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Decentralized adaptive partitioned approximation control of high degrees-of-freedom robotic manipulators considering three actuator control modes*

Hayder F. N. Al-Shuka^{1,**}, R. Song¹

Abstract Partitioned approximation control is avoided in most decentralized control algorithms; however, it is essential to design a feedforward control term for improving the tracking accuracy of the desired references. In addition, consideration of actuator dynamics is important for a robot with high-velocity movement and highly varying loads. As a result, this work is focused on decentralized adaptive partitioned approximation control for complex robotic systems using the orthogonal basis functions as strong approximators. In essence, the partitioned approximation technique is intrinsically decentralized with some modifications. Three actuator control modes are considered in this study: (i) a torque control mode in which the armature current is well controlled by a current servo amplifier and the motor torque/current constant is known, (ii) a current control mode in which the torque/current constant is unknown, and (iii) a voltage control mode with no current servo control being available. The proposed decentralized control law consists of three terms: the partitioned approximation-based feedforward term that is necessary for precise tracking, the high gain-based feedback term, and the adaptive sliding gain-based term for compensation of modeling error. The passivity property is essential to prove the stability of local stability of the individual subsystem with guaranteed global stability. Two case studies are used to prove the validity of the proposed controller: a two-link manipulator and a six-link biped robot.

Keywords Adaptive Approximation Control · Orthogonal Basis Functions · Actuator Dynamics · Electrically Driven Robots

1 Introduction

For complex robotic systems, such as humanoid robots or any robot having a number of degrees of freedom (DOFs) larger than 6-DOFs, difficulties are encountered in the implementation of the control algorithms. Therefore, over 30 years, the robotics researchers have focused on the problem of computational efficiency. Many efficient $O(n)$ algorithms have been developed for inverse [1-6] and forward dynamics [7-10] of robotic systems. For more details on the efficient dynamic algorithms, we refer to [11,12].

However, the adaptive control algorithm that deals with controlling the robotic systems despite their uncertain parameters may decrease the computational efficiency of the dynamics $O(n)$ algorithms. K. S. Fu et al. [13] have shown that the combined identification and control algorithms can be computed in $O(n^3)$ time despite using recursive Newton-Euler (NE) formulation. Therefore, most researchers have used a local controller (decentralized controller) such as PID family for controlling complex dynamic systems [14-19]. However, Liu [20] proved that decentralized PD control for robotic manipulator cannot ensure global stability. Therefore, the author proposed decentralized PD + nonlinear cubic term for ensuring global stability. A modified version of Liu's work has been proposed by Hsu and Fu. Yang et al. [21] have used a disturbance observer and an adaptive sliding mode control strategy for compensation of low-pass and high-pass coupled uncertainties respectively. In effect, most available

control strategies are based on designing PD control plus a robust and/or adaptive term(s) for compensation of modeling errors and the interconnections. Jasim and Plapper [22] used the sliding common Lyapunov function and sliding mode-based decentralized control for robotic manipulators in free and constrained spaces.

Apart from conventional robust control, the adaptive approximation control is a powerful tool to control robotic manipulator with unknown dynamics. There are two possible categories of adaptive approximation techniques: lumped approximation-based adaptive control [23-26,30,61,62] and partitioned approximation-based adaptive control [27-29,60]. The former deals with collecting the uncertainty in one term and using the linear-in-the-parameters property to represent the uncertainty in terms of weighting and basis-function matrices. Then designing control law based on updating the weighting-coefficient matrix using Lyapunov stability. This strategy requires nominal (ideal) estimation for the unknown parameters. On the other hand, partitioned approximation-based adaptive control approximates each dynamic matrix of the equation of motion of the target robot separately using the linear-in-the-parameters property. The control law is then designed based on updating the weighting-coefficient matrices of the corresponding basis-function matrices using Lyapunov stability. To our knowledge, the partitioned approximation does not require nominal estimation for the dynamic coefficients that eases the control task. However, most decentralized approximation control [20-22,31-37]

* This paper is an extended version of the conference paper [60] in which the actuator dynamics were not considered and the focus was on torque control mode only.

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have used lumped approximation term combined with either PD term or nominal values. Banagi and Polycarpou [31] used the decentralized neural network (NN) control for adaptive control of the large-scale system. The unknown dynamics and the unknown interconnections are approximated by using linear parameterized NN assuming that the interconnection term is bounded by summation of analytic functions in terms of error. The dynamics are formulated in state space model. Tan et al. [32] used decentralized NN adaptive control for robotic manipulators. It is a model-free control strategy that is based on NN control, but the authors did not consider the actuator dynamics in their proposed strategy.

Consideration of actuator dynamics is of importance for a robot with high-velocity movement and highly varying loads [27]. Fateh and Fateh [33] used decentralized fuzzy voltage control for robotic manipulators. The lumped uncertainty of interconnections at voltage level is approximated using fuzzy compensator. The authors considered the actuator dynamics using the voltage control mode. Therefore, Huang and Chen [27] investigated the effect of actuator dynamics on the performance of the actuator dynamics assuming that the motor torque/current constant is known; however, their control strategy was fully centralized. Zhu [42] proposed three motor control modes using the distributed virtual decomposition control. According to his strategy, an electric motor can be in the motor torque control mode when the armature current is well controlled by a current servo amplifier and the motor torque/current constant is known. Otherwise, an electric motor should be in the motor current control mode when only the armature current is well controlled but the torque/current constant is unknown. Finally, an electric motor must be in the motor voltage control mode when no current servo control is available. For more details on actuators dynamics, the reader is referred to [41]. In addition, for more work on decentralized approximation control techniques, see [34-38].

In view of above, the partitioned approximation control is avoided in most decentralized control algorithms. However, it can be a strong solution to control high DOF robots since it is decentralized in nature but with some modifications. In this study, two important issues are considered: i) design of decentralized control law exploiting the feature of partitioned approximation technique, and ii) consideration of three actuator control modes: motor torque mode, motor current mode, and motor voltage mode. The proposed control law consists of three terms: partitioned approximation-based feedforward term, high-gain-based feedback term, and adaptive sliding mode term for compensating for modeling errors. The second issue coincides with virtual decomposition control proposed by Zhu [42]. These control modes are applied in sequence for adaptive control purposes without considering prior information of robot dynamics parameters, friction, actuator dynamics parameters, etc. In effect, this work is an extended version of the conference paper [60] in which the actuator dynamics are not considered and the focus was on torque control mode only.

The remainder of the paper is organized as follows. Section 2 introduces preliminaries and motivation for the problem. The methodology is presented in Section 3. Section 4

described simulation results and discussions. Section 5 concludes.

2 Preliminaries and Motivation

2.1 Preliminaries

As stated previously, the function approximation technique (FAT) is an essential tool for approximating the unknown parameters of a dynamic system for adaptive control purposes. In general, there are two essential adaptive approximation techniques: lumped approximation technique (global approximation) [23-26,30,61,62], and partitioned approximation (local approximation) techniques [27-29,60]. The former approximation technique collects all uncertain parameters in one lumped vector term. The lumped vector can then be represented as linear combinations of basis functions using miscellaneous approximators (splines, orthogonal functions, intelligent techniques etc.). Its main drawback is the need for nominal (simplified) model. On the other hand, the partitioned approximation technique attempts to approximate (linearly parameterize) each dynamic matrix and vector (mass and Coriolis matrices, gravity vector, friction vector etc.) separately. In general, the procedure employed in constructing the FAT-based adaptive control includes a selection of a suitable approximator for the uncertain parameters, choosing an appropriate adaptation law for the weighting coefficient and designing the controller structure [29,30]. There are different classes of approximators such as polynomials, splines, radial basis functions, Cerebellar Model Articulation Controller (CMAC), multilayer perceptron, fuzzy approximation and wavelets. In this paper, we will focus on orthogonal functions due to their simplicity and capability of achieving minimum approximation errors [27-30].

Theorem 1 (Weierstrass Approximation Theorem). [31] Let $f(x)$ be a continuous function on $[a, b]$, then for all $\epsilon > 0$, there exists a sequence of polynomials $p_\beta(x)$ of degree $< \beta$ that converges uniformly to $f(x)$ on $[a, b]$, i.e. $\|f(x) - p_\beta(x)\| < \epsilon$ where

$$p_\beta(x) = \sum_{j=1}^{\beta} w_j \phi_j(x) \quad (1)$$

where w_j and ϕ_j are the weighting coefficients and basis functions respectively.

