper signatory. The aim of the inquiry proposed to the students it is the same of the original aim research: discovering with which kind of building the ruins could be identified. In this sense, the context used to design the didactic sequence was an *authentic* situation (Vos, 2011) because the situation introduced to the students is clearly not created for educational purposes even though some elements are included for educational purposes.

Therefore the students had to investigate —from real data, archaeological reports, and canons of Roman architects— what type of building the discovered Roman ruins could have been and its features. In the first session, they could explore the map of the zone where the ruins were located by the link in the blog. They also visited later the place of the discovery accompany by one of the archaeologists of the Museum of Badalona and asked questions, took photographs, and measurements. The current constructions in this zone, houses and streets, followed a curious curved shape—easily perceptible in the map. It indicated, surely, that all these constructions were built on top of the ancient structures.

The role of the context in the students' process of enquiring

Students looked up and investigated information about the Roman architecture to find what type of buildings had a curved part of their perimeter. They found few buildings that showed this feature at least. For instance, theatres were circular, amphitheatres were elliptic or circuses had a part circular

and other part quadrangular. This fact generated some other questions that promoted the developing of the inquiry process because the students could formulate their first hypothesis about that kind of building the ruins could belong to: *Which Roman building (theatre, circus, amphitheatre, etc.) shape would concur with the shape of the part of the Roman wall found? What do the geometrical shape of the partial Roman wall discovered determine?*

In the first stage of the process, the problematic situation was introduced to the students by the History teacher and the first author. Then, first questions emerged and the students started to look up information about the public Roman buildings in the links, maps and books. At the end of this stage most of the students should have understood the context of problem. So, they could formulate their preliminary hypothesis and conjectures about which kind of public building the ruins could have been only based on the historical information found. So that, at this stage when they had only taken into account this historical information, their assumptions included more than one building (all the buildings that could have a curved part). Each inquiry team had to write in their report the agreed hypotheses they reached.

Actually, the inquiry was based on a mainly discovery: a part of a Roman curved wall, a metre and half high. This partial wall belonged to a building that was the centre of the inquiry. It was the external wall of the public building and determining its shape could mean to know which type of Roman building it was. The students had to follow their inquiry from the study of this element and the context where it was found. The number of types of Roman buildings existing limited the quantity of different types of shapes the students had to consider; but other information deduced from the context directly as the orography and the dimensions of the place where the ruins were discovered, or the dimensions of the curved Roman wall discovered, also limited the options that the students could choose in order to formulate their hypothesis about the building the ruins would fit with. In this case, the context influence on the inquiry helps students to deal with a problem that would be very difficult to resolve from an only mathematical point of view and completely unachievable to their level.

Improving the first hypothesis: Building up models to systematise and mathematize the archaeological system

The second stage started with an important session that had the objective of discovering the geometrical shape that the Roman wall described (a circumference, an ellipsis, etc.) and test their first hypothesis. From this stage the teachers of Mathematics, Technology and Catalan Literature had involved in the sequence management. The results obtained from this central task provided enough data that allowed formulate sufficient plausible hypothesis about the type of public building.

The session in which the problematic situation began to be systematised and mathematized were placed on the public square next to the school, and was recorded and afterwards analysed. The students work with an exact representation of the part of the Roman wall drawing on the ground. They could experiment with different ways to prove what was the geometrical shape that fit with the Roman wall. After few tasks in which students had to construct and drawing different curves on the ground, all the teams could check that the perimeter of the curve Roman wall fits with a circumference and so, they could conjecture about the building would have been a theatre. Also,

this evidence lead students to think that it was necessary to find the radius of the circumference in order to know how large the building was. The students find the radius of the circumference (16 m.) with graphical methods, always on the ground of the square. Then they spread out on the square, drawing with their bodies the perimeter, to notice the likely real dimensions of the Roman theatre.

After this session they returned to the classroom and tried to explain the experimentation in their reports but it emerge the necessity to get a tool to draw the theatre properly. Moreover, the teacher introduced the book by *Vitruvius* (available in the blog), classical Roman architect who wrote several canons that Roman people followed to construct each type of public building. In this stage the technology had an essential role in order to allow students drawing their geometrical model of the theatre following the *Vitruvius*' instructions. The students constructed their model of the theatre using the software *Geogebra*.

Interpreting and validating mathematical models within the archaeological context

When each team had their model of the theatre finished could export the file as an image and pasted it on the map of the area studied, fitting it properly in the exactly site where the ruins was discovered (Figure 1). This task allowed students interpret their model considering the specific context and verify if their construction and hypothesis were suitable.

