



HAL
open science

Finding a specific permutation of jobs for a single machine scheduling problem with deadlines

Thanh Thuy Tien Ta, Jean-Charles Billaut

► **To cite this version:**

Thanh Thuy Tien Ta, Jean-Charles Billaut. Finding a specific permutation of jobs for a single machine scheduling problem with deadlines. 19ème congrès de la Société Française de Recherche Opérationnelle et d'Aide à la Décision (ROADEF'18), Feb 2018, Lorient, France. hal-01932838

HAL Id: hal-01932838

<https://hal.science/hal-01932838v1>

Submitted on 9 Feb 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Finding a specific permutation of jobs for a single machine scheduling problem with deadlines

Thanh Thuy Tien TA¹, Jean-Charles BILLAUT¹

Université François-Rabelais de Tours, Laboratoire d'Informatique EA6300, ERL CNRS 6305,
64 Avenue Jean Portalis, 37200 Tours
jean-charles.billaut@univ-tours.fr

Mots-clés : *single machine, maximum lateness, characterization, new objective function*

1 Introduction

Many scheduling problems have a huge number of optimal solutions, such as $1||L_{max}$, $1|r_j|C_{max}$, $F2||C_{max}$, etc. A general methods to characterize the set of optimal solutions of such a problem has been presented in [1, 2, 3, 5, 6].

It is assumed that jobs are renumbered in EDD order (can be done in $O(n \log n)$ time) [4] and sequence (J_1, J_2, \dots, J_n) is the root sequence of the lattice. Based on the framework of the lattice, the objective is to find a sequence (permutation) σ as deep as possible in the lattice. A property is that all the predecessors of σ are also optimal sequences and these predecessors can be characterized very easily. The distance to the bottom sequence (reverse EDD sequence), also called 'level', is denoted by $\sum N_j$, which is to minimize.

2 Definition of function $\sum N_j$ and first results

We consider a set of n jobs J_j , $1 \leq j \leq n$. To each job is associated a processing time p_j and a due date d_j . An $O(n \log n)$ time pre-treatment allows to renumber the jobs in EDD order and to transform due dates into deadlines \tilde{d}_j , so that sequence $EDD = (J_1, J_2, \dots, J_n)$ is feasible.

Definition of $\sum N_j$

Consider a sequence σ . The level of σ is the number of pairs (J_j, J_k) such that $j < k$ and J_j precedes J_k . N_j denotes the number of jobs after J_j with an index greater than j . We have $N_j = \sum_{i=j+1}^n \sum_{h=k+1}^n x_{i,h}$, if $x_{j,k}$ is a binary variable equal to 1 if job J_j is in position k . The level of σ is equal to $\sum N_j$. This objective function has other denominations in the literature : it is the so called Kendall's tau distance and it is also the crossing number between the considered sequence with the inverse EDD sequence.

First results

- Problem $1||\sum N_j$ (without due date or deadline) is trivial. The optimal sequence is the reverse order of index, with level $\sum N_j = 0$.
- Problem $1|p_j = p, \tilde{d}_j|\sum N_j$ can be solved in $O(n \log n)$ time by an exact Backward algorithm or Forward algorithm.
- The complexity of problem $1|\tilde{d}_j|\sum N_j$ remains open.

3 Properties and resolution methods for $1|\tilde{d}_j|\sum N_j$ problem

Property : An optimal solution can always be decomposed in a succession of batches defined as follows (proof admitted) :

- the "head" of the batch is the last job of the batch,

- the jobs in each batch are in decreasing numbering order,
- the index of the heads are increasing, starting with index 1.

Exact resolution methods

Two MIP models were presented in [3], one using positional variable, the other using relative position variables.

A branch-and-bound algorithm (*BB*) with some dominance rules is proposed. The above property is one of very useful dominance rules used to cut branches. The initial solution is the best result of two heuristic methods Backward algorithm and Forward algorithm.

Heuristic and Metaheuristic methods

Two polynomial time heuristic methods are proposed :

- Backward algorithm (*BW*) builds the solution by the end, putting in last position the feasible job with the smallest index ;
- Forward algorithm (*FW*) takes the jobs in EDD order, put each job as late as possible and inserts feasible and most suited jobs with the biggest index before it.

Two metaheuristic methods : Tabu search (denoted Tabu) and Simulated Annealing (denoted SA), with some (common) neighborhoods operators, have been developed to improve the heuristic solutions. The initial solution of Tabu search and SA are the best solution of *BW*, *FW*.

Computational experiments

Two types of instances have been generated. One type of classical random instances, one type of "difficult" instances. The computational experiments have been performed for all the methods above. They show that *BB* outperforms CPLEX and that *BW* is the best heuristic method for difficult data.

4 Conclusions and perspectives

In this paper, a new category of scheduling problems has been identified. Some trivial problems are solved in polynomial time, but the general problem with arbitrary processing time and deadlines remains open. Some resolution methods are proposed and compared.

We are continuing to improve the resolution methods by introducing cuts and more dominance conditions. But the most important – to our point of view – is to investigate the complexity of the general problem $1|\tilde{d}_j|\sum N_j$, which is supposed to be NP-hard.

Références

- [1] Billaut, J-C., Lopez, P. and Takouti, L,. *A method for characterizing the set of optimal sequences for some scheduling problems*. 12th Congress of the French OR Society (ROADEF'2011), Saint-Etienne, March 2011.
- [2] Billaut, J-C. and Lopez, P. *Characterization of all ρ -approximated sequences for some scheduling problems*. IEEE International Conference on Emerging Technologies and Factory Automation, ETFA, art. No. 6059026.
- [3] Billaut, J-C., Hebrard, E. and Lopez, P. *Complete Characterization of Near-Optimal Sequences for the Two Machine Flow Shop Scheduling Problem*. Ninth International Conference on Integration of Artificial Intelligence and Operations Research Techniques in Constraint Programming (CPAIOR'2012), Nantes, June 2012.
- [4] Jackson, J-R. *Scheduling a production line to minimize maximum tardiness*. Research report 43, Management Science Research Project, Los Angeles, 1955.
- [5] Ta,T-T-T., Billaut, J-C. *Characterization of optimal solutions for some problems scheduling*. ROADEF Conference, Metz, France, February 2017.
- [6] Ta,T-T-T., Ringard,K., Billaut, J-C. *New objective functions based on jobs positions for single machine scheduling with deadlines*. 7th IESM Conference, Germany, October 2017.