This theorem indicates that each continuous function on a compact interval can be approximated by polynomials with any degree. The scope of this paper focuses on orthogonal basis functions.

Definition 1. [27] For any set of orthonormal functions $\{\phi_j(x)\}$, ($\forall j = 1, 2, \dots, \beta$) on $[a, b]$, an arbitrary function $f(x)$ can be approximated using Eq. (1) under conditions of Theorem 1 with the following weighting coefficients being calculated as

$$w_j = \frac{\int_a^b f(x)\phi_j(x)dx}{\int_a^b \phi_j^2(x)} \quad (2)$$

The following important points should be noted [27, 39]:

- Equation (2) is computed using the orthogonal property $\int_a^b f(x) \phi_i \phi_j dx \begin{cases} = 0 & i \neq j \\ \neq 0 & i = j \end{cases}$
- To guarantee convergence of the approximating series, the orthogonal set should be complete.
- There are many orthogonal basis functions: Chebyshev polynomials, Legendre polynomials, Laguerre polynomials, etc.
- The Chebyshev approximation formula is very close to the minimax polynomial since the approximation error is spread smoothly over the period $[-1, 1]$. Accordingly, this orthogonal approximation polynomials will be used as strong approximators in the current study.

2.2 Motivation

To motivate the core of this paper, let us consider the following equation of motion of n -joint robotic manipulators in free space

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}_l \quad (3)$$

where $\mathbf{M} \in \mathbb{R}^{n \times n}$ is the inertia matrix of the links, $\mathbf{C} \in \mathbb{R}^{n \times n}$ is the Coriolis and centripetal matrix of the links, $\mathbf{g} \in \mathbb{R}^{n \times 1}$ is the gravity vector, $\mathbf{q} \in \mathbb{R}^{n \times 1}$ represents the displacement output joint variables after gear transmission, and $\boldsymbol{\tau}_l \in \mathbb{R}^{n \times 1}$ denotes to the output joint torques.

However, the detailed analysis of robotic manipulators dynamics considering actuator dynamics will be introduced in the next section.

The following properties assumptions are necessary for adaptive approximation control (centralized and decentralized version) [28,40].

Property 1. The inertia matrix, Coriolis and centrifugal matrix, and the gravity vectors are uniformly bounded.

The last property can be extended to include that the dynamic coefficients of the elements of each matrix/vector are uniformly bounded and satisfying theorem 1.

Property 2 (Passivity property). The matrix $\mathbf{N} = \dot{\mathbf{M}} - 2\mathbf{C}$ is a skew-symmetric matrix, i.e., $\mathbf{s}^T \mathbf{N} \mathbf{s} = 0$, if $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is defined using the Christoffel symbols.

According to the property 2, each diagonal element of \mathbf{N} , with $k, j \in n$, is equal to zero. This property is essential to prove the stability of the proposed decentralized control law.

Assumption 1. The state variables of the target robots are measurable and bounded.

Assumption 2. Each entry of dynamic matrices of $\mathbf{M}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, and $\mathbf{g}(\mathbf{q})$ satisfies conditions of Theorem 1, i.e., they can be represented as linear combinations of orthogonal basis functions.

The last assumption includes that each entry of the dynamic matrices can be bounded by analytic and continuous functions. Please for more details on properties of the

dynamic matrices, the gravity vector and the residual dynamics, the reader is referred to [28,40,41].

In view of the partitioned approximation technique introduced in [27-29], Eq. (3) can be reformulated as

$$\mathbf{W}_M^T \Phi_M \ddot{\mathbf{q}} + \mathbf{W}_C^T \Phi_C \dot{\mathbf{q}} + \mathbf{W}_g^T \Phi_g + \boldsymbol{\epsilon} = \boldsymbol{\tau}_l \quad (4)$$

where $\mathbf{W}_M \in \mathbb{R}^{n \times n}$, $\mathbf{W}_C \in \mathbb{R}^{n \times n}$ and $\mathbf{W}_g \in \mathbb{R}^{n \times n}$ are the weighting matrices, while $\Phi_M \in \mathbb{R}^{n \times n}$, $\Phi_C \in \mathbb{R}^{n \times n}$, and $\Phi_g \in \mathbb{R}^{n \times n}$ are the basis-function matrices. $\boldsymbol{\epsilon} \in \mathbb{R}^n$ is the accumulated modeling error vector.

For a 2-DOF robot, Eq. (4) can be written as

$$\begin{bmatrix} \mathbf{w}_{m11}^T \Phi_{m11} & \mathbf{w}_{m12}^T \Phi_{m12} \\ \mathbf{w}_{m21}^T \Phi_{m21} & \mathbf{w}_{m22}^T \Phi_{m22} \end{bmatrix} \ddot{\mathbf{q}} + \begin{bmatrix} \mathbf{w}_{c11}^T \Phi_{c11} & \mathbf{w}_{c12}^T \Phi_{c12} \\ \mathbf{w}_{c21}^T \Phi_{c21} & \mathbf{w}_{c22}^T \Phi_{c22} \end{bmatrix} \dot{\mathbf{q}} + \begin{bmatrix} \mathbf{w}_{g1}^T \Phi_{g1} \\ \mathbf{w}_{g2}^T \Phi_{g2} \end{bmatrix} = \boldsymbol{\tau}_l \quad (5)$$

According to Eq. (5), the followings are noted:

- The decoupled dynamics of each DOF can be represented as

$$\underbrace{\mathbf{w}_{m11}^T \Phi_{m11} \ddot{q}_1 + \mathbf{w}_{c11}^T \Phi_{c11} \dot{q}_1 + \mathbf{w}_{m12}^T \Phi_{m12} \ddot{q}_2 + \mathbf{w}_{c12}^T \Phi_{c12} \dot{q}_2 + \mathbf{w}_{g1}^T \Phi_{g1}}_{\text{coupling torque}=\tau_{c1}} = \tau_{l1}$$

$$\underbrace{\mathbf{w}_{m22}^T \Phi_{m22} \ddot{q}_2 + \mathbf{w}_{c22}^T \Phi_{c22} \dot{q}_2 + \mathbf{w}_{m21}^T \Phi_{m21} \ddot{q}_1 + \mathbf{w}_{c21}^T \Phi_{c21} \dot{q}_1 + \mathbf{w}_{g2}^T \Phi_{g2}}_{\text{coupling torque}=\tau_{c2}} = \tau_{l2} \quad (6)$$

- The whole dynamics of the target robot is intrinsically decoupled by using the partitioned approximation technique, i.e.

$$\begin{aligned} \mathbf{w}_{m12}^T \Phi_{m12} \ddot{q}_2 + \mathbf{w}_{c12}^T \Phi_{c12} \dot{q}_2 + \mathbf{w}_{g1}^T \Phi_{g1} &= \tau_{c1} \\ \mathbf{w}_{m11}^T \Phi_{m11} \ddot{q}_1 + \mathbf{w}_{c11}^T \Phi_{c11} \dot{q}_1 + \tau_{c1} &= \tau_{l1} \\ \mathbf{w}_{m21}^T \Phi_{m21} \ddot{q}_1 + \mathbf{w}_{c21}^T \Phi_{c21} \dot{q}_1 + \mathbf{w}_{g2}^T \Phi_{g2} &= \tau_{c2} \\ \mathbf{w}_{m22}^T \Phi_{m22} \ddot{q}_2 + \mathbf{w}_{c22}^T \Phi_{c22} \dot{q}_2 + \tau_{c2} &= \tau_{l2} \end{aligned} \quad (7)$$

These equations can recursively be solved; however, this technique results in a number of equations that are equal to n^2 which can be computationally tedious.

- The partitioned approximation technique has flexibility in decentralization, e.g., a 6-DOF robot can be decoupled into two 3-DOF subsystems and so on.

