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Methodology for designing didactical activities as an engineering project of a tactile sensorial therapeutic ramp

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The objective of this investigation is designing an activity research study in mathematical modeling for the training of future engineers in the subjects of linear algebra, and structural mechanics of composite materials, in an application of stress and strain calculation. Considering elements of the Anthropological Theory of Didactics (ATD) implementing a methodology within the mentioned theory. This allows the analysis of mathematical models in use, as basis for designing didactical activities, in order to create a link between these two subjects, showing the future engineers that mathematics can be used to solve problems in an extra-mathematical context. This took place in the Research Center for Applied Science and Advance Technology of the National Polytechnic Institute (CICATA-IPN) and in the School of Science of Engineering and Technology of the Autonomous University of Baja California (ECITEC-UABC).

Keywords: Mathematical models, linear algebra, engineering training, ATD.

Introduction

The objective of this work is to design modeling activities for a mathematical training of engineers. The design is based on mathematical modeling analysis on specialty training courses, focusing on laminated composite materials. This collaborative work involves aerospace, mechanical engineers and mathematicians that teach in the carriers of Aerospace engineering and mechanical engineering, having an opportunity to analyze a real context of modeling; namely Calculation of stress and strain of composite materials. This project was proposed on a structural mechanics of composite materials course; because the students wanted to know where they could use the mathematics they were learning. For this work we have considered elements of the Anthropological Theory of Didactics proposing a methodology associated to this theory that permits the analysis of mathematical models in use based on the design of didactic activities.

Elements of the Anthropological Theory of Didactic

The ATD is an epistemological model that allows the study of human activity in its institutional dimension. An institution is a stable social organization that defines the human activities generating resources that make them possible. These materials or intellectual resources, which are made available to the subjects, have been produced by communities along the confrontation of problematic situations with the objective of solve them in a regularly and effectively way (Castela and Romo, 2011). The classic praxeological model, proposed by Chevallard (1999), recognizes the praxeology $[T, \tau, \theta, \Theta]$ as a minimal unit of analysis of human activity. Its four components are: the task type (T), the technique (τ), the technology (θ); and the theory (Θ). The ‘task’ refers to what is to be done; the ‘technique’ is how it is to be done; the ‘technology’ is a discourse that produces, justifies and explains the ‘technique’; while the ‘theory’ produces, justifies and explains the ‘technology’.

The training of engineers through institutions

The training of engineers can be seen through institutions, Romo (2009) distinguish three types: production of knowledge $P(S)$, teaching of knowledge $E(S)$ and use of knowledge or practices Ip . This distinction seeks to recognize the first vocation of the institutions and the production of knowledge correspond to scientific disciplines, such as mathematics or engineering sciences, are validated in these the existence of knowledge and the relations between them. The teaching of knowledge $E(S)$ are responsible for displaying and disseminating praxeologies, meanwhile in the institutions of usage Ip , the praxeologies are used to solve problems of practice. This does not mean that within the institutions $P(S)$ there is no teaching or usage praxeologies nor in teaching $E(S)$ and practice Ip there is no production of knowledge. In this investigation linear algebra is considered an institution of teaching mathematics $E(AL)$, and in teaching engineering, structural mechanics of composite materials $E(MC)$ and connect them through a Study and Research Activity (SRA). The SRA constitute didactic devices for the construction of a praxeology through three didactic moments that are: first encounter with T, exploration of T and the emergence of the τ technique and construction of the technological-theoretical block, in this case coming from the structural mechanics of composite materials as outlined below

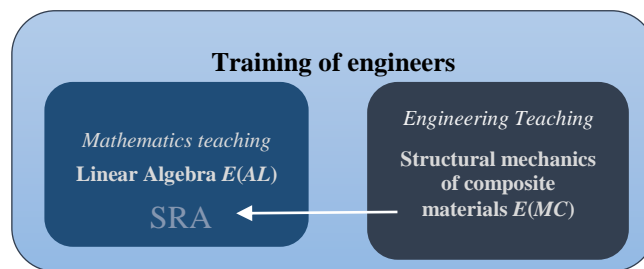


Figure 1: The SRA as an element linking educational institutions

In order to initiate the transition from the traditional paradigm in mathematics "visiting works" (to the teaching of pre-existing mathematical objects) to questioning the world according to inquiry-based mathematics education (IBE).

