

# The electric location-routing problem with heterogeneous fleet: Formulation and Benders decomposition approach

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## Abstract

In this paper, we focus on a problem that requires location of recharging stations and routing of electric vehicles in a goods distribution system. The goods are disseminated from a depot and distributed to the customers via electric vehicles with limited capacity. Differently from the classical vehicle routing problem, the vehicles have battery restrictions that need to be recharged at some stations if a trip is longer than their range. The problem reduces to finding the optimal location of the recharging stations and their number to minimize the total cost, which includes the routing cost, the recharging cost, and the fixed costs of opening stations and operating vehicles. We propose a novel mathematical formulation and an efficient Benders decomposition algorithm to solve this environmental logistics problem. Our methods solve the problem in a general setting with non-identical stations and vehicles by allowing multiple visits to the stations and partial recharging.

*Keywords:* Environmental Logistics, Electric Vehicle Routing, Recharging Station Location, Integer Programming, Benders Decomposition

## 1 Introduction

The transport sector is responsible, to a large extent, for energy consumption and greenhouse gas emissions. According to the European Environment Agency (2018), the energy consumption of road transport increases by 32% from 1990 to 2016 in the EEA-33. To tackle environmental and energy challenges, several countries are considering the prospect of carbon neutrality over the next 30 years, with the objective of discouraging the sale of vehicles emitting greenhouse gases. The implementation of such a strategy has already begun with the introduction of low-emission

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zones (LEZ), where vehicles with higher emissions either cannot enter the area or have to pay a high penalty. For instance, the traffic pollution charge in London LEZ is £100 per day for larger vans and minibuses and rises to double this amount for lorries, buses, and coaches. Vehicles with alternative fuels, such as electric vehicles and hydrogen vehicles, provide credible solutions for achieving the carbon neutrality target.

Unlike the hydrogen vehicle, which is currently at the experimental stage, and consequently having an exorbitant cost, the electric vehicle has reached an industrial maturity that makes it competitive compared to the combustion vehicle. However, as indicated by Davis and Figliozzi (2013) and Sassi and Oulamara (2017), electric vehicle (EV) is still facing weaknesses related to their availability, purchase price, and battery management. From a logistics point of view, there are still weaknesses that are worth pointing at. These include

(i) The limited choice of light duty electric vehicles offered by the car industry. These vehicles are mainly needed in the last mile logistics.

(ii) The limited electric vehicle driving range. For instance, for light duty electric vehicles, the range is between 120km and 180km. Note that the range can depend on topology of the road as well as weather and driving conditions.

(iii) The long charging time. The time to fully charge a vehicle can take up to 8 hours depending on the capacity of the battery pack and chargers level.

(iv) The lack of availability of charging infrastructures in existing road networks.

Although all these weaknesses are manageable in practice, the cost of electric vehicle presents a barrier to their extensive use. An opportunity to reduce vehicle price is focusing on the developments on those markets that are ready to adopt such a green based strategy. Such markets allow a large-scale production of electric vehicles which can consequently lead to the reduction of vehicle costs. Last miles logistic transportation provides this opportunity to speed up the market penetration of electric vehicles. In such markets, an electric vehicle has the advantage meeting the requirement of low-emission zones that are mainly located in city centers. Here, the distances covered in last mile logistic are either within its limit range or requires one charging session along the route only. Furthermore, even though the acquisition cost for electric vehicles is usually higher than the combustion engine vehicles, this difference can be offset at the operational cost of usage of electric vehicle. This is because a high utilization of electric vehicle favors their TCO (Total Cost Ownership) since their operating costs (maintenance, tax, fuel, and depreciation) are low compared to those of their counterparts.

In this paper, we consider a goods distribution system that utilizes electric vehicles. This is a system where the operating companies have access to their own recharging stations (private) or subscribing a contract to access without accessibility constraints (no queue or waiting times) to some recharging stations that have to be selected. Similar business models are considered by Yang and Sun (2015) for battery swap stations and by Schiffer and Walther (2017a,b) for recharging stations. In these types of business models, the operators need to decide on both location and routing aspects. As location and routing decisions are interdependent, they need to be handled simultaneously to operate an overall system in the most profitable way (Salhi and Rand, 1989). It may be argued that it is difficult to integrate operational decision such as routing into strategic decision like locating facilities. Though this is a critical issue, studies dealing with this dilemma showed that an intelligent way of incorporating the results of the integration can be very useful. For instance, Salhi and Nagy (1999) conduct a robustness analysis leading to a conclusion that integrated models constantly provide higher quality solutions and they are as reliable as 'locate

first - route second' methods. Very recently, Schiffer and Walther (2018) present a methodology to produce a robust location-routing solution.

In our study, we consider the vehicles to depart from a single depot. We also assume there is a sufficient number of charging stations and electrical grid capacity. This is to ensure that all vehicles are fully charged before their departure from the depot. However, we may need to recharge them during their trips if the total energy consumption to visit certain customers is larger than the battery capacity. Once a station is open, it might be visited multiple times by any vehicle. As we allow partial recharging, the vehicles do not need to be fully recharged. Besides, we do not impose any restrictions on the types of stations or vehicles. In other words, we allow the utilization of slow or fast charging stations as well as the use of heterogeneous vehicles.

The problem is to decide on the number and location of stations, the number of vehicles needed, the amount of recharging needed for each vehicle, and the route(s) for visiting all the customers. The objective is to minimize the total cost which includes the variable cost of routing and recharging as well as the fixed costs of opening stations and operating vehicles.

In this study, we develop exact methods by formulating the problem as a constrained multi-commodity flow problem. There are several applications in the literature where multi-commodity flow based formulations with capacity constraints are successfully solved using a Benders decomposition approach. These include the hub location-routing problem studied by de Camargo et al. (2013) and network design problems by Fortz and Poss (2009), Botton et al. (2013), and Calik et al. (2017). See also other relevant Benders decomposition applications for location of EV recharging stations in car sharing systems (Çalik and Fortz, 2019), under probabilistic travel range (Lee and Han, 2017), and with plug-in hybrid EVs (Arslan and Karışan, 2016); in the survey by Costa (2005) for fixed-charge network design problems; and in the book by Birge and Louveaux (2011) for stochastic programming problems. This motivates us to apply a Benders decomposition algorithm leading to very successful results for solving the small size instances which are shown to be challenging by the preceding study of Schiffer and Walther (2017b).

Our contributions are twofold:

- to propose a new mixed integer programming formulation for this strategic electric location-routing problem and
- to develop a Benders decomposition algorithm to solve the problem to proven optimality.

The rest of the paper is organized as follows: Section 2 gives an informative review on the related works. In Section 3, we provide the notation used throughout the paper and present our mathematical formulation. In Section 4, we propose our Benders decomposition algorithm followed by Sections 5 and 6 describing the implementation and the intermediate processing, respectively. In Section 7, we provide the setting and present the results of our computational study. We conclude in Section 8 with a summary of our findings and a highlight of some future research directions.

## 2 Related work

Location of recharging stations can be seen as a facility location problem. The purpose is then to decide on the optimal number and location of facilities given the position of customers to serve. In this vein, He et al. (2016) present a case study in Beijing, China. Their objectives are to incorporate the local constraints of supply and demand on public electric vehicle charging stations into facility location models, and to compare the optimal locations from three different location models: the set covering model, the maximal covering location model, and the p-median model.

Liu and Wang (2017) address the optimal location of multiple types of charging facilities, including dynamic wireless charging facilities and different levels of plug-in charging stations. Their tri-level programming first treats the model as a black-box optimization, which is then solved by an efficient surface response approximation model based solution algorithm. He et al. (2019) investigate strategies for locating fast charging stations to maximize long-distance trip completions in the United States. The mixed integer programming based analysis conducted on the nation’s long distance data provides interesting policy implications.

However, as raised in Salhi and Rand (1989), facility location and routing decisions are interdependent and should be tackled simultaneously. In the general case where both vehicles and depots are capacitated, the problem is known as the capacitated location routing problem (CLRP). The aim here is to i) define which depots must be opened, ii) assign each serviced node (customer) to one and only one depot and, iii) route the vehicle to serve the nodes, in such a way that the sum of the depot cost and the total routing cost is minimized. Many papers appeared in the subject and more particularly during the last decade, as shown in surveys by Nagy and Salhi (2007); Prodhon and Prins (2014), and Schneider and Drexl (2017). To solve this NP-hard problem, exact methods such as branch-and-cut algorithm (Belenguer et al., 2011) and set partitioning based exact methods (Akca et al., 2009; Contardo et al., 2013), are limited to medium-scale instances. To solve larger instances, new efficient metaheuristics have been proposed. These include a cooperative Lagrangean relaxation-granular tabu search heuristic by Prins et al. (2007), an adaptive large-neighborhood search (ALNS) by Hemmelmayr et al. (2012), and a three-phase matheuristic by Contardo et al. (2014). Other studies cover a multiple ant colony optimization algorithm (Ting and Chen, 2013) and a two-phase hybrid heuristic (Escobar et al., 2013). Very recently, a tree-based search algorithm by Schneider and Löffler (2017) and a Genetic Algorithm by Lopes et al. (2016) are proposed.

Despite the interest of LRP, integrating issues from electric vehicles is scarce and worth exploring. One reason may come from the fact that in LRP, a route should end at the same depot as its departure. However, this assumption is no longer valid for recharging stations as these now act as intermediate stops in the route. Similar models that use such satellite depots can be found in the Truck and Trailer problem (Villegas et al., 2013; Parragh and Cordeau, 2017). In this particular problem, when a vehicle leaves a satellite, it should go back to the same place before continuing its route, leading to a long trip including sub-tours. A closer model is the vehicle routing problem (VRP) with intermediate depots as described by Schneider et al. (2015) but the location aspect is not considered as part of the decisions.