3 Methodology

3.1 Dynamics of contact-free motion robots

The dynamics of electrically driven n -joint robot consist of three subsystems: the link dynamics, the joint dynamics, and armature controlled DC motor dynamics (Fig. 1). The classical Euler-Lagrangian formulation of the whole system dynamics can be expressed as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{g} = \boldsymbol{\tau}_l \quad (8a)$$

$$\mathbf{I}_m \ddot{\mathbf{q}}_m + \boldsymbol{\tau}'_f = \boldsymbol{\tau}'_m - \mathbf{G}\boldsymbol{\tau}_l \quad (8b)$$

$$\mathbf{L}\dot{\mathbf{i}} + \mathbf{R}\mathbf{i} + \mathbf{B}'\dot{\mathbf{q}}_m = \mathbf{u} \quad (8c)$$

where $\mathbf{q}_m \in \mathbb{R}^n$ is the angular motor displacement before the gear transmission and equal to $\mathbf{G}^{-1}\mathbf{q}$ with $\mathbf{G} \in \mathbb{R}^{n \times n}$ referring to the diagonal gear-ratio matrix, $\mathbf{I}_m \in \mathbb{R}^{n \times n}$ is a diagonal effective inertia matrix of the actuator, $\boldsymbol{\tau}'_f$ is the joint friction torques, and $\boldsymbol{\tau}'_m$ is the motor torque vector which is equal to $\mathbf{K}_\tau \boldsymbol{\iota}$, with $\mathbf{K}_\tau \in \mathbb{R}^{n \times n}$ denoting to the torque constant, and $\boldsymbol{\iota} \in \mathbb{R}^n$ referring to the armature current, $\mathbf{L} \in \mathbb{R}^{n \times n}$ is a diagonal inductance matrix, $\mathbf{R} \in \mathbb{R}^{n \times n}$ is a diagonal resistance matrix, $\mathbf{B}' \in \mathbb{R}^{n \times n}$ is a diagonal matrix representing the EMF constant, and $\mathbf{u} \in \mathbb{R}^n$ is the input voltage control.

Substituting Eqs. (8b) into Eq. (8a) leads to the simplified full dynamics of robotic manipulators

$$\mathbf{D}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{g} + \boldsymbol{\tau}_f = \boldsymbol{\tau}_m \quad (9a)$$

$$\boldsymbol{\tau}_m = \mathbf{H}\boldsymbol{\iota} \quad (9b)$$

$$\mathbf{L}\dot{\boldsymbol{\iota}} + \mathbf{R}\boldsymbol{\iota} + \mathbf{B}'\dot{\mathbf{q}} = \mathbf{u} \quad (9c)$$

with $\mathbf{D} = \mathbf{M} + \mathbf{G}^{-2}\mathbf{I}_m$, $\boldsymbol{\tau}_f = \mathbf{G}^{-1}\boldsymbol{\tau}'_f$, $\mathbf{H} = \mathbf{G}^{-1}\mathbf{K}_\tau$, $\mathbf{B} = \mathbf{B}'\mathbf{G}^{-1}$

Remark 1. Equation (9) neglects the coupling effects between rotor and link motion [59]. This assumption has been made for electrically driven robots provided with high gear ratio. Due to the high gear ration, the motor rotor moves faster along its axis than other directions and hence its motion is a pure rotation with respect to an inertial frame [42].

Remark 2. It is well-known that the electric time constant is smaller than the mechanical time constant leading to neglecting the inductance matrix. This assumption leads to simplified robot dynamics with reduced actuator dynamics. In other words, the full dynamics of robots can be simplified to one equation, please see chapter 7 of [15] for more details.

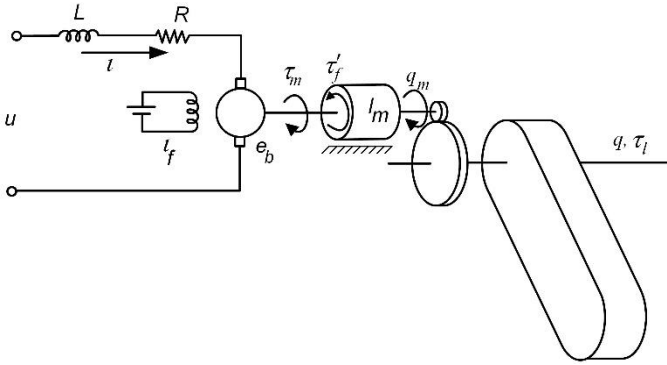


Fig. 1. Schematic diagram showing the dynamics of the k -th subsystem consisting of a link, joint, and armature controlled DC motor with I_f being denoted to the constant field current.

3.2 Decomposition approach

Based on Eq. (9), the k -th subsystem of the target robot, in which $k = 1, 2, \dots, n$, can be expressed as

$$\sum_{j=1}^n d_{kj}(q)\ddot{q}_j + \sum_{j=1}^n c_{kj}(q, \dot{q})\dot{q}_j + g_k(q) + \tau_{fk}(q, \dot{q}) = \tau_{mk}(10a)$$

$$\tau_{mk} = h_{kk}l_k \quad (10b)$$

$$L_{kk}l_k + R_{kk}l_k + B_{kk}\dot{q}_k = u_k \quad (10c)$$

where $d_{(\cdot)}$, $c_{(\cdot)}$, g_k , $\tau_{fk}(q, \dot{q})$, τ_{mk} , h_{kk} , l_k , L_{kk} , R_{kk} , B_{kk} , \dot{q}_k , and u_k are the corresponding elements of the target matrix/vector.

Eq. (10a) is a highly coupled nonlinear system, whereas Eq. (10b,c) is fully decoupled. Accordingly, Eq. (10a) should be modified to meet the conditions of decomposition. As aforementioned, the objective of decoupled control is to control every DOF individually; therefore, Eq. (10a) can be further re-written as

$$d_{kk}(q)\ddot{q}_k + c_{kk}(q, \dot{q})\dot{q}_k + \sum_{j=1, j \neq k}^n d_{kj}(q)\ddot{q}_j + \sum_{j=1, j \neq k}^n c_{kj}(q, \dot{q})\dot{q}_j + g_k(q) + \tau_{fk}(q, \dot{q}) = h_{kk}l_k \quad (11)$$

or alternatively,

$$d_{kk}(q)\ddot{q}_k + c_{kk}(q, \dot{q})\dot{q}_k + \Delta_k(q, \dot{q}) = \tau_{mk} \quad (12a)$$

with

$$\Delta_k = \sum_{j=1, j \neq k}^n d_{kj}(q)\ddot{q}_j + \sum_{j=1, j \neq k}^n c_{kj}(q, \dot{q})\dot{q}_j + g_k(q) + \tau_{fk}(q, \dot{q})$$

$$\tau_{mk} = h_{kk}l_k \quad (12b)$$

$$L_{kk}l_k + R_{kk}l_k + B_{kk}\dot{q}_k = u_k \quad (12c)$$

Equation (12) represents the standard dynamic representation that will be adopted to solve the problem of decoupled control.

Assumption 3. The interconnection/disturbance term $\Delta_k(q, \dot{q})$ is bounded with analytic and continuous functions satisfying theorem 1.

Remark 3. The interconnection/disturbance term is assumed bounded with smooth analytic functions, e.g., see [31,32]. Accordingly, this assumption is enough to decompose this term as a linear combination of basis functions without using the inverse analysis proposed in most work.

Let us recall the partitioned approximation technique described in details in [29]. Accordingly, the approximation-based dynamics can be expressed as

$$\begin{bmatrix} \mathbf{w}_{D11}^T \boldsymbol{\varphi}_{D11} & \cdots & \mathbf{w}_{D1n}^T \boldsymbol{\varphi}_{D1n} \\ \vdots & \ddots & \vdots \\ \mathbf{w}_{Dn1}^T \boldsymbol{\varphi}_{Dn1} & \cdots & \mathbf{w}_{Dnn}^T \boldsymbol{\varphi}_{Dnn} \end{bmatrix} \ddot{\mathbf{q}} + \begin{bmatrix} \mathbf{w}_{C11}^T \boldsymbol{\varphi}_{C11} & \cdots & \mathbf{w}_{C1n}^T \boldsymbol{\varphi}_{C1n} \\ \vdots & \ddots & \vdots \\ \mathbf{w}_{Cn1}^T \boldsymbol{\varphi}_{Cn1} & \cdots & \mathbf{w}_{Cnn}^T \boldsymbol{\varphi}_{Cnn} \end{bmatrix} \dot{\mathbf{q}} + \begin{bmatrix} \mathbf{w}_{\Delta 1}^T \boldsymbol{\varphi}_{\Delta 1} \\ \vdots \\ \mathbf{w}_{\Delta n}^T \boldsymbol{\varphi}_{\Delta n} \end{bmatrix} + \boldsymbol{\epsilon} = \boldsymbol{\tau} \quad (13)$$

where $\mathbf{w}_{(\cdot)} \in \mathbb{R}^\beta$, $\boldsymbol{\varphi}_{(\cdot)} \in \mathbb{R}^\beta$ represent the weighting-coefficient and basis-function vectors respectively, and $\boldsymbol{\epsilon} \in \mathbb{R}^n$ denotes to the accumulated modeling error vector.