Didactical design methodology of an SRA

The methodology initially proposed in Macias (2012) allows the design of SRA that involves non mathematical elements. Its four stages are: 1) Election of an extra-mathematical context; 2) Praxeological analysis and identification of a mathematical model; 3) Analysis of the identified mathematical model and their relationship with $E(M)$ and 4) design of the SRA.

Election of an extra-mathematical context

To choose an appropriate extra-mathematical context for the design of an SRA, the following elements were considered.

1) Generation of surveys aimed at teachers and students about mathematical needs of engineers in training:

Students. Which of these subjects you thought more important, and why? Have you used or adapted a mathematical model? Which model and for what?

Teachers. In any of your subjects taught in an engineering career you use mathematics? Have you used a mathematical model or adapted one in any of your courses? Which model and for what?

67 students from 3rd to the 7th semester were surveyed, finding that the most useful subjects are calculus and linear algebra. 54% recognizes the work with mathematical models. All teachers who teach subjects of MC, integral calculus CI and AL were surveyed, 92% of them recognize the use of matrices in their courses

2) Interviews with the coordinator of the career of mechanical engineering, who noted that the subject of MC required early in the course of a review of linear algebra.

3) Approach with MC teachers, who pointed out that in general, students do not recall procedures or operations previous courses, AL for example.

4) Joint work with engineers-researchers in the area of materials.

Praxeological analysis and identification of a mathematical model

The praxeology that is identified and analyzed is the calculation of stress, strain and elastic modulus in laminated materials. The type of task is to calculate the stress or strain of a laminated material, the technique is associated with the use of the matrix S (strain) or matrix Q (stiffness), the technology is Hooke's law and the theory is the mechanics of materials. The analysis is based on a technical report of basic mechanics of laminated composite plates (Nettles, 1994), suggested by an engineer-researcher, who indicated it, as a heavily used reference material. In this section the theoretical technological-block is shown, which displays how the stiffness matrix associated with the technique of calculating the stress and strain is presented. This will provide the basis to show in the next section full praxeology from the analysis of an exercise presented in a class of structural mechanics of composite materials.

Technology: Generalized Hooke's law for anisotropic materials¹

Nettles explains that the relationship between stress and strain is independent of the direction of the force, and is provided by the constant of elasticity (Young's modulus), this is for isotropic² materials. In nonisotropic materials it should use two elastic constants at least. The relationship stress / strain for isotropic materials appears as follows:

$$\sigma = E\varepsilon \quad (1)$$

Where σ : is the stress, E : Denotes the Young's modulus and ε : is the strain.

For orthotropic materials³, the direction must be specified in the stress/strain relationship:

$$\sigma_1 = E_1\varepsilon_1; \sigma_2 = E_2\varepsilon_2 \quad (2)$$

where

σ_1 : Denotes the stress in the longitudinal direction

E_1 : Denotes the stiffness in the longitudinal direction (Young's modulus)

ε_1 : Denotes the strain in the longitudinal direction

σ_2 : Denotes the stress in the transversal direction

¹ Anisotropic materials: is the material that its mechanical properties differ according to the load direction

² Isotropic materials: It is the material that has identical mechanical properties in all directions regardless of the direction of the load

³ Orthotropic materials: is the material in which mechanical properties are different in three perpendicular directions

E_2 : Denotes the stiffness in the transversal direction
(Young's modulus)

$$S_{11} = \frac{1}{E_1}$$

$$S_{22} = \frac{1}{E_2}$$

ε_2 : Denotes the strain in the transversal direction

$$S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2}$$

$$S_{66} = \frac{1}{G_{12}}$$

$E_1 = E_L$ Defines the stiffness in the longitudinal direction

and $E_2 = E_T$ is the stiffness in the transversal direction. This law produces different techniques, Nettles initiated by the special orthotropic plates and is why we analyze them below..

Stress and strain for special orthotropic plates

The author begins by explaining that on a plate, stress can be given in more than one direction. Immediately he defines Poisson's ratio as the strain perpendicular to a given loading direction, showing the relationship for different loads.