The integration of the location of recharging stations with the routing decision, also called electric location-routing problem (ELRP), is relatively recent though it can lead to a massive environmental benefit. To the best of our knowledge, the first study of simultaneous vehicle routing and charging station siting for commercial electric vehicles is presented in a conference paper in 2012 by Worley et al. (2012). Then, Yang and Sun (2015) introduce the interesting battery swap station location-routing problem, where the charge is completely fulfilled at each stop. The authors develop two heuristic approaches. The problem is revisited by Hof et al. (2017) who adapt an interesting and powerful adaptive variable neighborhood search (AVNS) heuristic originally dedicated to the VRP with intermediate depots. Recently, Zhang et al. (2019) introduce a battery swap station location-routing problem with stochastic demand and solve this problem by developing a hybrid algorithm combining binary particle swarm optimization and variable neighborhood search.

The first paper dealing with partial recharge may come from Felipe et al. (2014), and is dedicated to a Green Vehicle Routing Problem (G-VRP). In G-VRP the fleet is composed of Alternative Fuel

Vehicles (AFV) where, in addition to the routing of each electric vehicle, the amount of energy recharged and the technology used must also be determined. However, the location aspect is not considered. Constructive and improving heuristics are embedded in a Simulated Annealing framework. The partial recharging policies are then reused showing that they may considerably improve the routing decisions as noted by Keskin and Çatay (2016). Thus, Schiffer and Walther (2017b) extend the problem by including siting charging stations which leads to the electric location routing problem with time windows and partial recharging (ELRP-TWPR). The authors focus on a problem with a single type of vehicle and multiple visits to the stations. They propose a mathematical formulation based on Miller-Tucker-Zemlin type constraints, supported by several preprocessing steps to eliminate the arcs that violate time windows, capacity, and battery restriction constraints. Later, the authors extend their work to consider a robust location-routing problem with strategic planning of electric logistics (Schiffer and Walther, 2018). The Location Routing Problem with Intraroute Facilities which is a generalization of the ELRP-TWPR is explored by Schiffer and Walther (2017a) where large instances are solved using an ALNS which is enhanced by local search and dynamic programming components.

Our problem can be considered as an electric vehicle routing problem with location decisions or an electric location-routing problem (ELRP) with a heterogeneous fleet, multi-type stations, multi-visit, and partial recharging. In the next section we provide the notation and a mathematical formulation of the problem.

### 3 Notation and Problem Formulation (PF)

Consider a network  $G = (N, A)$  with arc set  $A$  and node set  $N = I \cup J \cup \{0\}$  where  $I$  is the set of customer locations,  $J$  is the set of potential locations for charging stations, and '0' is a depot node. We are required to select a subset of  $J$  to locate recharging stations. Each customer should be served by a vehicle originating from the depot and each vehicle can perform a single trip. The vehicles have a battery restriction and they have to visit charging stations before the battery is depleted if a trip longer than their range is to be traversed. In addition, we consider the vehicles to have restricted capacities.

Below we list the following parameters:

- $K$ : the set of vehicles
- $d_i > 0$ : the demand of client  $i \in I$
- $c_{ij}$ : the routing cost of traversing arc  $(i, j) \in A$
- $e_{ij}$ : the energy consumption on arc  $(i, j) \in A$  expressed in kWh
- $f_j$ : the fixed cost of opening a charging station at node  $j \in J$
- $r_k$ : the unit cost of recharging for vehicle  $k \in K$
- $v_k$ : the fixed cost of operating vehicle  $k \in K$
- $Q^k$ : the load capacity of vehicle  $k \in K$
- $\beta^k$ : the battery capacity of vehicle  $k \in K$  expressed in kWh.

We further define the following decision variables:

- $y_j = 1$  if station  $j \in J$  is open, 0 otherwise
- $x_{ij}^k = 1$  if arc  $(i, j)$  is traversed by vehicle  $k \in K$ , 0 otherwise

$z_j^k$  is the amount of energy recharged at station  $j \in J$  for vehicle  $k \in K$

$b_{ij}^k$  is the battery level of vehicle  $k \in K$  at node  $i \in N$  before leaving for node  $j \in N$  expressed in kWh

$l_{ij}^k$  is the load of vehicle  $k \in K$  at node  $i \in N$  before leaving for node  $j \in N$ .

In the remaining of this paper, we assume  $I \subset J$  but all the methods can be easily adapted to the case where  $I \setminus J \neq \emptyset$  by simply defining  $y_j$  and  $z_j^k$  variables for all  $j \in I \cup J, k \in K$  and setting  $y_j = z_j^k = 0, \forall j \in I \setminus J$ .

### 3.1 Catering for multiple visits

In order to allow multiple visits to a station, we perform the following modification on our input network:

Step 1: Let  $|I|$  be the number of demand nodes.

Step 2: Create  $|I|$  copies of each station.

Step 3: Form set  $J_j^A = \{j_1, j_2, \dots, j_{|I|}\}$  for each  $j \in J$  and  $J^A = \bigcup_{j \in J} J_j^A$ .

Step 4: For each  $j \in J$ , set  $f_{j_1} = f_j; d_{j_1} = d_j$  and  $f_{j_i} = d_{j_i} = 0, i = 2, \dots, |I|$  where  $j_1, \dots, j_{|I|} \in J_j^A$ .

Step 5: Let  $N^E = J^A \cup \{0\}$  and  $A^E = A \cup \{(i, j) : i, j \in N^E; i \neq j; \neg(i, j \in J_l^A \text{ for some } l \in J)\}$ .

Step 6: Define  $A^k = \{(i, j) \in A^E : e_{ij} \leq \beta^k; d_i + d_j \leq Q^k\}$  for  $k \in K$ .

### 3.2 Mathematical formulation PF

$$\min \sum_{k \in K} \sum_{(i,j) \in A^k} c_{ij} x_{ij}^k + \sum_{k \in K} \sum_{j \in J^A} r_k z_j^k + \sum_{j \in J^A} f_j y_j + \sum_{k \in K} \sum_{(0,i) \in A^k} v_k x_{0i}^k \quad (1)$$

$$\text{s.t. } y_i \leq y_j, \quad i \in J_j^A : i \neq j \quad (2)$$

$$\sum_{i \in J^A} x_{0i}^k \leq 1, \quad k \in K \quad (3)$$

$$\sum_{k \in K} \sum_{(j,i) \in A^k} x_{ji}^k = 1, \quad i \in J^A : d_i > 0 \quad (4)$$

$$\sum_{(j,i) \in A^k} x_{ji}^k \leq y_i, \quad i \in J^A : d_i = 0, k \in K \quad (5)$$

$$\sum_{(i,j) \in A^k} x_{ij}^k - \sum_{(j,i) \in A^k} x_{ji}^k = 0, \quad i \in J^A, k \in K \quad (6)$$

$$\sum_{(i,j) \in A^k} (l_{ij}^k - d_i x_{ij}^k) = \sum_{(j,i) \in A^k} l_{ji}^k, \quad i \in J^A, \forall k \in K \quad (7)$$

$$\sum_{j \in J^A} l_{0j}^k = 0, \quad k \in K \quad (8)$$

$$l_{ij}^k \leq Q^k x_{ij}^k, \quad k \in K, (i, j) \in A^k \quad (9)$$

$$\sum_{(i,j) \in A^k} e_{ij} x_{ij}^k - \sum_{j \in J^A} z_j^k \leq \beta^k, \quad k \in K \quad (10)$$

$$\sum_{(i,j) \in A^k} b_{ij}^k = \sum_{(j,i) \in A^k} (b_{ji}^k - e_{ji} x_{ji}^k) + z_i^k, \quad i \in I \cup J^A, k \in K \quad (11)$$

$$z_j^k \leq \beta^k y_j, \quad j \in J^A, k \in K \quad (12)$$

$$z_j^k \leq \beta^k \sum_{(i,j) \in A^k} x_{ij}^k, \quad j \in J^A, k \in K \quad (13)$$

$$b_{0j}^k = \beta^k x_{0j}^k, \quad k \in K, (0, j) \in A^k \quad (14)$$

$$b_{ij}^k \leq \beta^k x_{ij}^k, \quad k \in K, (i, j) \in A^k \quad (15)$$

$$b_{ij}^k \geq e_{ij} x_{ij}^k, \quad k \in K, (i, j) \in A^k \quad (16)$$

$$y_j \in \{0, 1\}, \quad j \in J^A \quad (17)$$

$$x_{ij}^k \in \{0, 1\}, \quad k \in K, (i, j) \in A^k \quad (18)$$

$$l_{ij}^k \geq 0, \quad k \in K, (i, j) \in A^k \quad (19)$$

$$b_{ij}^k \geq 0, \quad k \in K, (i, j) \in A^k \quad (20)$$

$$z_j^k \geq 0, \quad j \in J^A, k \in K. \quad (21)$$

The objective function (1) minimizes the total sum of routing costs, charging costs, fixed costs of opening stations, and fixed cost of using vehicles. If a zero-demand copy of a station is opened, Constraints (2) force the original copy of this node to be opened and therefore, ensure that the costs of the stations are counted in the objective function. By Constraints (3), we restrict the number of trips by each vehicle to at most one. Constraints (3)-(6) together ensure that each client is served by a unique vehicle trip that starts at the depot and the capacities of vehicles are respected. Constraints (5) ensure that a zero-demand copy of any station is visited only if that station is open. We ensure the elimination of sub-tours for each vehicle trip via the load balance constraints (7)-(9). Battery restriction on the vehicles are imposed by Constraints (10) and (11). Constraints (12) and (13) avoid recharging of a vehicle at a node that has no station and that is not visited by that vehicle, respectively.

We initialize the battery level for each vehicle to 100% by Constraints (14). For each arc-vehicle pair, Constraints (15) restrict the amount of battery level with full battery level if the arc is traversed by the vehicle and set it to zero otherwise; Constraints (16) make sure that the battery level is larger than the energy consumption on the arc that will be traversed by the vehicle. Finally, Constraints (17)-(21) represent the binary and non-negativity restrictions on the decision variables.