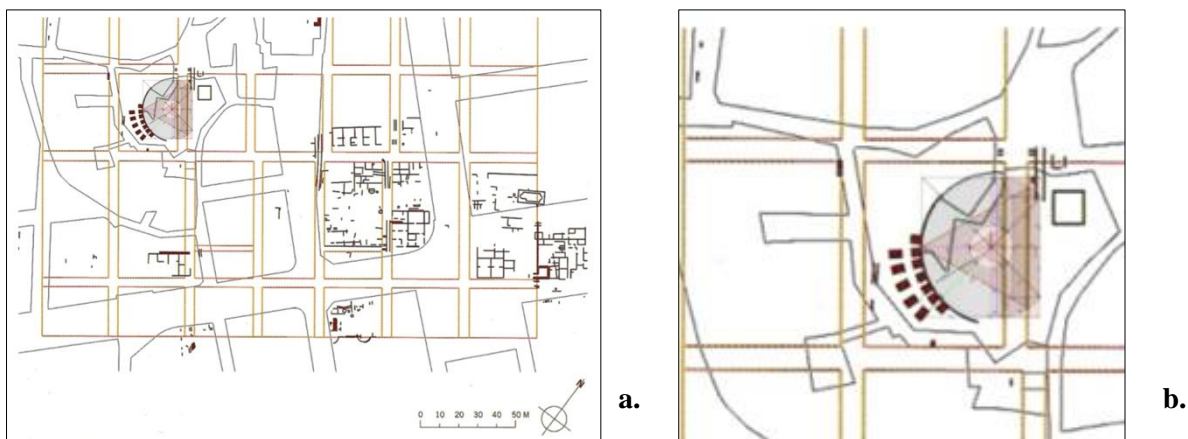


Figure 1. (a) Application the theatre model, which is construct with Geogebra, on the map studied from one of the inquiry teams report; (b) Detail enlarged

To do the tasks in this second stage the students had to deal with information from the Mathematics —data related of geometrical shape of the wall, extracted from the activity on the square—and from the History —considering the features of the context in where the ruins were discovered—. Modelling, in that moment, became an essential tool that facilitated find answers.

Then, when the students were became competent researchers yet, it started the third stage of the sequence. They arrived at that point with an important collection of questions and doubts but in the following session they had the opportunity, firstly, to share it with other groups and, thereupon, preparing an interview to ask the archaeologist of the Museum. During this interview students could contrast and validate their results about the model selected. It was another especial time you could notice that in the real world different disciplines interacts in a natural way to find solutions of real

problems. After the session outdoors talking to the archaeologist, they could apply their geometrical model construct with *Geogebra* on maps with other ruins of theatres in Europe and thus could check how the same model fits there, too. The model also was useful counting how many people would fit in the theatre, the last task of this stage. Finally, the students finished writing the final report of their inquiry describing their process, the mathematical tools used, the result of verifying their hypothesis, new opened questions, etc.

Conclusions

We would like to stress the importance that the context and the initial question studied had to deeply involved students in the inquiry activity. Moreover, the possibility to work with real data, the facilities to get access to the ruins and to other real archaeological investigation facilitated that students faced the inquiry questions from a wider approach. What the design and implementation of the activity shows is that the context easily offered different kind of information, such as the one coming from the history, from orography, from geometrical models used to measure and drew plans. It facilitated that students considered all these kind of different nature information and tried to systematise it and to work with it in order to provide answers.

On the one hand, we have shown how the enquiring process carried out in the implementation nests a sub-process of modelling that appears as a tool to contemplate information emerged from the context that could become mathematized (such as: the measure of the curved wall, of the ratio, of the building perimeter or the possible use of the Vitruvius canon) in order to be able to progress in looking for specific answers. Thus, at some stages, mathematics appears as an essential modelling tool to look into systems, build up mathematical models, simulate and test them.

More concretely, in the task done on the square next to the school, students wondering how could know the geometrical shape of the whole Roman wall from the partial wall in the map. But this is a difficult mathematical problem to resolve—to find the geometrical curve from a part of it— because there could be a lot of solutions. Besides, the methods to finds these solutions only from the mathematical point of view, are totally beyond the powers of secondary school students’.

On the other hand, the dialogue with History limits the possible answers because the Roman buildings only had three relevant shapes: ellipsis (amphitheatre), circle (circus) or semi-circle (theatre). Due to the contribution of the historical information the problem became achievable at the students’ level of mathematical knowledge and allows the beginning of modelling.

Last but not least, it was also very important the possibility that students know about the experts’ work (the real research of archaeologists) and to realise that the process they follow are quiet similar to the process of enquiring and modelling that they were following. It also had an important impact on their motivation and on the perception they had about the usefulness of Mathematics and of modelling. The interaction with the archaeologists allowed students to validate their whole process of enquiring and their results.

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