For loading along the fibers:

$$\text{Poisson's ratio} = \nu_{12} = \frac{\varepsilon_T}{\varepsilon_L} = \frac{\varepsilon_2}{\varepsilon_1} \quad (3a)$$

For loading perpendicular to the fibers

$$\text{Poisson's ratio} = \nu_{21} = \frac{\varepsilon_L}{\varepsilon_T} = \frac{\varepsilon_1}{\varepsilon_2} \quad (3b)$$

The strain is equal to the difference between the stretched component deformation due to an applied force and contraction of the Poisson's effect due to other forces perpendicular to the applied force, thus:

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \nu_{21}\varepsilon_2 \quad y \quad \varepsilon_2 = \frac{\sigma_2}{E_2} - \nu_{12}\varepsilon_1 \quad (4a)$$

aplicando la ecuación (2)

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \nu_{21}\frac{\sigma_2}{E_2} \quad y \quad \varepsilon_2 = \frac{\sigma_2}{E_2} - \nu_{12}\frac{\sigma_1}{E_1} \quad (4b)$$

Subsequently, the author considers the presence of shear forces. The shear stress and shear strain are related by a constant called shear modulus, denoted by G.

$$\tau_{12} = \gamma_{12}G_{12} \quad (5)$$

Where: τ_{12} : Shear stress, γ_{12} : Shear strain y G_{12} : Shear modulus

Equation (5) is similar to equation (1) it only considers shear stress and strain, where the indices 1-2 indicate shear in the 1-2 plane. The author mentions that a relationship exists between the Poisson constant and the Young's modulus in both directions, in the longitudinal direction and the transverse direction to the material, and then it holds that:

$$\nu_{21}E_1 = \nu_{12}E_2 \quad (6)$$

Equations (4b) and (5) can be written in their matrix form obtaining

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \quad (7) \quad \text{where,}$$

Here it is where we can see a relationship between the subjects of linear algebra and structural mechanics of composite materials as a matrix model for calculating stress or strain of laminated

materials whether they are isotropic or orthotropic. Calculating the inverse stress matrix S , we obtain the stiffness matrix Q turning out to be:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (8)$$

where,

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} \qquad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} \qquad Q_{66} = G_{12}$$

We can appreciate in equation 7 to 8, the calculation of the inverse matrix was made, to switch the stress matrix to the stiffness matrix, with basic operations taught in linear algebra. The author explains, broadly, the mathematical model based on the algebraic work, having left the reader the task of verifying the connections between the different equations, until reaching the mathematical model that relates the stress, strain and Young's modulus (which are the elastic properties of the material). All this is the technological component of the praxeology. To illustrate the types of tasks that can be solved and the associated techniques to the praxeology, we analyze below a classroom exercise from the subject of structural mechanics of composite materials.

Analysis of the identified mathematical model and its relationship with $E(M)$

In the class of structural mechanics of composite materials (MC), where we analyzed and identified the praxeology of the stress calculation of a laminate material, with a task type T , calculate the modulus of elasticity of a laminated material in a particular direction, with two tasks, t_1 y t_2 :

t_1 : Calculate the modulus of elasticity of a laminated material in the X direction.

t_2 : Calculate the modulus of elasticity of a laminated material in the Y direction.

For laminate fiberglass polyester matrix that is laid up in a $[45_2/-45_2/0]_s$ stacking sequence.

τ_1 : Find the stiffness matrix of a laminate material, with the following information

$$E_1 = 40 \text{ GPa}$$

$$E_2 = 9.8 \text{ GPa}$$

$$G_{12} = 2.8 \text{ GPa}$$

$$\nu_{21} = 0.3$$

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$$

$$[Q] = \begin{bmatrix} 40.90 & 3.01 & 0 \\ 3.01 & 10.02 & 0 \\ 0 & 0 & 2.80 \end{bmatrix} \text{ GPa}$$

Determine the stiffness matrix of the lams in 45° .