## 4 Benders Decomposition Algorithm (BDA)

Our mathematical formulation can be solved by using a Benders decomposition (Benders, 1962) framework that we briefly describe here before presenting the details of our algorithm. The classical Benders decomposition method aims to solve a mixed integer program (MIP) with a group of integer variables and a group of continuous variables by decomposing the MIP into a master problem (MP) with all integer variables and a series of subproblems of continuous variables. For each feasible solution of MP, a subproblem (SP) is constructed by fixing the values of all the integer variables in the MIP to the value obtained from the master problem. Each extreme ray and extreme point of the dual of this SP provides a so called feasibility and an optimality cut, respectively, for the MP. Since enumeration of the extreme points and extreme rays is impractical, the cutting plane

procedures are usually employed for the generation and the addition of these cuts.

The classical Benders decomposition method might suffer from slow convergence especially if the subproblem is large in size. On the other hand, the method might perform relatively efficiently if the subproblem can be decomposed further into smaller and easy-to-solve problems as in multi-commodity, multi-period, or multi-scenario problems (Birge and Louveaux, 2011). Motivated by this fact, we aim to further decompose our subproblem into  $|K|$  smaller problems, each one corresponding to a single vehicle trip. For this purpose, we decide to keep  $\mathbf{y}$ ,  $\mathbf{x}$ ,  $\mathbf{l}$  variables in the master problem and  $\mathbf{z}$ ,  $\mathbf{b}$  variables in the subproblems. To speed up our implementation, we remove the optimality cuts since they slow down the convergence. In order to achieve this, we introduce an additional non-negative decision variable  $w^k, \forall k \in K$  and make a slight modification to our model to ensure that  $w^k$  takes value  $\sum_{j \in J^A} z_j^k, \forall k \in K$ . The modified formulation (PF2), as given below, is defined by Constraints (2)-(21) and (22)-(24):

$$(PF2) \quad \min \sum_{k \in K} \sum_{i \in N^E} \sum_{j \in N^E: i \neq j} c_{ij} x_{ij}^k + \sum_{k \in K} r_k w^k + \sum_{j \in J^A} f_j y_j + \sum_{k \in K} \sum_{i \in J^A} v_k x_{0i}^k \quad (22)$$

$$\text{s.t. } w^k = \sum_{j \in J^A} z_j^k, \quad \forall k \in K \quad (23)$$

$$w^k \geq 0, \quad \forall k \in K \quad (24)$$

$$(2) - (21).$$

When solving PF2 in a Benders fashion, we employ a branch-and-cut framework which keeps  $\mathbf{y}$ ,  $\mathbf{x}$ ,  $\mathbf{l}$ ,  $\mathbf{w}$  variables in the master problem (MP) and  $\mathbf{z}$ ,  $\mathbf{b}$  variables in the subproblems.

$$(MP) \quad \min (22) \\ \text{s.t. } (2) - (9), (17) - (19), (24) \\ w^k \geq \sum_{i \in N^E} \sum_{j \in N^E: j \neq i} e_{ij} x_{ij}^k - \beta^k, \quad \forall k \in K. \quad (25)$$

Let  $(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$  be the vector of variable values in the solution obtained from the master problem. One can easily observe that if  $\bar{w}^k = 0$ , then, no recharging is needed for the corresponding vehicle trip and  $(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$  is feasible for PF2. On the other hand, if  $\bar{w}^k > 0$ , we construct and solve the dual of the subproblem  $SP_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$  for every  $k \in K$ .

$$SP_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}}) \quad \min 0 \quad (26)$$

$$\text{s.t. } \sum_{j \in J^A} z_j^k = \bar{w}^k, \quad (27)$$

$$z_j^k \leq \beta^k \bar{y}_j, \quad \forall j \in J^A \quad (28)$$

$$z_j^k \leq \beta^k \sum_{i \in N^E: i \neq j} \bar{x}_{ij}^k, \quad \forall j \in J^A \quad (29)$$

$$\sum_{j \in N^E: j \neq i} b_{ij}^k = \sum_{j \in N^E: j \neq i} (b_{ji}^k - e_{ji} \bar{x}_{ji}^k) + z_i^k, \quad \forall i \in J^A \quad (30)$$

$$b_{0j}^k = \beta^k \bar{x}_{0j}^k, \quad (31)$$



$$b_{j0}^k = e_{j0}\bar{x}_{j0}^k, \quad (32)$$

$$b_{ij}^k \leq \beta^k \bar{x}_{ij}^k, \quad i, j \in N^E : i \neq j \quad (33)$$

$$b_{ij}^k \geq e_{ij}\bar{x}_{ij}^k, \quad i, j \in N^E : i \neq j \quad (34)$$

$$b_{ij}^k \geq 0, \quad i, j \in N^E : i \neq j \quad (35)$$

$$z_j^k \geq 0, \quad \forall j \in J \quad (36)$$

Note that Constraints (10) of PF is ensured by Constraints (25) and (27) as  $\sum_{j \in J^A} z_j^k = \bar{w}^k \geq \sum_{i \in N^E} \sum_{j \in N^E: j \neq i} e_{ij}x_{ij}^k - \beta^k, \forall k \in K$ . Moreover, we can replace equality (27) with inequality  $\sum_{j \in J^A} z_j^k \geq \bar{w}^k$  due to Lemma 4.1.

**Lemma 4.1.** *If (28)-(36) is non-empty and  $\bar{w}^k > 0$ , then  $\sum_{j \in J^A} z_j^k = \sum_{i \in N^E} \sum_{j \in N^E: j \neq i} e_{ij}\bar{x}_{ij}^k - \beta^k$ .*

*Proof.*  $z_i^k = \sum_{j \in N^E: j \neq i} b_{ij}^k - \sum_{j \in N^E: j \neq i} (b_{ji}^k - e_{ji}\bar{x}_{ji}^k), \forall i \in J^A$  by (30). Moreover,  $\sum_{i \in J^A} \beta^k \bar{x}_{0i}^k = \beta^k$  since  $\bar{w}^k > 0$ .

$$\begin{aligned} \sum_{i \in J^A} z_i^k &= \sum_{i \in J^A} \sum_{j \in N^E: j \neq i} b_{ij}^k - \sum_{i \in J^A} \sum_{j \in N^E: j \neq i} (b_{ji}^k - e_{ji}\bar{x}_{ji}^k) \\ &= \sum_{i \in J^A} b_{i0}^k - \sum_{i \in J^A} b_{0i}^k + \sum_{i \in J^A} \sum_{j \in J^A: i \neq j} (b_{ij}^k - b_{ji}^k) + \sum_{i \in J^A} \sum_{j \in N^E: j \neq i} e_{ji}\bar{x}_{ji}^k \\ &= \sum_{i \in J^A} e_{i0}\bar{x}_{i0}^k - \sum_{i \in J^A} \beta^k \bar{x}_{0i}^k + \sum_{i \in J^A} \sum_{j \in N^E: j \neq i} e_{ji}\bar{x}_{ji}^k \\ &= \sum_{i \in N^E} \sum_{j \in N^E: j \neq i} e_{ji}\bar{x}_{ji}^k - \beta^k \end{aligned}$$

□

After elimination of equality constraints and necessary rearrangements on the remaining subproblem, we obtain the following  $SP_k$  in canonical maximization form for each  $k \in K$ :

$$\max 0 \quad (37)$$

$$\text{s.t. } -\sum_{j \in J} z_j^k \leq -\bar{w}^k, \quad (38)$$

$$z_j^k \leq \beta^k \bar{y}_j, \quad \forall j \in J^A \quad (39)$$

$$z_j^k \leq \beta^k \sum_{i \in J^A: i \neq j} \bar{x}_{ij}^k, \quad \forall j \in J^A \quad (40)$$

$$z_j^k + \sum_{i \in J^A: i \neq j} b_{ij}^k - \sum_{i \in J^A: i \neq j} b_{ji}^k \leq e_{j0}\bar{x}_{j0}^k - \beta^k \bar{x}_{0j}^k + \sum_{i \in N^E: i \neq j} e_{ij}\bar{x}_{ij}^k, \quad \forall j \in J^A \quad (41)$$

$$-z_j^k - \sum_{i \in J^A: i \neq j} b_{ij}^k + \sum_{i \in J^A: i \neq j} b_{ji}^k \leq \beta^k \bar{x}_{0j}^k - e_{j0}\bar{x}_{j0}^k - \sum_{i \in N^E: i \neq j} e_{ij}\bar{x}_{ij}^k, \quad \forall j \in J^A \quad (42)$$

$$b_{ij}^k \leq \beta^k \bar{x}_{ij}^k, \quad \forall i, j \in J^A : i \neq j \quad (43)$$

$$-b_{ij}^k \leq -e_{ij}\bar{x}_{ij}^k, \quad \forall i, j \in J^A : i \neq j \quad (44)$$

$$b_{ij}^k \geq 0, \quad \forall i, j \in J^A : i \neq j \quad (45)$$

$$z_j^k \geq 0, \quad \forall j \in J^A \quad (46)$$

Let  $\alpha, \delta_j, \gamma_j, \pi_j, \rho_j, \phi_{ij}, \epsilon_{ij}$  be the dual variables associated with constraints (38)-(44), respectively. Then, we can write the equivalent dual problem  $D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$  for each  $k \in K$  as follows:

$$\begin{aligned} D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}}) \quad \min \quad & -\bar{w}^k \alpha + \sum_{j \in J^A} \beta^k \bar{y}_j \delta_j + \sum_{i \in N^E} \sum_{j \in J^A : i \neq j} \beta^k \bar{x}_{ij}^k \pi_j \\ & + \sum_{j \in J^A} e_{j0} \bar{x}_{j0}^k \gamma_j - \sum_{j \in J^A} \beta^k \bar{x}_{0j}^k \gamma_j + \sum_{i \in N^E} \sum_{j \in J^A : i \neq j} e_{ij} \bar{x}_{ij}^k \gamma_j \\ & + \sum_{j \in J^A} \beta^k \bar{x}_{0j}^k \rho_j - \sum_{j \in J^A} e_{j0} \bar{x}_{j0}^k \rho_j - \sum_{i \in N^E} \sum_{j \in J^A : i \neq j} e_{ij} \bar{x}_{ij}^k \rho_j \\ & + \sum_{i \in J^A} \sum_{j \in J^A : i \neq j} \beta^k \bar{x}_{ij}^k \phi_{ij} - \sum_{i \in J^A} \sum_{j \in J^A : i \neq j} e_{ij} \bar{x}_{ij}^k \epsilon_{ij} \end{aligned} \quad (47)$$

$$\text{s.t.} \quad -\alpha + \delta_j + \pi_j + \gamma_j - \rho_j \geq 0, \quad \forall j \in J^A \quad (48)$$

$$-\gamma_i + \gamma_j + \rho_i - \rho_j + \phi_{ij} - \epsilon_{ij} \geq 0, \quad \forall i, j \in J^A : i \neq j \quad (49)$$

$$\alpha \geq 0, \quad (50)$$

$$\delta_j, \gamma_j, \pi_j, \rho_j \geq 0, \quad \forall j \in J^A \quad (51)$$

$$\phi_{ij}, \epsilon_{ij} \geq 0, \quad \forall i, j \in J^A : i \neq j \quad (52)$$