According to Eq. (13), the partitioned approximation technique has inherently decentralization features that can be exploited for decentralized control. In view of properties 1,2, assumptions 1,2, and Eq. (13), Eq. (12a) can be reformulated as

$$\mathbf{w}_{Dk}^T \boldsymbol{\varphi}_{Dk} \ddot{q}_k + \mathbf{w}_{Ck}^T \boldsymbol{\varphi}_{Ck} \dot{q}_k + \mathbf{w}_{\Delta k}^T \boldsymbol{\varphi}_{\Delta k} + \epsilon_k = \tau_k \quad (14)$$

where

$$d_{kk}(q) = \mathbf{w}_{Dk}^T \boldsymbol{\varphi}_{Dk} + \epsilon_{Dk}$$

$$c_{kk}(q) = \mathbf{w}_{Ck}^T \boldsymbol{\varphi}_{Ck} + \epsilon_{Ck}$$

$$\Delta_k(q, \dot{q}) = \mathbf{w}_{\Delta k}^T \boldsymbol{\varphi}_{\Delta k} + \epsilon_{\Delta k}$$

with $\epsilon_{(\cdot)}$ being denoted to the corresponding modeling error. Accordingly, the dynamics of the coupled robotic system is transformed to decoupled subsystems based on linear combinations of basis function with constant weighting coefficients that should be updated using the Lyapunov's theory; the details are next.

3.3 Controller design

The objective of the proposed controller is to control the k -th subsystem individually considering the possibly accompanied uncertainties. Accordingly, let us consider the case that $d_{kk}, c_{kk}, \Delta_k, h_{kk}, L_{kk}, R_{kk}, B_{kk}$ are unknowns. In addition, consideration of actuator dynamics is important for a robot with high-velocity movement and highly varying loads. Three possible actuator control modes are possible for modeling and control of the motor [42]: (i) the torque control mode in which the armature current is well controlled by a current servo amplifier and the motor torque/current constant is known, (ii) current control mode in which only the armature current is well controlled but the torque/current constant is unknown, and (iii) voltage control mode in which no current servo control is available. The solution steps can be summarized in Fig. 2.

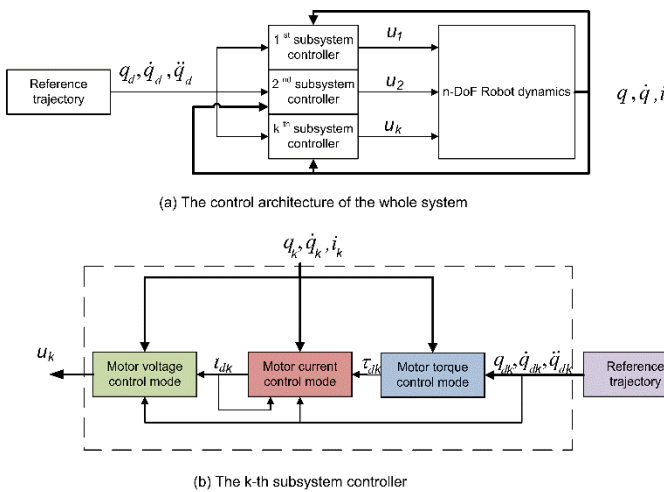


Fig. 2. (a) The control architecture of the whole system, (b) the k -th subsystem controller. There is an algebraic loop at the motor current control mode due to the dependence of the input signal on the output signal.

Assumption 2. For controller design, it is assumed that there is no servo current amplifier and the current-to-torque constant is unknown. Accordingly, the mentioned three control modes will be introduced in sequence.

3.3.1 Motor torque control mode

The aim of this mode is to generate the desired output torque with guaranteed precise tracking for the desired angular joint displacements. The control law for this mode (see Eq. (12a)) can be designed as

$$\tau_{dk} = \hat{d}_{kk}(q) \dot{v}_k + \hat{c}_{kk}(q) v_k + \hat{\Delta}_k - K_k s_k - \hat{\Gamma}_k(t) \text{sgn}(s_k) \quad (15)$$

with

$$\hat{d}_{kk} = \hat{\mathbf{w}}_{Dk}^T \boldsymbol{\varphi}_{Dk}$$

$$\hat{c}_{kk} = \hat{\mathbf{w}}_{Ck}^T \boldsymbol{\varphi}_{Ck}$$

$$\hat{\Delta}_k = \hat{\mathbf{w}}_{\Delta k}^T \boldsymbol{\varphi}_{\Delta k}$$

$$v_k = \dot{q}_{dk} - \Lambda_k e_k, s_k = \dot{q}_k - v_k = \dot{e}_k + \Lambda_k e_k, e_k = q_k - q_{dk}$$

where K_k and Λ_k are positive feedback gains, $\hat{\Gamma}_k(t)$ denotes to the adaptive sliding gain that should be updated in order to avoid the demand of modeling error bounds. Please for more details on adaptive sliding mode control see [43], and

$$\text{sgn}(s) = \frac{|s|}{s} \quad (16)$$

The mathematical relationship of Eq. (16) is very useful in the stability proof discussed later. Substituting Eq. (15) into Eq. (14) leads to the output position closed loop dynamics

$$d_{kk} \dot{s} + c_{kk} s + K_k s_k + \hat{\Gamma}_k(t) \text{sgn}(s_k) = -(\tilde{\mathbf{w}}_{Dk}^T \boldsymbol{\varphi}_{Dk} \dot{v}_k + \tilde{\mathbf{w}}_{Ck}^T \boldsymbol{\varphi}_{Ck} v_k + \tilde{\mathbf{w}}_{\Delta k}^T \boldsymbol{\varphi}_{\Delta k}) - (\tau_{kd} - \tau_k) + \epsilon_k \quad (17)$$

From Eq. (17), the closed-loop control system is stable if $\tilde{\mathbf{w}}_{(\cdot)} \rightarrow 0$, $\tau_{kd} - \tau_k \rightarrow 0$, $\epsilon_k \rightarrow 0$, and $\hat{\Gamma}_k(t) \rightarrow \delta_k \geq |\epsilon_k|$. This requires finding suitable update laws for the weighting coefficients, and the adaptive sliding gain. Let us select the following updating adaptive laws for the weighting vectors and adaptive sliding gain.

$$\hat{\mathbf{w}}_{Dk} = -\mathbf{Q}_{Dk} \boldsymbol{\varphi}_{Dk} \dot{v}_k s_k$$

$$\hat{\mathbf{w}}_{Ck} = -\mathbf{Q}_{Ck} \boldsymbol{\varphi}_{Ck} v_k s_k$$

$$\hat{\mathbf{w}}_{\Delta k} = -\mathbf{Q}_{\Delta k} \boldsymbol{\varphi}_{\Delta k} s_k$$

$$\hat{\Gamma}_k(t) = \frac{1}{\rho_k} |s_k| \quad (18)$$

where $\mathbf{Q}_{(\cdot)} \in \mathbb{R}^{\beta \times \beta}$ is a positive-definite adaptation matrix, and ρ_k is a positive adaptation gain.

Theorem 2. The k -th subsystem of the robotic manipulator ($k = 1, 2, \dots, n$) described by Eq. (12a) combined with the

desired applied torque control law of Eq. (15), and with the adaptation laws of the weighting vectors, and the adaptive sliding gain of Eq. (18), subject to

$$\tau_{dk} = \tau_k \quad (19)$$

is stable in the sense of Lyapunov stability [15,43,44].

Proof.