Using the expressions for the calculation of the stiffness matrix, for any fiber orientation it can be written by:

$$[Q] = \begin{bmatrix} \bar{Q}_{xx} & \bar{Q}_{xy} & \bar{Q}_{xs} \\ \bar{Q}_{yx} & \bar{Q}_{yy} & \bar{Q}_{ys} \\ \bar{Q}_{sx} & \bar{Q}_{sy} & \bar{Q}_{ss} \end{bmatrix}$$

xy axes: global axes; $1,2$ axes : material axes. Thus the stiffness matrix turns to be: $[Q] =$

$$\begin{bmatrix} 17.03 & 11.43 & 7.72 \\ 11.43 & 17.03 & 7.72 \\ 7.72 & 7.72 & 7.01 \end{bmatrix} \text{ GPa}$$

For the -45° the stiffness matrix is $[Q] = \begin{bmatrix} 17.03 & 11.43 & -7.72 \\ 11.43 & 17.03 & -7.72 \\ -7.72 & -7.72 & -7.01 \end{bmatrix} GPa$

Stiffness matrix in the flat tension of the laminate material

$$[A] = \sum_i [Q]_i \cdot h_i \text{ where } h \text{ is the thickness of the material } [A] = \begin{bmatrix} 218.04 & 97.46 & 0 \\ 97.46 & 156.28 & 0 \\ 0 & 0 & 61.68 \end{bmatrix} 10^9 h \frac{N}{m}$$

Normalized stiffness matrix in the plane stress of the laminate is written as $[A^*] = \frac{[A]}{10^9 h} = \begin{bmatrix} 21.8 & 9.75 & 0 \\ 9.75 & 15.63 & 0 \\ 0 & 0 & 6.17 \end{bmatrix} GPa$

τ_1 : Applying a tensile stress in the X direction $\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{Bmatrix} N_x \\ 0 \\ 0 \end{Bmatrix}$. As mean stress acting on the

laminate material $\begin{Bmatrix} \sigma_x^0 \\ \sigma_y^0 \\ \tau_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \sigma_x \\ 0 \\ 0 \end{Bmatrix}$; The relation between the average stress and the strain of the material

is given by: $\{\sigma\} = [A^*] \cdot \{\varepsilon\}$; Calculating the strain state for the loading state. $\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} 0.06362 \\ -0.03969 \\ 0 \end{Bmatrix} 10^9 \sigma_x$

In the direction of the X axes the relation between the strain and stress is given by: $\varepsilon_x = 0.06362 (10^9) \sigma_x$, on the other hand the apparent elasticity modulus in the X direction is presented as $E_x = 1.572 GPa$

τ_2 : Applying a tensile stress in the Y direction

Analogously the mean stress acting on the laminate material is

$$\begin{Bmatrix} \sigma_x^0 \\ \sigma_y^0 \\ \tau_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \sigma_y \\ 0 \end{Bmatrix} \text{ Therefore the strain state is } \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} -0.03969 \\ 0.08874 \\ 0 \end{Bmatrix} 10^9 \sigma_y$$

Particularly in the Y axis direction the relationship between the stress and strain is: $\varepsilon_y = 0.08874 (10^9) \sigma_y$. Furthermore the apparent elasticity modulus in the X direction is given by $E_y = 11.27 GPa$. The praxeological analysis of the technical report and the classroom exercise presented here very briefly, allowed to identify the matrix as a mathematical model for calculating strain of a laminate (isotropic, anisotropic and orthotropic) material. Also, the matrix operations and calculating the inverse of the matrix are used to determine the modulus of elasticity of a composite material, specifically laminated materials.

SRA design proposed for a tactile sensorial therapeutic ramp (RTEST)

For the SRA design, we considered the construction of a product that requested the use of a laminated material and calculating the stress and strain of the material to ensure the usefulness of the product. Therefore, it was intended that the use of the mathematical model appear the way it happens in engineering projects. Determining that for students in the early college years, more than a structure was necessary to think of a product consisting of plates, which also would be useful for the

community. Thus, it came to the proposal of a tactile sensory therapeutic ramp (RTEST) with laminate material to help children from three to ten years to correct gait problems.