In order to ensure that the dual problem is bounded, we further bound variables  $\alpha, \gamma_j, \rho_j, \forall j \in J^A$ , and  $\epsilon_{ij}, \forall i, j \in J^A : i \neq j$  by 1 from above. If the optimal value of  $D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$  is negative valued, we add the feasibility cut (53) to MP to cut the current solution  $(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$ .

$$\begin{aligned} & -\bar{\alpha} w + \sum_{j \in J^A} \beta^k \bar{\delta}_j y_j + \sum_{i \in N^E} \sum_{j \in J^A : i \neq j} \beta^k \bar{\pi}_j x_{ij}^k + \sum_{j \in J^A} e_{j0} \bar{\gamma}_j x_{j0}^k - \sum_{j \in J^A} \beta^k \bar{\gamma}_j x_{0j}^k + \sum_{i \in N^E} \sum_{j \in J^A : i \neq j} e_{ij} \bar{\gamma}_j x_{ij}^k \\ & + \sum_{j \in J^A} \beta^k \bar{\rho}_j x_{0j}^k - \sum_{j \in J^A} e_{j0} \bar{\rho}_j x_{j0}^k - \sum_{i \in N^E} \sum_{j \in J^A : i \neq j} e_{ij} \bar{\rho}_j x_{ij}^k \\ & + \sum_{i \in J^A} \sum_{j \in J^A : i \neq j} \beta^k \bar{\phi}_{ij} x_{ij}^k - \sum_{i \in J^A} \sum_{j \in J^A : i \neq j} e_{ij} \bar{\epsilon}_{ij} x_{ij}^k \geq 0 \end{aligned} \quad (53)$$

## 5 Implementation Details - General Framework

Our decomposition algorithm mainly consists of two phases. In Phase I, we solve the problem with at most one visit to each station ( $BDA^1$ ) and in the second phase, we focus on the general problem that allows multiple visits to stations. Between the two phases, we perform an intermediate reduction procedure (See Section 6) to decrease the size of the problem in Phase II. The aim is to cut as much as possible without eliminating any potential solution that is better than the one in

Phase I. We provide a brief summary of the general framework at the end of this section. For clarity of presentation, we use the notation ‘BDA’ throughout the paper to refer to both the algorithm and the formulation.

Through our preliminary experiments, we observe that our algorithm has a better convergence behavior if we introduce a high quality initial feasible solution to our master problem. In order to achieve this, we first perform a ‘Step 0’ process where we solve our BDA formulation via a CPLEX option that allows stopping after finding the first integer feasible solution. We also introduce a partial *warm start* solution to CPLEX by opening all potential stations. In our experiments, CPLEX usually finds a solution with all stations opened. We then improve this solution by closing some of the stations. This removal process is a greedy approach based on checking the energy consumption between three consecutive stations and then closing the intermediate one if the battery level is sufficient to go from the first one to the third one. Finally, we introduce the set of open stations of this improved solution as a partial *warm start* solution for our Phase I problem and solve  $BDA^1$  with the valid inequalities given next in Section 5.1.

### 5.1 Valid Inequalities for Phase I

Let  $N_{min}^V$  be a lower bound on the number of vehicles needed for any feasible solution. We can obtain such a lower bound by solving a bin packing problem (BPP) as follows. Define  $v_k = 1$  if vehicle  $k$  is used, 0 otherwise and  $a_{ik} = 1$  if the request of customer  $i$  is provided by vehicle  $k$ , otherwise. Constraints (55) assign each customer to a vehicle while Constraints (56) ensure that these assignments respect the capacities of vehicles.

$$(BPP) \quad N_{min}^V = \min \sum_{k \in K} v_k \quad (54)$$

$$\text{s.t.} \quad \sum_{k \in K} a_{ik} = 1, \quad \forall i \in I \quad (55)$$

$$\sum_{i \in I} d_i a_{ik} \leq Q^k v_k, \quad \forall k \in K \quad (56)$$

$$v_k \in \{0, 1\}, \quad \forall k \in K \quad (57)$$

$$a_{ik} \in \{0, 1\}, \quad \forall i \in I, k \in K. \quad (58)$$

We can detect the infeasibility due to insufficient freight capacity by solving BPP. Our preliminary experiments revealed that introducing Constraint (59), which enforces using at least  $N_{min}^V$  vehicles, usually reduces the solving time. This observation has led us to include this constraint in our computations for every model of Phase I and Phase II.

$$\sum_{k \in K} \sum_{j \in J^A} x_{0j}^k \geq N_{min}^V \quad (59)$$

When we solve BDA to optimality with at most one visit to each station ( $BDA^1$ ), we include the following sets of valid inequalities to our master problem:

$$\sum_{k \in K} \sum_{j: (i,j) \in A^k} x_{ij}^k \leq 1, \quad i \in J^A : d_i > 0 \quad (60)$$

$$\sum_{j:(i,j) \in A^k} x_{ij}^k - \sum_{j:(0,j) \in A^k} x_{0j}^k \leq 0, \quad i \in N^E, \forall k \in K \quad (61)$$

$$\sum_{i \in J^A} x_{i0}^k \leq 1, \quad \forall k \in K \quad (62)$$

$$\sum_{j:(j,i) \in A^k} x_{ij}^k \leq y_i, \quad i \in J^A : d_i = 0, \forall k \in K \quad (63)$$

$$y_i \leq \sum_{k \in K} \sum_{j \in N^E} x_{ij}, \quad \forall i \in J^A : d_i = 0 \quad (64)$$

$$y_j \leq \sum_{k \in K} \sum_{i \in N^E} x_{ij}, \quad \forall j \in J^A : d_j = 0 \quad (65)$$

$$w^k \leq \sum_{j \in J^A} \beta^k y_j \quad \forall k \in K. \quad (66)$$

Constraints (60) restrict the number of arcs entering a demand node to one. Constraints (61) ensure that an arc is visited by a vehicle only if that vehicle leaves the depot. Constraints (62) make sure that each vehicle enters the depot at most once. Constraints (63) forbid leaving a zero-demand copy of a station if it is not open while Constraints (64) and (65) forbid opening these type of stations if they are not visited by any vehicle. Constraints (66) limit the total recharging for each vehicle by full battery charging times the number of open stations.

Even though most of these constraints are implied by the original constraints, their inclusion improves the time performance of our algorithm considerably.

Let  $(\bar{y}^1, \bar{x}^1, \bar{1}^1, \bar{w}^1)$  be the solution with value  $Z^1$  that we obtain from Phase I. After the intermediate process which will be explained in Section 6, we proceed to Phase II to solve a reduced problem via BDA with valid inequalities of Section 5.2 below. We introduce  $\bar{y}^1$  as a partial *warm start* solution to the Phase II problem.

## 5.2 Valid Inequalities for Phase II

When we apply BDA for the last time with all possible copies of potential stations, in addition to the valid inequalities (59),(61)-(66), we also introduce the following set of valid inequalities to break the symmetry between the copies of stations:

$$\sum_{i:(i,j) \in A^k} x_{ij}^k \leq \sum_{i:((j-1),i) \in A^k} x_{(j-1)i}^k, \quad \forall k \in K, j \text{ is the } m^{\text{th}} \text{ copy of some } j_1 : d_{j_1} > 0, m \geq 3 \quad (67)$$

$$\sum_{i:(i,j) \in A^k} x_{ij}^k \leq \sum_{i:((j-1),i) \in A^k} x_{(j-1)i}^k, \quad \forall k \in K, j \text{ is the } m^{\text{th}} \text{ copy of some } j_1 : d_{j_1} = 0, m \geq 2. \quad (68)$$

Constraints (67) and (68) make sure that an additional copy of any station is visited by a vehicle only if the preceding copy is visited by the same vehicle. Exceptionally, the second copy (the first non-original copy), might be visited by a vehicle not serving the original copy if it is a demand node.

Below we give a brief summary of the general framework of our algorithm:

Step 0: Solve  $BDA^1$  to obtain a feasible solution  $(\bar{y}^0, \bar{x}^0, \bar{1}^0, \bar{w}^0)$  (not necessarily optimal).

Close the redundant stations of  $(\bar{y}^0, \bar{x}^0, \bar{1}^0, \bar{w}^0)$  in a greedy manner and obtain  $(\bar{y}, \bar{x}, \bar{1}, \bar{w})$ .

Step 1: Phase I: Solve  $BDA^{1\cup}(59)-(66)$  with partial *warm start*  $\bar{y}$  to obtain the optimal solution  $(\bar{y}^1, \bar{x}^1, \bar{l}^1, \bar{w}^1)$ .

Step 3: Apply the intermediate process (see Section 6) to reduce the size of  $BDA\cup(59)-(68)$ .

Step 4: Phase II: Solve the reduced  $BDA\cup(59)-(68)$  with partial *warm start*  $\bar{y}^1$  to obtain the optimal solution.

## 6 Intermediate Reduction Process

Creating multiple copies of stations leads to a large-size formulation and excessive solving times. We develop a two phase method that solves our Benders formulation initially for a single copy of each station. Based on the value  $Z^1$  of the solution obtained at this stage, we apply an intermediate processing that checks the availability of a solution with multiple copies of stations that has a smaller objective value than  $Z^1$ . This is an iterative procedure that proceeds by increasing the number of copies considered, say  $m$ , one by one and applies lower bound checking steps.