Consider the following Lyapunov-like function candidate (V_τ) along the trajectory of Eq. (17)

$$V_{\tau k} = \frac{1}{2}d_{kk}s_k^2 + \frac{1}{2}\tilde{\mathbf{w}}_{Dk}^T \mathbf{Q}_{Dk}^{-1} \tilde{\mathbf{w}}_{Dk} + \frac{1}{2}\tilde{\mathbf{w}}_{Ck}^T \mathbf{Q}_{Ck}^{-1} \tilde{\mathbf{w}}_{Ck} + \frac{1}{2}\tilde{\mathbf{w}}_{\Delta k}^T \mathbf{Q}_{\Delta k}^{-1} \tilde{\mathbf{w}}_{\Delta k} + \frac{1}{2}\rho_k \tilde{\Gamma}_k^2 \quad (20)$$

Taking the derivative of the last equation leads to

$$\dot{V}_{\tau k} = d_{kk}s_k\dot{s}_k + \frac{1}{2}\dot{d}_{kk}s_k^2 - \tilde{\mathbf{w}}_{Dk}^T \mathbf{Q}_{Dk}^{-1} \dot{\hat{\mathbf{w}}}_{Dk} - \tilde{\mathbf{w}}_{Ck}^T \mathbf{Q}_{Ck}^{-1} \dot{\hat{\mathbf{w}}}_{Ck} - \tilde{\mathbf{w}}_{\Delta k}^T \mathbf{Q}_{\Delta k}^{-1} \dot{\hat{\mathbf{w}}}_{\Delta k} - \rho_k \dot{\tilde{\Gamma}}_k(t) \tilde{\Gamma}_k(t) \quad (21)$$

By substituting Eq. (17) into Eq. (21), using the adaptation laws of Eq. (18), and applying the passivity property 2 lead to

$$\dot{V}_{\tau k} = -K_k s_k^2 + s_k \epsilon_k - s_k \hat{\Gamma}_k(t) \text{sgn}(s_k) - (\delta_k - \hat{\Gamma}_k(t)) |s_k| - s_k (\tau_{dk} - \tau_k) \quad (22)$$

Substituting Eq. (19) into Eq. (22) and using Eq. (16) yield

$$\dot{V}_{\tau k} = -K_k s_k^2 + s_k \epsilon_k - \delta_k |s_k| < 0 \quad (23)$$

Both s_k and $\tilde{\Gamma}_k(t)$ reach zero in finite time resulting in position error approaching to zero provided that $\delta_k \geq |\epsilon_k|$.

3.3.2 Motor current control mode

As aforementioned, the motor current control mode is necessary if the motor current can be controlled directly, and the current-to-torque constant (h_{kk}) is not available. Accordingly, the desired torque associated with the unknown h_{kk} can be expressed as

$$\tau_{dk} = \hat{h}_{kk} \iota_{dk} \quad (24)$$

The updating adaptive law for unknown \hat{h}_{kk} can be designed as

$$\dot{\hat{h}}_{kk} = Q_{hk} \iota_{dk} s_k \quad (25)$$

Theorem 3. The k -th subsystem of the robotic manipulator ($k = 1, 2, \dots, n$) described by Eqs. (12a) and (12b) combined with the desired applied torque control law of Eq. (15), the adaptation laws of the weighting vectors, the adaptive sliding gain of Eq. (18), and with updating law of current-to-torque constant of Eq. (25), subject to

$$\iota_{dk} = \iota_k \quad (26)$$

is stable in the sense of Lyapunov stability [15,43,44].

Proof.

The Lyapunov-like function candidate along the trajectory of Eq. (17) can be expressed as

$$V_{ck} = V_{\tau k} + \frac{1}{2} Q_{hk}^{-1} \tilde{h}_{kk}^2 \quad (27)$$

Taking the time derivative of Eq. (27) and considering Eq. (22) lead to

$$\dot{V}_{ck} = -K_k s_k^2 + s_k \epsilon_k - s_k \hat{\Gamma}_k(t) \text{sgn}(s_k) - (\delta_k - \hat{\Gamma}_k) |s_k| + s(\tau_{dk} - \tau_k) - Q_{hk}^{-1} \dot{\tilde{h}}_{kk} \hat{h}_{kk} \quad (28)$$

However, the fifth term can be manipulated as

$$\tau_{dk} - \tau_k = h_{kk}(\iota_{dk} - \iota_k) - \tilde{h}_{kk} \iota_{dk} \quad (29)$$

Substituting Eq. (29) into Eq. (28) and considering the simplification of the third and the fourth terms as made in Eq. (23) yield

$$\begin{aligned} \dot{V}_{ck} &= -K_k s_k^2 + s_k \epsilon_k - \delta_k |s_k| - s_k h_{kk}(\iota_{dk} - \iota_k) \\ &\quad + s_k \tilde{h}_{kk} \iota_{dk} - Q_{hk}^{-1} \dot{\tilde{h}}_{kk} \hat{h}_{kk} \\ \dot{V}_{ck} &= -K_k s_k^2 + s_k \epsilon_k - \delta_k |s_k| - s_k h_{kk}(\iota_{dk} - \iota_k) + \\ &\quad \tilde{h}_{kk}(s_k \iota_{dk} - Q_{hk}^{-1} \dot{\tilde{h}}_{kk}) \quad (30) \end{aligned}$$

Substituting Eq. (25) and Eq. (26) into Eq. (30) results in the same stability results of Eq. (23).

3.3.3 Motor voltage control mode

The motor voltage control mode is a complementary control mode necessary if a servo current amplifier is unavailable. The intuitive control law for the armature current dynamics described in Eq. (12c) can be selected as

$$u_{dk} = \hat{\mathbf{w}}_{uk}^T \boldsymbol{\varphi}_{uk} - \hat{h}_{kk} s_k - K_{vk}(\iota_k - \iota_{dk}) \quad (31)$$

where

$$L_{kk} \dot{\iota}_k + R_{kk} \iota_k + B_{kk} \dot{q}_k = \mathbf{w}_{uk}^T \boldsymbol{\varphi}_{uk} + \epsilon_{uk}$$

, \hat{h}_{kk} can be calculated from Eq. (25), and

$$\dot{\hat{\mathbf{w}}}_{uk} = -\mathbf{Q}_{uk} \boldsymbol{\varphi}_{uk} (\iota_{dk} - \iota_k) \quad (32)$$

Substituting Eq. (31) into Eq. (12c) results in the following closed-loop dynamics (neglecting the approximation error associated with (31))

$$\begin{aligned} \tilde{\mathbf{w}}_{uk}^T \boldsymbol{\varphi}_{uk} + K_{vk}(\iota_k - \iota_{dk}) + h_{kk} s_k &= L_k(\iota_{dk} - \iota_k) + \\ &\quad (u_k - u_{dk}) \quad (33) \end{aligned}$$

Theorem 4. The k -th subsystem of the robotic manipulator ($k = 1, 2, \dots, n$) described by Eqs. (12a)-(12c) combined with the desired motor torque control law of Eq. (7), the adaptation laws of the weighting vectors, the adaptive sliding gain of Eq. (10), updating law of current-to-torque constant of Eq. (25), The desired voltage control input of Eq. (31), updating adaptive law of weighting vector of Eq. (32), and subject to

$$u_{dk} = u_k \quad (34)$$

is stable in the sense of Lyapunov stability.

Proof.

To prove the validity of controller law of Eq. (31) associated with adaptation law of Eq. (32), consider the following Lyapunov-like function candidate

$$V_{uk} = V_{ck} + \frac{1}{2}L_k(l_{kd} - l_k)^2 + \frac{1}{2}\tilde{\mathbf{w}}_{uk}^T \mathbf{Q}_{uk}^{-1} \tilde{\mathbf{w}}_{uk} \quad (35)$$

By taking the time derivative of Eq. (35) and considering Eq. (30) with adaptation law of Eq. (25), we can get

$$\dot{V}_{uk} = -K_k s_k^2 + s_k \epsilon_k - \delta_k |s_k| - s_k h_{kk} (l_{dk} - l_k) + L_k (l_{dk} - l_k) (\dot{l}_{dk} - \dot{l}_k) - \tilde{\mathbf{w}}_{uk}^T \mathbf{Q}_{uk}^{-1} \dot{\tilde{\mathbf{w}}}_{uk} \quad (36)$$

Substituting Eqs. (32) and (33) into Eq. (36) results in

$$\dot{V}_{uk} = -K_k s_k^2 + s_k \epsilon_k - \delta_k |s_k| - K_{vk} (l_k - l_{dk})^2 - (l_{dk} - l_k) (u_k - u_{dk}) \quad (37)$$

In view of Theorem 4, Eq. (37) can be expressed as

$$\dot{V}_{uk} = -K_k s_k^2 - K_{vk} (l_k - l_{dk})^2 + s_k \epsilon_k - \delta_k |s_k| \quad (38)$$

The variables s_k , \dot{l}_k and $\tilde{\Gamma}_k(t)$ reach zero in finite time resulting in position error approaching zero provided that $\delta_k \geq |\epsilon_k|$.

Remark 4. The stability of the individual subsystems can guarantee the stability of the whole system.

Remark 5. The function $sgn(\cdot)$ is a hard nonlinear function that can be approximated by a smoother function such as $sat(\cdot)$ [27]. However, this can result in a modeling error due to this approximation. Therefore, in order to avoid the possible parameter drift, the robust adaptive laws can be used alternatively such as project modification, σ -modification, ϵ -modification, etc. [45].