Moment of the first meeting with T. To design the ramp students must develop three basic tasks, t_1) design the ramp, t_2) choose the laminate and stress calculations, t_3) determine the type of material with which the ramp will stimulate the sensory part. *Moment of the exploration of T and technical emergency τ .* Students should find the technique or techniques to solve t_1 , t_2 y t_3 . For t_1 , they should investigate the types of ramps, analyze and choose one. For t_2 , they should investigate laminated materials, choose one and make the stress calculation (the technic, p. (3-5)). And for t_3 , investigate the materials that promote sensory stimulation and choose one, justifying the reasons for their choice. *Moment of construction of the technological-theoretical block.* The third moment intersects with the second because here students must build the ramp, using drawings and stress calculations previously made, as well as the preparation of the composite material. In t_2 students should know and apply Hooke's law, for the problem they are solving, they have to know how to build the stress matrix and determine the stiffness matrix. To perform these calculations students can use computer programs such as MathLab, Scilab and SolidWorks.

First implementation of the SRA

Presenting a first implementation of the evolution of the SRA of one team:

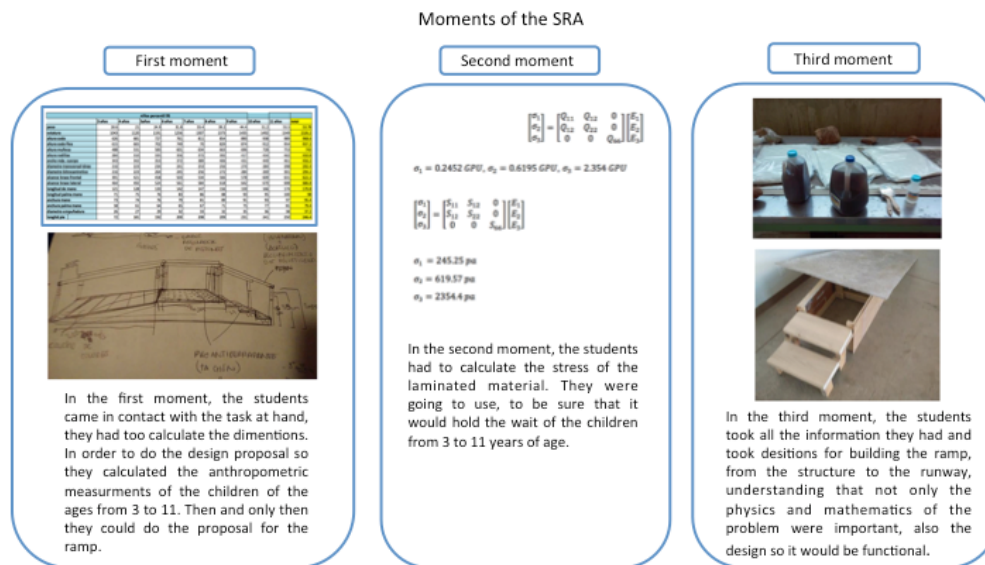


Figure 2: Momentum schematics of the SRA

The implementation with students of the core curriculum of engineering, designers and aerospace engineering; Of three different semesters (2, 6 and 7), trying to mimic the form of work of the industry: as different specialties as well as novices and experts engineers. The SRA was proposed to each teacher of the course to see if the assignment was pertinent to their subject and if they could make it part of their class and grade it. The development of the SRA was parallel to the classes of the teachers that agreed to work on the project assigning a certain time in each course for doubts they might have. A report was requested for each of the three phases. Phase 1: Proposal of a design for a RTEST ramp (3 weeks), phase 2: Strain calculation of the laminate material (3 weeks) and phase 3: Elaborate and choose materials for the RTEST ramp: Laminate materials and for the tactile and

sensorial part of the project (4 weeks). The SRA had three phases associated with the first three moments described above.

Conclusion

The SRA is proposed within the framework of the paradigm of questioning the world in the training of engineers. In this SRA unlike the commonly proposed projects of engineering the mathematical topics are highlighted. In addition the engineering topics are shown in a more important roll in the mathematics subjects. The SRA involved students and teachers from different specializations. The design of the material and the RTEST requires the calculation of stress -matrix model-, knowledge of materials and design. To do this the students must investigate and study elements from different disciplines as well as practical knowledge, students learn to do research, model, use available knowledge to create new, teamwork, communicate their ideas and justify the practice with theoretical elements of different levels. The analysis of the development of the SRA would allow us to understand the institutional necessary conditions for designing SRA in a more complex environment.

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