The aim of this procedure is to check whether there exists a solution of BDA with exactly  $m$  copies for some station  $j$  whose cost is lower than  $Z^1$ .

**Lemma 6.1.** *Let  $Z_{(m,j,k)}^{LB}$  be a lower bound on the cost when exactly  $m$  copies of station  $j$  is visited by vehicle  $k$ . If  $Z_{(m,j,k)}^{LB} \geq Z^1, \forall j, k$ , then, there exists no solution with  $m$  copies of any station whose value is less than  $Z^1$ .*

*Proof.* Any feasible solution to a minimization problem provides an upper bound. Therefore, the value of any feasible solution as described in Lemma 6.1 has to be greater than or equal to  $Z_{(m,j,k)}^{LB} \geq Z^1$ .  $\square$

**Lemma 6.2.** *If there exists some lower bound  $Z_{(m,j,k)}^{LB}$  such that  $Z_{(m',j,k)}^{LB} \geq Z_{(m,j,k)}^{LB}, \forall m' \geq m$  and if Lemma 6.1 holds for such  $Z_{(m,j,k)}^{LB}$  of  $m$  for all  $j, k$ , then, there exist no solution with more than or equal  $m$  copies of any stations whose value is less than  $Z^1$ .*

*Proof.*  $Z_{(m',j,k)}^{LB} \geq Z_{(m,j,k)}^{LB} \geq Z^1, \forall j, k, m'$  such that  $m' \geq m$  by Lemma 6.1. Then, there exists no solution with  $m'$  copies of any station whose value is less than  $Z^1$ .  $\square$

Below we give the details on how we obtain a lower bound that satisfies Lemma 6.2.

Let us consider a potential station  $j$ . If we use exactly  $m$  copies of this station, it means that we visit at least  $m - 1$  different customers with some vehicle  $k$ . This leads to a partial network structure as  $0 \dots j \dots i_1 \dots j \dots i_2 \dots \dots i_{m-1} \dots j \dots 0$ .

Let  $E^m$  and  $R^m$  be the amount of energy consumption and the amount of recharging needed, respectively, when we visit station  $j$  exactly  $m$  times by some vehicle  $k_1$ . Now, we consider two cases:

Case 1: all customers are visited by  $k_1$ .

Case 2: some customers are visited by other vehicle(s).

For any of Case 1 or Case 2, the following observation holds:

*Observation:*

(i)  $E^m > E_m^{base} = (m - 1)\beta^{k_1} + e_{j_0}$  and  $R^m > R_m^{base} = (m - 2)\beta^{k_1} + e_{j_0}$  if  $m$  is even.

(ii)  $E^m > E_m^{base} = (m - 1)\beta^{k_1}$  and  $R^m > R_m^{base} = (m - 2)\beta^{k_1}$  if  $m$  is odd.

In Figure 1, we illustrate this observation for  $m = 2$  and  $m = 3$ . In this figure, if  $E^2 \leq \beta^{k_1} + e_{j_0}$ , we would not need to visit  $j$  twice. Similarly, if  $E^3 \leq 2\beta^{k_1}$ , it would be redundant to visit  $j$  three times.

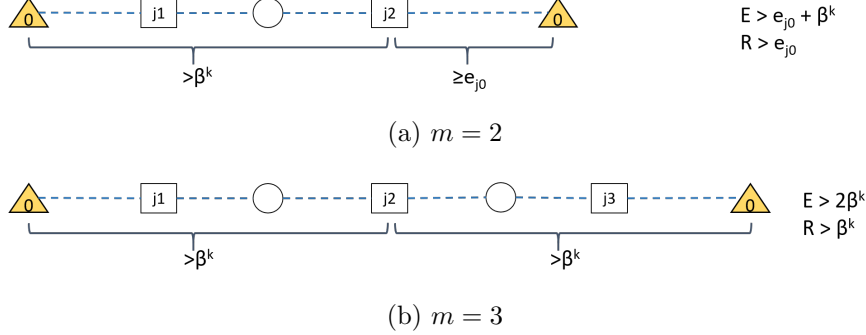


Figure 1: Illustration of minimal energy consumption and recharging need for visiting  $m$  copies of  $j$  with vehicle  $k$  for  $m = 2, 3$ .

We further check whether  $E^m$  and  $R^m$  values are much larger than  $E^{base}$  and  $R^{base}$ , respectively. This is performed as follows:

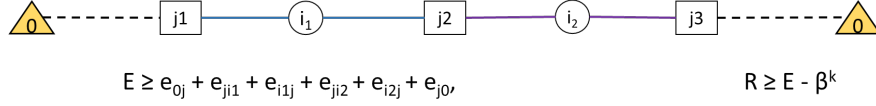


Figure 2: Calculation of minimal energy consumption  $E^1$  on the partial network for visiting  $m = 3$  copies of  $j$  with vehicle  $k$ .

In Case 1, we calculate the minimal possible energy consumption on such a partial network. More explicitly, we define  $E^1 = e_{0j} + e_{j_1} + e_{i_1j} + e_{j_2} + e_{i_2j} + \dots + e_{j_{m-1}} + e_{j_0}$  where  $i_1, \dots, i_{m-1}$  are  $m - 1$  closest customers to  $j$  (e.g. see Figure 2). For this particular case, we can further obtain a lower bound on the total energy consumption by constructing a 1-tree obtained via a minimum spanning tree which spans the union set of all customers and  $m$  copies of  $j$  and that is connected to the depot node with two minimal edges. Let the energy consumption on this 1-tree be  $E^T$  and  $E = \max\{E^1, E^T, E^m\}$ . We obtain a lower bound  $C$  on the total routing cost similarly. Then,  $R = \max\{E - \beta^{k_1}, 0, R^m\}$  gives us the amount of recharging needed for this partial network and  $Z^{LB} = R \times r_{k_1} + C + f_j + v_{k_1}$  gives us a lower bound on the cost of routing all customers by vehicle  $k$  via visiting  $j$   $m$  times or more.

When we look at Case 2, we investigate all possible vehicle combinations that need to be considered by iteratively increasing the number of additional vehicles. If we find a combination with  $k$  vehicles whose lower bound is less than  $Z^1$ , we do not check the combinations with more than  $k$  vehicles. Let us assume that in addition to  $k_1$ , we use  $K^* = \{k_2, \dots, k_l\}$ . This means that we are visiting a different customer by each additional vehicle. Therefore, we add  $0 \dots i_{k_h} \dots 0$  as a connected component to our partial network for each vehicle  $k_h \in K^*$  where  $i_{k_h}$  is the closest customer to depot which is not served by preceding vehicles. In a similar fashion to that of Case 1, we calculate the total energy consumption  $E^h$ , the amount of recharging needed  $R^h$ , the

total routing cost and hence a lower bound  $Z_{K^*}^{LB1}$  on the total cost with the corresponding vehicle combination.

In order to obtain another lower bound  $Z_{K^*}^{LB2}$  from the 1-tree constructed, this time, we use  $\beta = \sum_{k \in K^* \cup k_1} \beta^k$  as the total battery available in our calculation for the amount of recharging needed, that is,  $R^2 = \max\{\max\{E^2, E^T\} - \beta, 0\}$  and  $r = \min_{k \in K^* \cup k_1} r_k$  as the unit recharging cost. Our bound  $Z_{K^*}^{LB}$  for the corresponding combination is defined as  $Z_{K^*}^{LB} = \max\{Z_{K^*}^{LB1}, Z_{K^*}^{LB2}\}$ . See Figure 3 for sample Hamiltonian cycles to construct 1-tree for visiting two copies of  $j$  with 1, 2, or 3 vehicles.

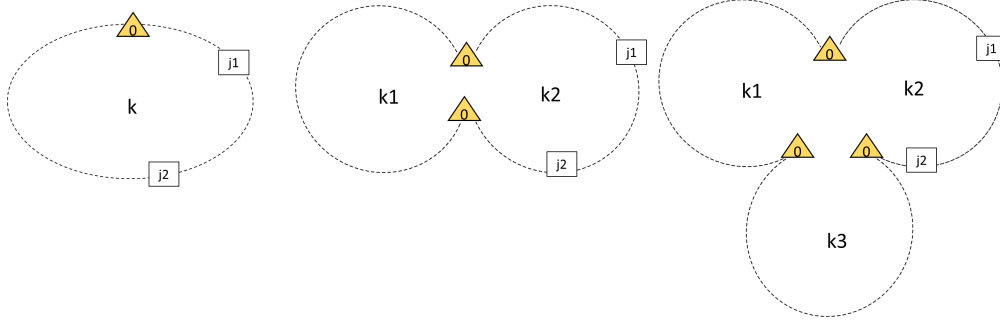


Figure 3: Illustrative Hamiltonian cycles for calculating the 1-tree lower bound LB2 on the cost of visiting  $m = 2$  copies of  $j$  with 1, 2, or 3 vehicles.

If  $\min\{Z^{LB}, Z_{K^*}^{LB}\} \geq Z^{m-1}$  for every  $(k_1, K^*)$  combination, then, the value of any solution visiting  $j$  no less than  $m$  times will be no better than  $Z^{m-1}$ . So, in further iterations, we do not need more than  $m - 1$  copies of  $j$ . If this holds for all stations, we can terminate the iterative checking procedure and solve our algorithm *BDA* with at most  $m - 1$  copies for each station.

*Additional speed-up mechanism:*

When solving *BDA* for this last time, we further apply variable fixing by using the information we obtained from this iterative procedure. More explicitly, if it is decided that we do not need more than  $l$  copies at a given station  $j$ , we fix all  $y$  values to zero for all those copies of  $j$ . Similarly, if we decided that visiting more than  $l$  copies of station  $j$  with vehicle  $k^1$  is not optimal, then we set all  $x$  variables of those copies to zero for vehicle  $k^1$ .