Remark 6. If it is supposed that a sufficient number of basis functions (β) are used, then the modeling error can ideally be neglected, and hence no adaptive sliding mode term could be required [27].

4. Simulation Results and Discussions

Two case studies have been simulated using MATLAB/SIMULINK to prove the validity of the proposed controller: a 2-link manipulator and a 6-link biped robot; the details are as follows.

4.1 Two-link manipulator

A fully actuated 2-link manipulator was simulated using MATLAB/SIMULINK to investigate the performance of the proposed controller (Figs. 3-5). The parameters of the simulated manipulators are borrowed from [27]. The target manipulator moves freely (without constrained motion) with the following desired trajectories [46]: $q_{d1} = 30^0 (1 - \cos(2\pi t))$ and $q_{d2} = 45^0 (1 - \cos(2\pi t))$.

However, there are some assumptions made for simulation implementation:

- The current-to-torque constant h_{kk} is known,
- The joint drive system is directly constructed with gear ratio equal to one. As a result, there are two-stage control modes for the actuators: the motor torque control mode (Eqs. (15) and (18)) and the motor voltage control mode (Eqs. (31) and (32)).
- It is supposed that a sufficient number of orthogonal Chebyshev basis functions ($\beta = 15$) are used for approximation purposes and hence the approximation error is neglected (it is a typical case used for simplicity in some previous work, see e.g. [27, 31]).
- The friction consists of two terms: Coulomb friction, $B_c \text{sign}(\dot{\theta})$, and viscous friction, $B_v \dot{\theta}$; with $B_c = B_v = 0.5$. See [49] and the references therein for other possible friction models.
- The saturated input control is not considered and it is assumed that the motors are strong to generate the required voltages.

The feedback and the adaptation gains used are:

$$\Lambda_k = 75, \mathbf{Q}_{Dk} = 10 \mathbf{I}_{15}, \mathbf{Q}_{Ck} = 10 \mathbf{I}_{15}, \mathbf{Q}_{\Delta k} = 25 \mathbf{I}_{15}, K_k = 100, K_{vk} = 50, \mathbf{Q}_{uk} = 50 \mathbf{I}_{15},$$

Figure (6) show the position error and the input control voltage for the target robot. The robot tracks well the desired trajectory. One important point should be mentioned is that although the proposed controller does not track precisely the dynamic coefficients of the equation of motion for each subsystem, the controller tracks well the desired references. The objective of adaptive control is to follow the desired trajectory rather than convergence of unknown parameters of the system to their actual values. The system signals should be persistently excited in order to ensure good estimation for unknown parameters, see [44] for more details.

A special case has been applied to the 2-link manipulator in order to test the superiority of the proposed controller comparing with the decoupled PD control. The PD control law can be described as: $\tau_k = -K_k s_k = -K_k (\dot{e}_k + e_k)$, with $K_k = 100$. Figures 7 and 8 show the position errors and the input control torques respectively considering both the decentralized adaptive partitioned approximation control (APAC) and PD control. The feedback and adaptive gain matrices are tuned by gradually increasing their values from zero to a limit at which the instability occurs, then the matrices should be scaled down [42]. Both controllers work well with small position error; however, the APAC shows more accuracy than the PD considering the same high gain.

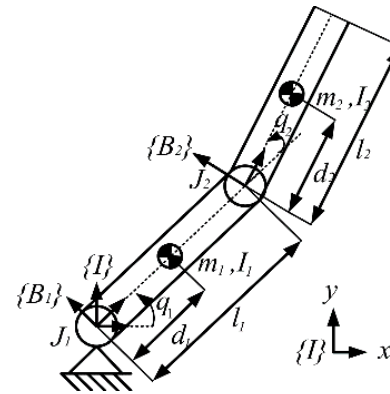


Fig. 3. The 2-link manipulator

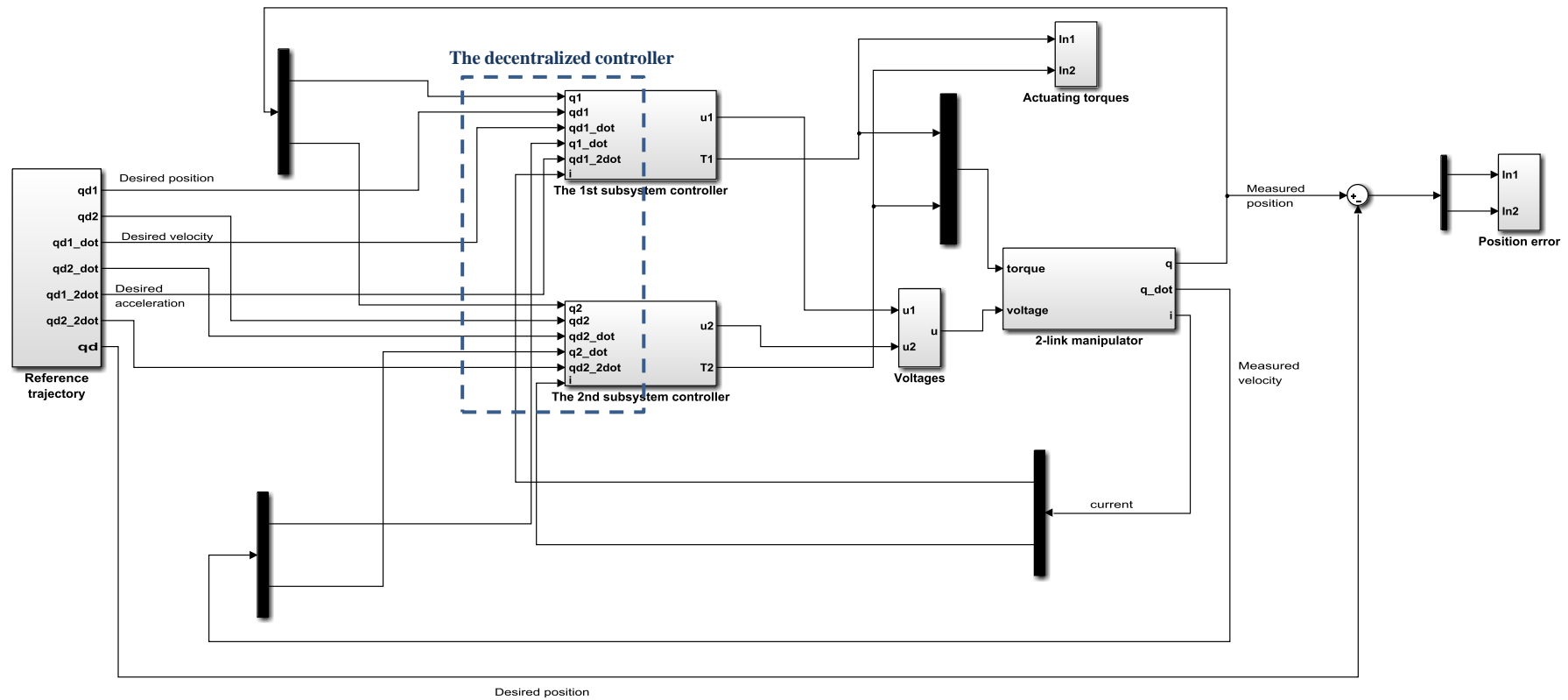


Fig. 4. A SIMULINK model describing the proposed control architecture for a two-link manipulator. As noted the control system is decomposed into two subsystems representing the degrees of freedom of the system. See Fig. 5 for more details on the block diagrams used for each subsystem controller.