## 7 Computational Study

In order to test our methods, we generated problem instances based on the data set provided by Schneider et al. (2014). This data set has 36 different instances with 5, 10, and 15 customers (12 instances for each customer size). From these instances, we retrieved the demand and network information (node coordinates). The vehicle freight capacities are equal to 200 in the original data. We introduced additional levels of capacities (80, 100), especially, to test relatively smaller instances. Similarly, for the battery capacities, we conducted tests for low, medium, and high capacities (10, 16, 22 kWh) to avoid extremely loose values on the tests of small problems (Sassi and Oulamara, 2017).

In our experiments, we use IBM ILOG CPLEX 12.8.1 in a Java environment. We run our tests on a PC with Intel(R) Core(TM) i7-7920HQ CPU at 3.10 GHz processor and 32 GB RAM. For

each experiment, we set a memory limit of 16 GB and a time limit of 3600, 5400, and 10800 seconds for instances with 5, 10, and 15 customers, respectively.

We assume that the system will be equipped with fast charging facilities. The lifetime of a charging facility is estimated to be 3 years and it is 5 years for the vehicles. When we calculate the fixed costs of opening stations and purchasing/leasing vehicles, we divide their costs by the number of days within their lifetime. We approximately obtain  $f_j = 8 \text{ €}$  as the fixed cost of opening stations and  $v_k = 16, 26, \text{ and } 36 \text{ €}$  as the fixed cost of low, medium, and high capacity vehicles, respectively. Let  $l_{ij}$  be the distance between nodes  $i, j \in N$ ; then, we set  $c_{ij} = l_{ij} \times 0.03$  (cents/km),  $r_k = r = 0.07$  (cents/km) for  $k \in K$ ,  $e_{ij} = l_{ij} \times 135$  (Wh/km). In order to better tackle the precision issues of CPLEX, we multiplied all the cost values by 100.

Table 1: Freight capacity, battery capacity, and cost values for the three vehicle types considered.

Vehicle type	1	2	3
$Q^k$	80	100	200
$\beta^k$	10	16	22
$v^k$	16	26	36

We conduct experiments on instances with 2, 3, and 4 vehicles of three different types, see Table 1. For each instance of Schneider et al. (2014), we test the problem with the vehicle combinations shown in Table 2.

Table 2: Vehicle combinations tested for each problem instance of Schneider et al. (2014).

Combination code	$K$	Vehicle types available
he1	2	1,2
he2	2	1,3
he3	2	2,3
he4	3	1,2,3
he5	3	1,1,2
he6	3	2,2,3
he7	4	1,1,2,3
he8	4	1,2,2,3
he9	4	1,2,3,3

We provide average results for each group of instances in Tables 3 and 4. The detailed results are presented in Appendix A. For illustration, we also display the results in Figures 4-9.

In these figures and tables,  $N^S$  and  $N^V$  show the number of stations opened and the number of vehicles used, respectively. The value ‘ $g_1$ ’ represents the gap provided by CPLEX at the end of the time limit (0.00 if the problem is solved to proven optimality) for the corresponding model solved. Similarly, ‘ $t(s)$ ’ represents the total time spent in seconds for the corresponding model or algorithm if it includes additional processes. Let ‘Obj’ be the value of the best solution obtained from a model. Then, the ‘ $g_2$ ’ values are the gap values between the objective value of the best solution found and the best dual bound ( $DObj$ ) of the second phase model of BDA, more explicitly,  $g_2 = (Obj - DObj)/Obj$ . Finally,  $t_2(s)$  is the total time (including  $t(s)$  of Phase I) spent by BDA. We use ‘NA’ to refer to the fact that no primal or dual bound is obtained within the time limit.



Our formulation PF can solve all but one instance of  $|I| = 5$  to optimality within one hour. The average time spent by PF on these instances is 178.68 seconds, which is much larger compared to that of BDA. On the other hand, we observe that our algorithm BDA can solve all the instances with  $|I| = 5, 10$  to optimality. In Figures 4 and 5, we show the optimal values of the instances of  $|I| = 5$  and  $|I| = 10$ , respectively. The average time spent by BDA for solving the instances with

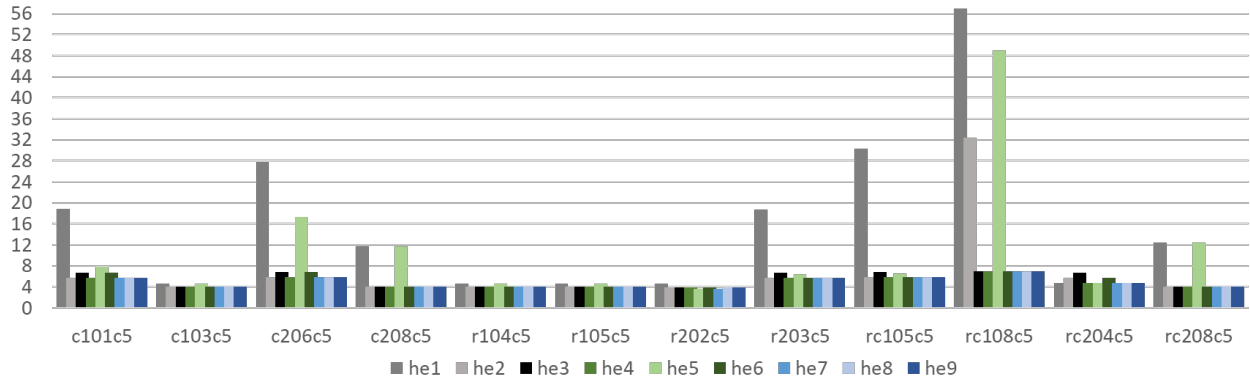


Figure 4: The optimal values (in thousands) for  $|I| = 5$ . The horizontal axis indicates the instance code given in Schneider et al. (2014).

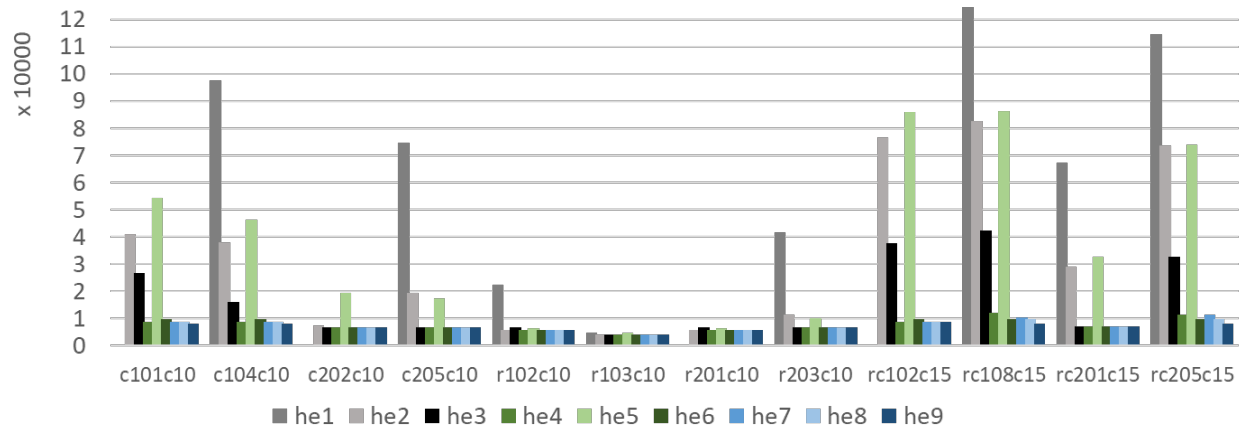


Figure 5: The optimal values for  $|I| = 10$ . The horizontal axis indicates the instance code given in Schneider et al. (2014).

$|I| = 5$  is 2.25 seconds and it is 146.39 seconds for the ones with  $|I| = 10$ . In Figure 6, we compare the average solving time of PF and BDA over 12 instances of Schneider et al. (2014) with  $|I| = 5$ . We can easily see that the performance of BDA is much better compared to PF. In fact, BDA is faster than PF in every single instance.

For larger problems ( $|I| = 10, 15$ ), optimality could not be guaranteed with PF within the time limit. In fact, PF could not even find a feasible solution for many instances. Based on these observations, for larger problems, we present the results for BDA only. In Figure 7, we record the average amount of time spent for Phase I ( $t_1(s)$ ) and the total time of BDA ( $t_2(s)$ ) for  $|I| = 10$  instances. These average values are calculated over 12 instances of Schneider et al. (2014). Phase

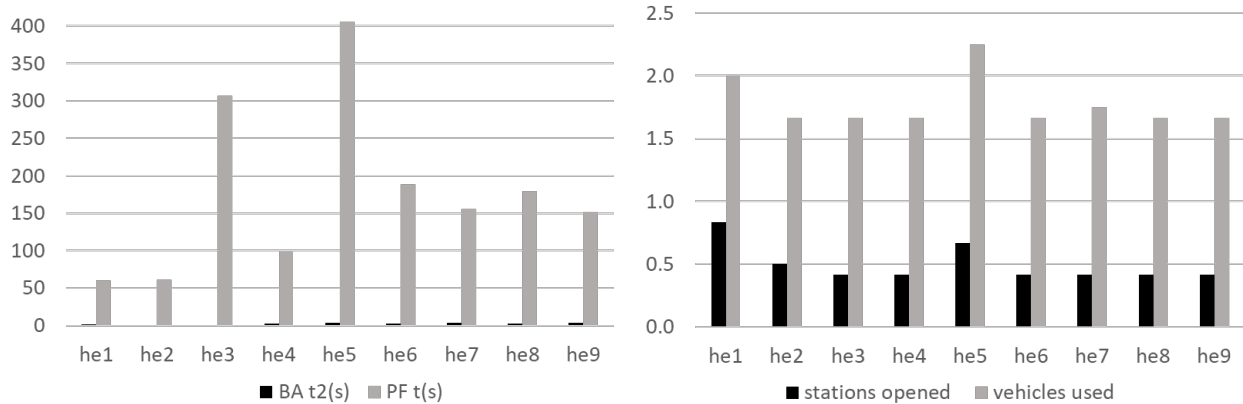


Figure 6: Average results (solving time, number of stations opened, and number of vehicles used) for PF and BDA on instances with  $|I| = 5$  for each vehicle combination.