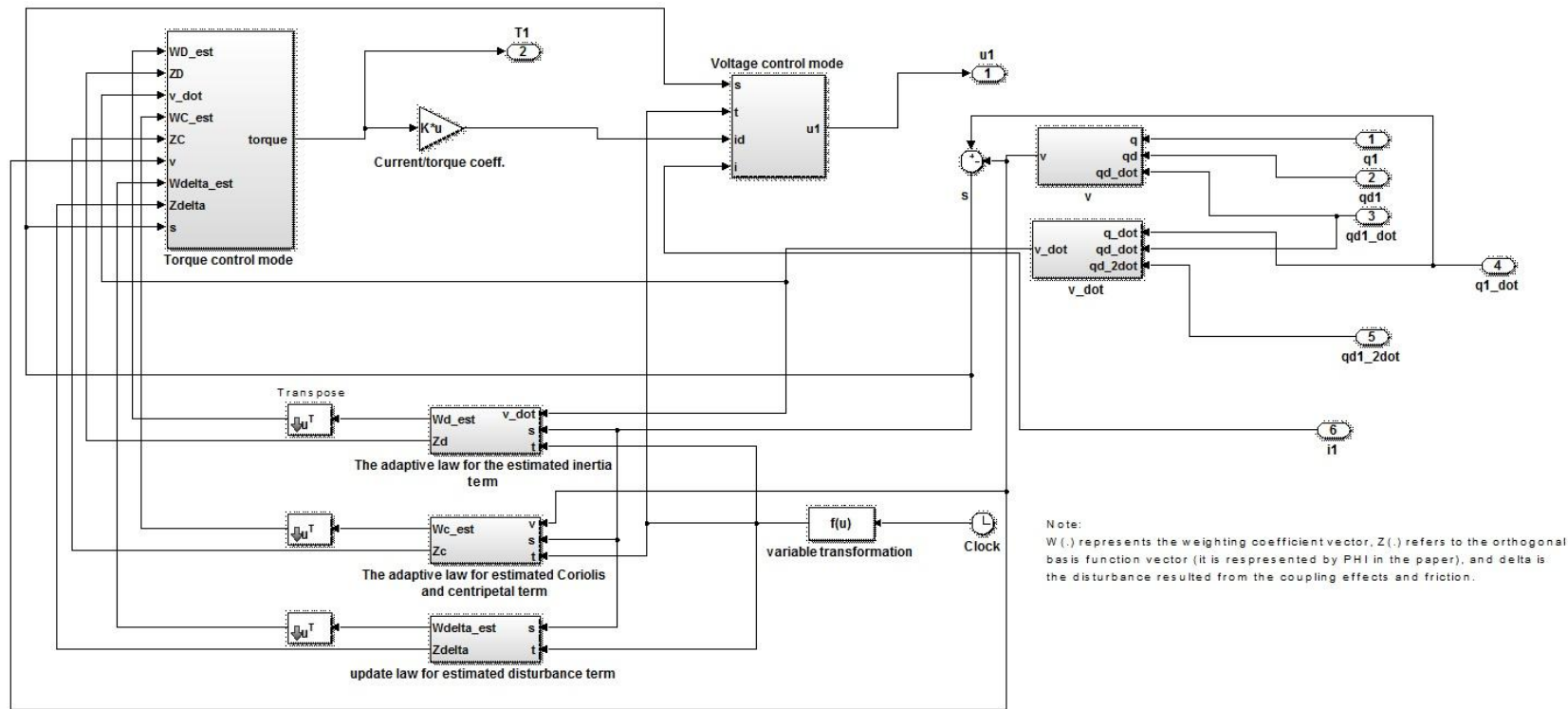


Fig. 5. A SIMULINK model describing the 1st subsystem decentralized adaptive approximation controller for joint 1. A similar control structure is used for the second subsystem.

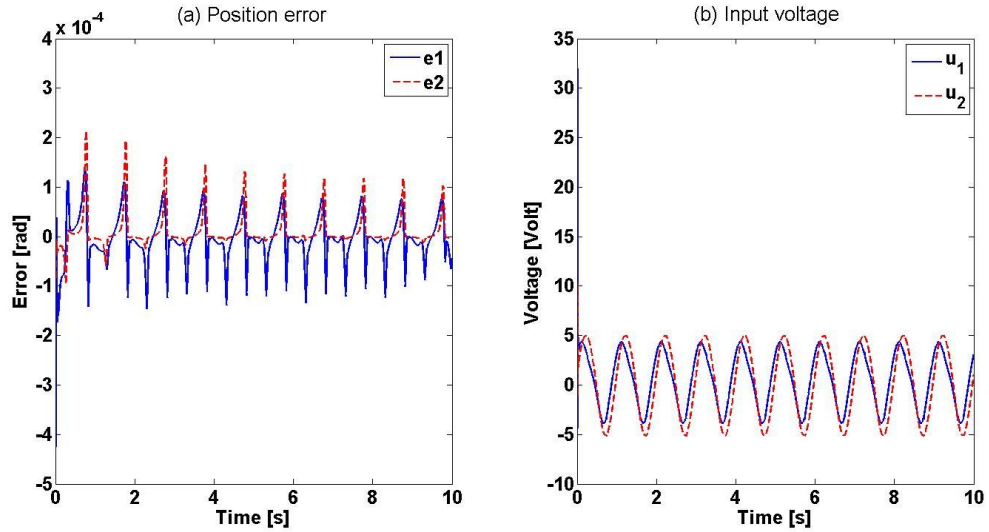


Fig. 6. Simulation results for 2-R robot

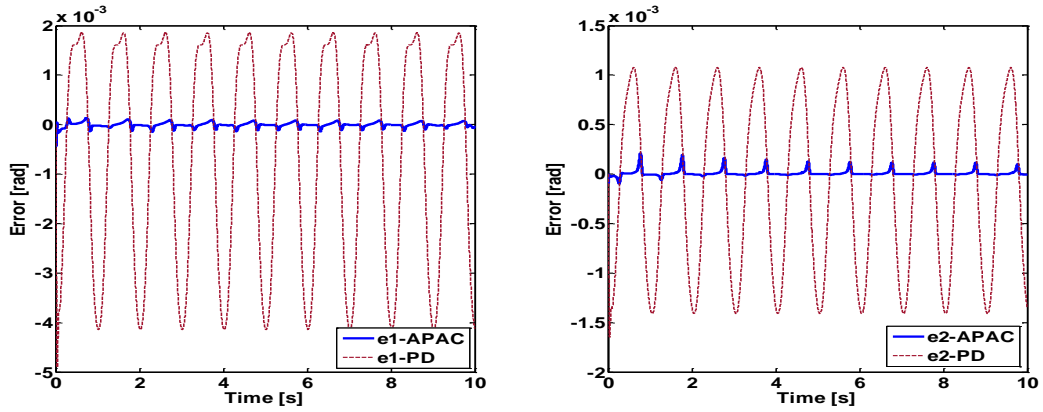


Fig. 7. The position errors for the 2-R robot using the PD and APAC controllers

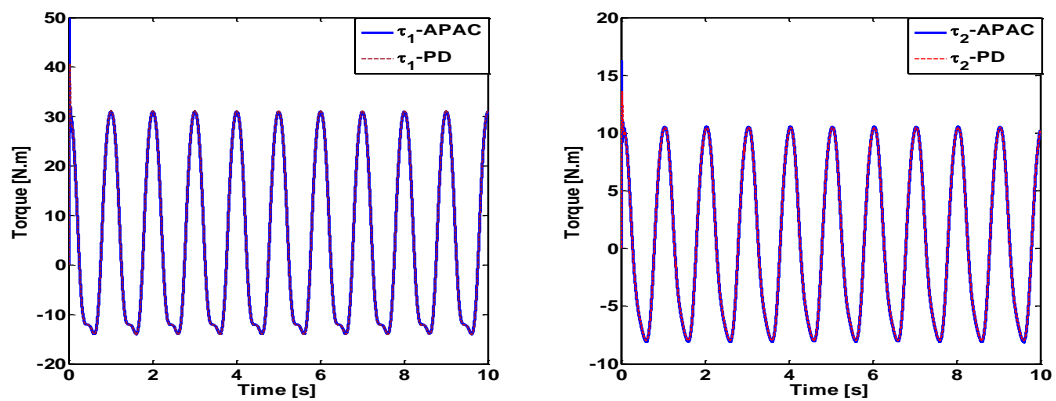


Fig. 8. The input control torques for the 2-R robot using the PD and APAC controllers

4.2 Six-link biped robot

In this subsection, a fully actuated 6-DOF biped robot is controlled by using the decentralized adaptive approximation control. There are different walking phases for biped

locomotion; however, this section concentrates on the single support phase (SSP). The biped is considered an open chain during this walking phase in which the supporting foot is in full contact with the plane ground. This means that the dynamics of the support foot are neglected. This assumption

coincides with the concept of zero-moment point (ZMP)-based balance criterion for biped locomotion*. If the dynamics of the stance foot are considered and it is allowed to rotate, the system will be underactuated and the stability of the biped mechanism cannot be governed by the ZMP and another balance criterion, e.g., Poincaré map, is used alternatively. See [48,49] for more details.