I consumes 48.85 seconds on average only with the longest time being 767.48 seconds.

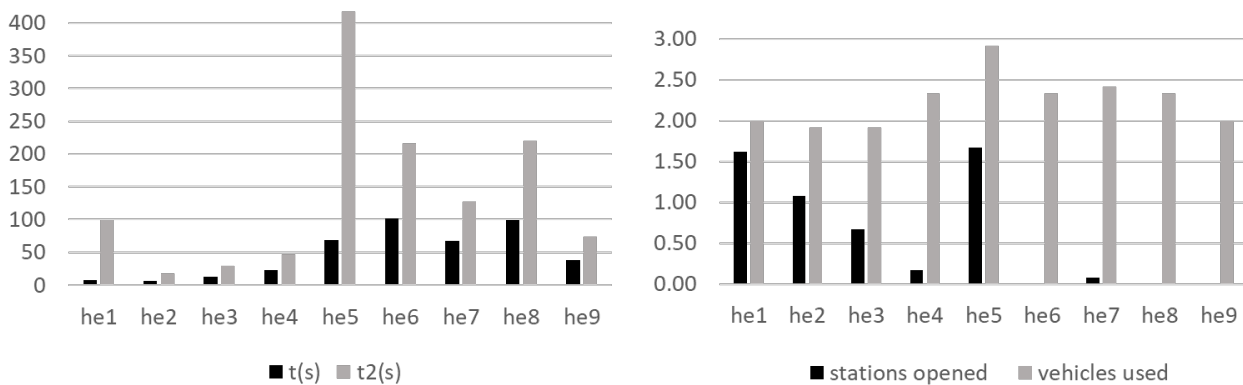


Figure 7: Average results (solving time, number of stations opened, and number of vehicles used) for BDA on instances with  $|I| = 10$  for each vehicle combination.

Moreover, BDA can solve 80% of the instances with  $|I| = 15$  and the average time spent for all instances with  $|I| = 15$  is just below 6500 seconds. In Figure 8, we show the values of the best solutions found for these instances. We further present the average values of  $t(s)$ ,  $t_2(s)$ ,  $g_1$ ,  $g_2$ ,  $N^S$ , and  $N^V$  over the 12 instances of Schneider et al. (2014) with  $|I| = 15$  in Figure 9. For these instances, we observe that BDA requires more time to reach optimality when  $K$  is larger.

It is interesting to note that small vehicles result in higher costs, an observation which is not that obvious. We can also note that the instances of ‘he1’ combination are frequently infeasible due to insufficient freight capacity. Here, the total capacity of the small and medium size vehicles is less than the total demand of those instances provided by Schneider et al. (2014). The ‘he5’ combination is also infeasible for some instances and for some others, the algorithm hits the memory limit in the second phase.

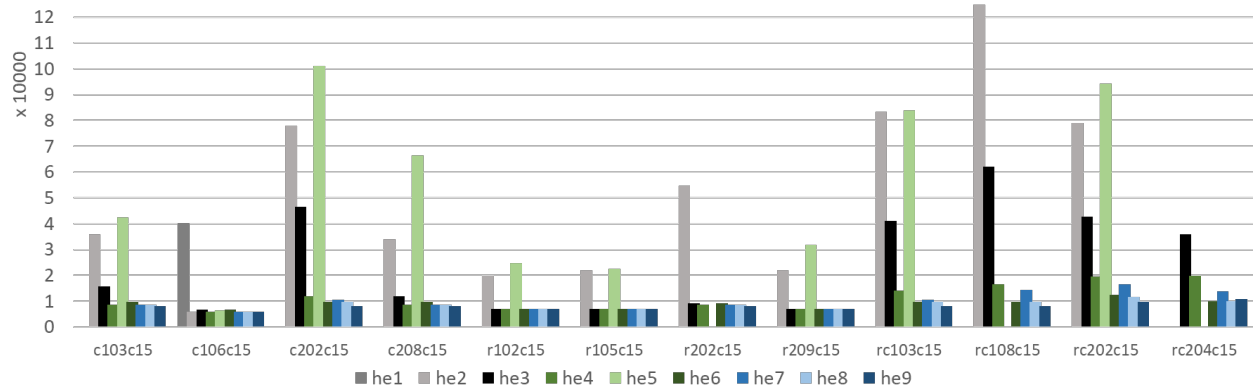


Figure 8: Best values for  $|I| = 15$ . The horizontal axis indicates the instance code given in Schneider et al. (2014).

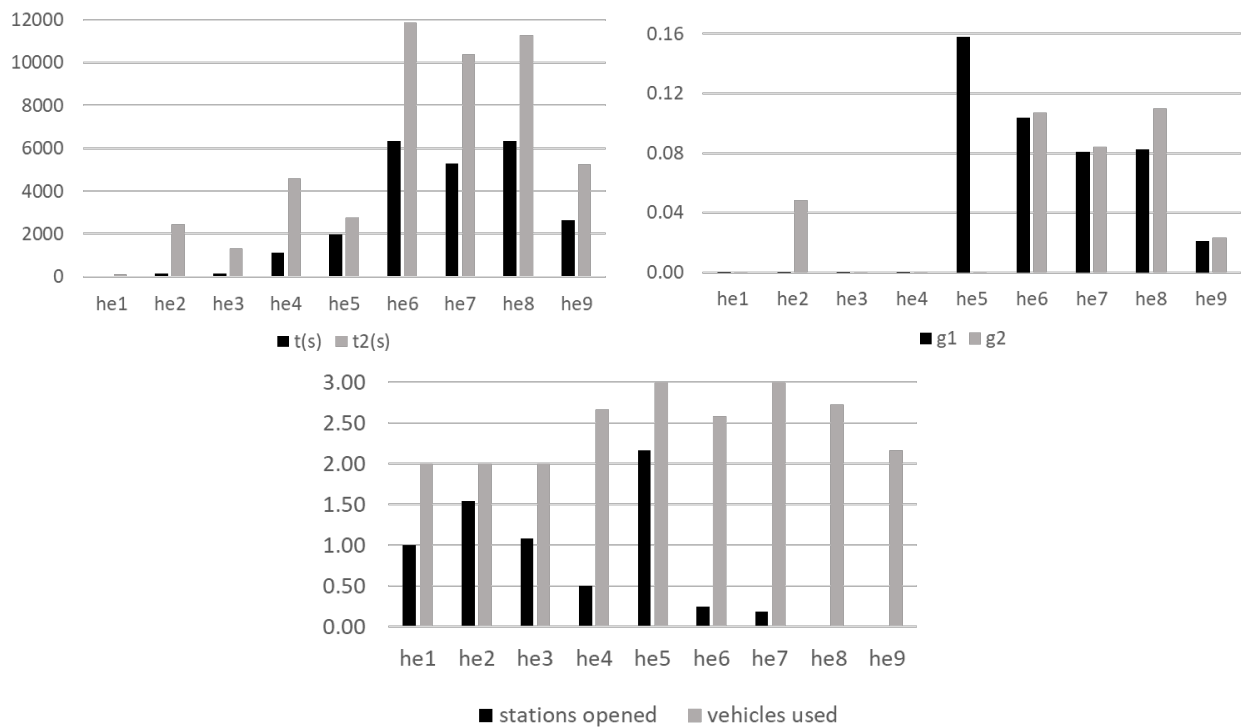


Figure 9: Average results (solving time, gaps, number of stations opened, and number of vehicles used) for BDA on instances with  $|I| = 15$  for each vehicle combination.

Table 3: Average results for c, r, and rc instances.

	BDA Phase I		BDA Phase II				PF	
	$g_1$	$t(s)$	$g_2$	$t_2(s)$	$N^S$	$N^V$	$g_1$	$t(s)$
c5	0.00	0.65	0.00	1.95	0.25	1.67	0.00	90.95
r5	0.00	0.77	0.00	2.17	0.03	1.47	0.00	96.10
rc5	0.00	1.06	0.00	2.64	0.22	1.86	0.01	348.98
c10	0.00	67.29	0.00	162.84	0.62	2.35		
r10	0.00	47.84	0.00	90.88	0.14	1.89		
rc10	0.00	32.12	0.00	181.85	0.89	2.52		
c15	0.03	3260.53	0.01	5802.79	0.54	2.51		
r15	0.04	1150.72	0.00	3100.75	0.50	2.17		
rc15	0.10	5091.34	0.14	11609.90	0.84	2.80		
*Avg.	0.02	1072.48	0.02	2328.42	0.45	2.14	0.00	178.68

\*Infeasible instances are excluded when calculating these average values.

## 8 Conclusion and Future Research Directions

In this paper, we introduce an electric location-routing problem with heterogeneous fleet and partial recharging. We initially propose a new mixed integer programming formulation for this problem. This is a flow-based formulation with three-index binary routing variables. The sub-tour elimination is enhanced via a group of load balancing constraints using these decision variables. We further utilize additional non-negative continuous variables to satisfy battery restrictions and energy-related constraints.

We test our formulation on small problem instances from the literature. Although the formulation is able to solve instances with 5 customers to optimality, we observe that its performance is limited when it comes to solving larger problems.

As we aim to solve this problem to optimality, we further develop a two-phase algorithm based on the Benders decomposition of our formulation. The first phase solves a restricted version of the problem that allows at most one visit to each station. By using the information obtained, the second phase problem, which is the generalized problem allowing multiple visits to any station, is reduced in size, making it relatively easier compared to the case with no a priori processing. This enhancement step allows us to solve all instances of 5 and 10 customers, and 80% of the instances with 15 customers to optimality. The average optimality gap over all other instances is negligible, just 0.02. In summary, our approach obtains very high quality solutions within the time limit.

We observe through our experimental study that the problem is usually harder to solve when the vehicle capacities are smaller. We also found that using small vehicles results in higher costs.

Though the main focus of this study is to present an exact method with proven optimality, this approach can be easily combined with additional procedures leading to powerful matheuristics to obtain near optimal solutions for larger instances, see Salhi (2017). This problem can also be tackled by powerful metaheuristics whose performance can be evaluated using lower bounds obtained from the proposed method.