Figure 9 shows the configuration of the biped robot during the SSP; for more details on modeling and control of biped robots see [47-57]. The physical parameters are borrowed from [58]. It should be noted that this short subsection is focused on low-level control of the biped robot. Control of the biped mechanism may require multi-level control architecture: the walking pattern generator control level, the stabilization controller for compensation of the deviation of the ZMP (as an example of balance criterion), and the low-level control strategy for precise tracking of the desired references (position, torque, etc.). One of the possible strategies for modeling the biped mechanism is to consider the support foot as fixed base and hence the biped mechanism can be dealt as fixed base robots. Therefore, the dynamics and control of robotic manipulators can be applied

efficiently. This assumption coincides with ZMP-based walking.

In similar to the previous simulation study, a Matlab/Simulink model used for implementing the simulation experiments. A similar control architecture described in Figs. 4,5 and the same assumptions are used for simulation implementation. Therefore, two-stage control modes are used for motion control of the target biped: the motor torque control mode and the motor voltage control mode. The decentralized adaptive approximation control is applied with the following feedback and adaptation gains:

$$\Lambda_k = 50 \quad Q_{Dk} = 5 I_{15} \quad Q_{Ck} = 5 I_{15} \quad Q_{\Delta k} = 15 I_{15}, \quad K_k = 100, \quad K_{vk} = 50, \quad Q_{uk} = 50 I_{15},$$

Figures 10 and 11 show the position error and the voltage input control for the simulated biped robot. The biped can track the desired references very well despite the presence of the unknown parameters of the system and the high degrees of freedom.

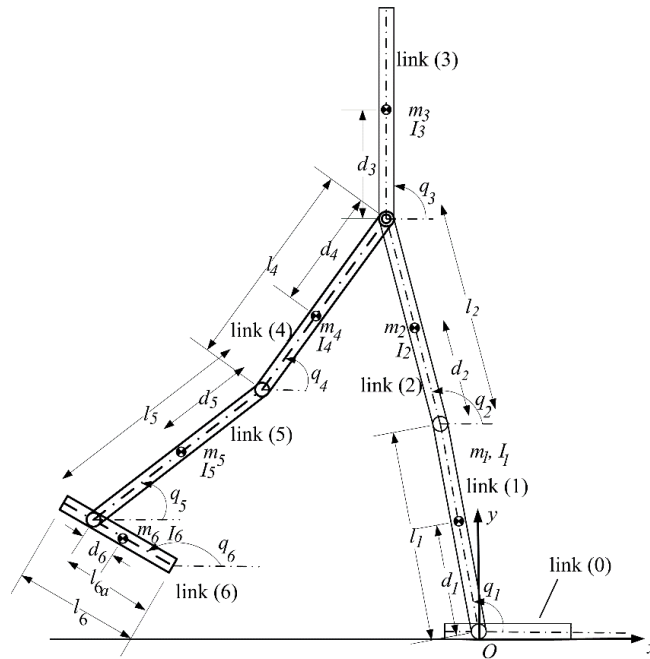


Fig. 9. The configuration of the biped robot during the single support phase

* ZMP can be defined as the ground reaction force wrench due to the foot-ground contact with zero horizontal moments. The ZMP location coincides with the location of the center of pressure for the balance walking; however, this is not the case for unbalanced walking. See [47-57] and the references therein for more details.

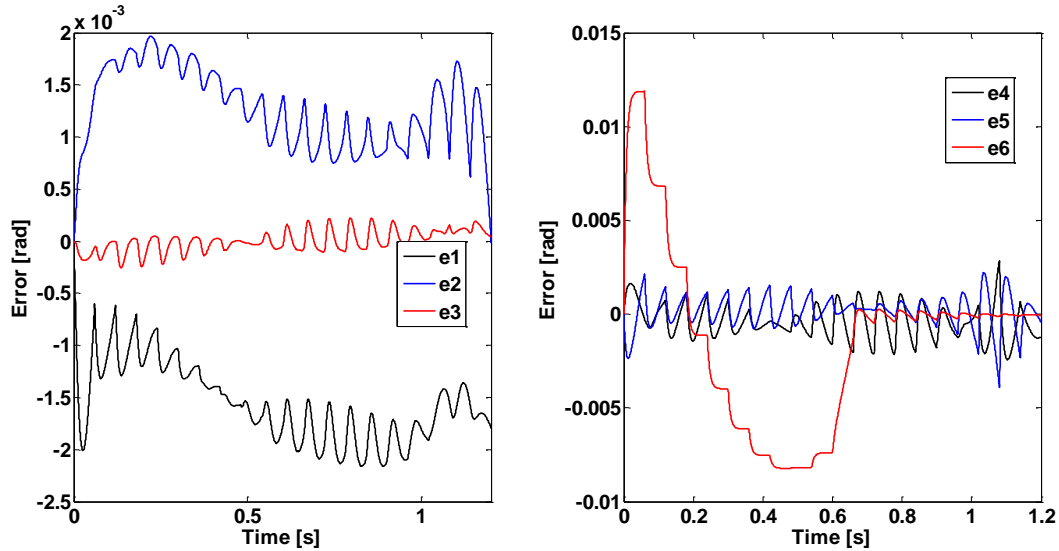


Fig. 10. The angular position errors associated with the angular joints of the biped robot depicted in Fig. 7.

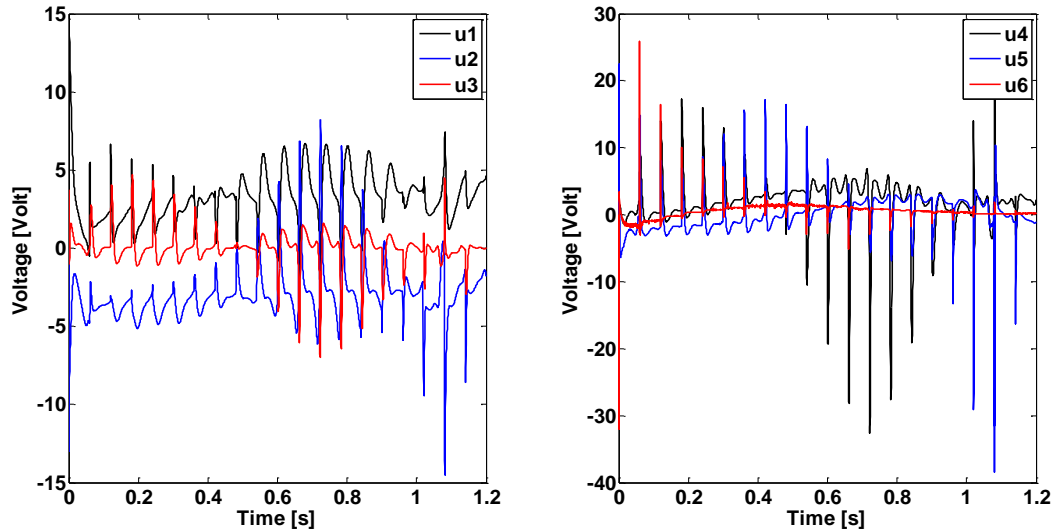


Fig. 11. The input voltage control for the biped robot during the SSP

Conclusion

This paper proposes a decentralized adaptive partitioned approximation control strategy for robotic manipulators considering actuator dynamics. Three actuator control modes are considered: torque control mode, current control mode, and voltage control mode. In the torque control mode, the armature current is well controlled by a current servo amplifier and the motor torque/current constant is known. Whereas in the current control mode only the armature current is well controlled but the torque/current constant is unknown. On the other hand, the voltage control mode is necessary when no current servo control is available. In general, the proposed control law consists of three terms: a feed-forward control term, a high gain feedback control term, and a robust sliding term. The proposed control architecture has been applied to fully actuated and fixed base robotic systems. Further work is required to consider underactuated and overactuated robotic systems and even the floating base

robots. In addition, the design of a decentralized control strategy for a robot moving in constrained space considering hybrid motion/force control or impedance control is necessary for different applications where the computational complexity inherently exists such as cooperating manipulators, robotic arm/hand systems and so on. Despite most decentralized control strategies are designed based on the Euler-Lagrange (E-L) formulation, exploiting the current strategy with the integration of the Newton-Euler (N-E) formulation is a powerful technique for dealing with modeling and control of complex robotic systems. Despite the work of [42] has been focused on this point for modeling and decoupled control of N-E based robotic systems, the proposed controller for the whole system is distributed (not fully decentralized). Future work may focus on this point using a fully decentralized control approach considering free and constrained motion with different applications.

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