The current problem can be extended to cater for several deterministic and stochastic variants

Table 4: Average results for each group of instances.

	BDA Phase I				BDA Phase II			PF		
	$ I $	$ J $	$g_1$	$t(s)$	$g_2$	$t_2(s)$	$N^S$	$N^V$	$g_1$	$t(s)$
c101c5		7	0.00	0.46	0.00	1.51	0.33	2.11	0.00	62.03
c103c5		7	0.00	0.45	0.00	1.82	0.00	1.22	0.00	25.59
c206c5		8	0.00	1.45	0.00	3.45	0.44	2.11	0.00	255.39
c208c5		7	0.00	0.26	0.00	1.02	0.22	1.22	0.00	20.80
r104c5		7	0.00	0.83	0.00	1.85	0.00	1.22	0.00	53.56
r105c5	5	7	0.00	0.33	0.00	1.17	0.00	1.22	0.00	22.66
r202c5		7	0.00	0.47	0.00	1.51	0.00	1.33	0.00	9.49
r203c5		8	0.00	1.45	0.00	4.15	0.11	2.11	0.00	298.69
rc105c5		8	0.00	1.42	0.00	3.33	0.11	2.11	0.00	337.14
rc108c5		8	0.00	1.13	0.00	2.98	0.56	2.11	0.00	52.25
rc204c5		8	0.00	1.49	0.00	3.26	0.00	2.00	0.00	553.52
rc208c5		7	0.00	0.21	0.00	0.98	0.22	1.22	0.05	453.00
Avg.			0.00	0.83	0.00	2.25	0.17	1.67	0.00	178.68
c101c10		14	0.00	59.06	0.00	148.43	0.75	2.63		
c104c10		13	0.00	161.25	0.00	360.62	0.67	2.56		
c202c10		14	0.00	28.45	0.00	60.51	0.50	2.13		
c205c10		12	0.00	20.38	0.00	81.81	0.56	2.11		
r102c10		13	0.00	14.74	0.00	26.78	0.11	2.11		
r103c10		12	0.00	2.74	0.00	7.77	0.00	1.22		
r201c10	10	13	0.00	122.25	0.00	223.78	0.00	2.13		
r203c10		14	0.00	51.61	0.00	105.20	0.44	2.11		
rc102c10		13	0.00	10.25	0.00	23.50	0.88	2.75		
rc108c10		13	0.00	44.17	0.00	510.58	0.89	2.67		
rc201c10		13	0.00	8.38	0.00	28.41	0.56	2.11		
rc205c10		13	0.00	65.68	0.00	164.90	1.22	2.56		
*Avg.			0.00	48.85	0.00	146.39	0.55	2.25		
c103c15		19	0.07	4799.07	0.03	6617.91	0.43	2.57		
c106c15		17	0.00	588.14	0.00	794.39	0.11	2.11		
c202c15		19	0.00	3479.35	0.00	6960.44	1.00	2.75		
c208c15		18	0.03	4175.56	0.02	8838.40	0.63	2.63		
r102c15		22	0.12	1454.12	0.00	692.95	0.29	2.00		
r105c15		20	0.00	521.89	0.00	1288.70	0.38	2.13		
r202c15	15	20	0.02	3639.25	0.02	10248.68	0.86	2.43		
r209c15		19	0.00	83.91	0.00	172.68	0.50	2.13		
rc103c15		19	0.16	4925.85	0.30	11494.95	1.00	2.75		
rc108c15		19	0.02	2109.19	0.00	6722.72	1.00	2.71		
rc202c15		19	0.13	6395.64	0.14	13771.28	0.86	3.00		
rc204c15		21	0.08	6934.69	0.10	14450.66	0.50	2.75		
*Avg.			0.05	3040.80	0.05	6469.88	0.63	2.48		

\*Infeasible instances are excluded when calculating these average values.

that are worth exploring. These include consideration of time windows, multiple depots and/or additional location decisions for the selection of depots, as well as periodicity or uncertainty in the customer demand.

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# Appendix A: Detailed computational results

Table 5: Results obtained from the instances in c101c5, c103c5, c206c5, and c208c5

	BDA Phase I			BDA Phase II			$N^S$	$N^V$	PF		
	Obj	$g_1$	$t(s)$	Obj	$g_2$	$t_2(s)$			Obj	$g_1$	$t(s)$
c101c5											
he1	18786.39	0.00	0.50	18786.39	0.00	1.33	2	2	18786.39	0.00	21.13
he2	5746.06	0.00	0.09	5746.06	0.00	0.98	0	2	5746.06	0.00	5.19
he3	6746.06	0.00	0.14	6746.06	0.00	0.66	0	2	6746.06	0.00	35.83
he4	5746.06	0.00	0.62	5746.06	0.00	1.30	0	2	5746.06	0.00	12.52
he5	7822.22	0.00	0.63	7822.22	0.00	1.96	1	3	7822.22	0.00	87.70
he6	6746.06	0.00	0.81	6746.06	0.00	2.89	0	2	6746.06	0.00	306.48
he7	5746.06	0.00	0.38	5746.06	0.00	1.25	0	2	5746.06	0.00	22.63
he8	5746.06	0.00	0.41	5746.06	0.00	1.55	0	2	5746.06	0.00	18.45
he9	5746.06	0.00	0.53	5746.06	0.00	1.64	0	2	5746.06	0.00	48.38
Avg.		0.00	0.46		0.00	1.51	0.33	2.11		0.00	62.03
c103c5											
he1	4661.24	0.00	0.16	4661.24	0.00	0.81	0	2	4661.24	0.00	29.67
he2	4039.60	0.00	0.05	4039.60	0.00	0.38	0	1	4039.60	0.00	4.80
he3	4039.60	0.00	0.03	4039.60	0.00	0.42	0	1	4039.60	0.00	4.64
he4	4039.60	0.00	0.56	4039.60	0.00	1.78	0	1	4039.60	0.00	22.78
he5	4661.24	0.00	0.83	4661.24	0.00	2.22	0	2	4661.24	0.00	86.23
he6	4039.60	0.00	0.09	4039.60	0.00	0.91	0	1	4039.60	0.00	16.61
he7	4039.60	0.00	0.47	4039.60	0.00	3.06	0	1	4039.60	0.00	22.63
he8	4039.60	0.00	1.30	4039.60	0.00	3.59	0	1	4039.60	0.00	22.64
he9	4039.60	0.00	0.52	4039.60	0.00	3.20	0	1	4039.60	0.00	20.30
Avg.		0.00	0.45		0.00	1.82	0.00	1.22	4177.74	0.00	25.59
c206c5											
he1	27764.86	0.00	0.69	27764.86	0.00	1.83	2	2	27764.86	0.00	33.48
he2	5877.58	0.00	0.30	5877.58	0.00	1.43	0	2	5877.58	0.00	58.53
he3	6845.46	0.00	0.44	6845.46	0.00	1.31	0	2	6845.46	0.00	134.61
he4	5877.58	0.00	1.93	5877.58	0.00	3.92	0	2	5877.58	0.00	219.89
he5	17313.27	0.00	2.89	17313.27	0.00	6.66	2	3	17313.27	0.00	481.19
he6	6845.46	0.00	2.41	6845.46	0.00	5.83	0	2	6845.46	0.00	499.47
he7	5877.58	0.00	1.58	5877.58	0.00	3.56	0	2	5877.58	0.00	158.25
he8	5877.58	0.00	0.84	5877.58	0.00	2.31	0	2	5877.58	0.00	568.69
he9	5877.58	0.00	2.00	5877.58	0.00	4.21	0	2	5877.58	0.00	144.42
Avg.		0.00	1.45		0.00	3.45	0.44	2.11	9795.22	0.00	255.39
c208c5											
he1	11746.53	0.00	0.19	11746.53	0.00	0.91	1	2	11746.53	0.00	24.42
he2	4056.05	0.00	0.05	4056.05	0.00	0.38	0	1	4056.05	0.00	5.23
he3	4056.05	0.00	0.05	4056.05	0.00	0.42	0	1	4056.05	0.00	5.72
he4	4056.05	0.00	0.11	4056.05	0.00	0.98	0	1	4056.05	0.00	13.56
he5	11746.53	0.00	0.61	11746.53	0.00	1.86	1	2	11746.53	0.00	77.89
he6	4056.05	0.00	0.12	4056.05	0.00	0.58	0	1	4056.05	0.00	11.58
he7	4056.05	0.00	0.23	4056.05	0.00	0.84	0	1	4056.05	0.00	14.61
he8	4056.05	0.00	0.53	4056.05	0.00	1.89	0	1	4056.05	0.00	16.72
he9	4056.05	0.00	0.44	4056.05	0.00	1.36	0	1	4056.05	0.00	17.48
Avg.		0.00	0.26		0.00	1.02	0.22	1.22	5765.05	0.00	20.80

Table 6: Results obtained from the instances in r104c5, r105c5, r202c5, and r203c5

	BDA Phase I			BDA Phase II			$N^S$	$N^V$	PF		
	Obj	$g_1$	$t(s)$	Obj	$g_2$	$t_2(s)$			Obj	$g_1$	$t(s)$
r104c5											
he1	4609.35	0.00	0.25	4609.35	0.00	0.88	0	2	4609.35	0.00	46.30
he2	3998.41	0.00	0.05	3998.41	0.00	0.41	0	1	3998.41	0.00	10.89
he3	3998.41	0.00	0.08	3998.41	0.00	0.54	0	1	3998.41	0.00	11.53
he4	3998.41	0.00	0.77	3998.41	0.00	2.00	0	1	3998.41	0.00	25.67
he5	4609.35	0.00	0.36	4609.35	0.00	1.48	0	2	4609.35	0.00	107.16
he6	3998.41	0.00	0.42	3998.41	0.00	1.40	0	1	3998.41	0.00	30.00
he7	3998.41	0.00	3.20	3998.41	0.00	4.40	0	1	3998.41	0.00	121.13
he8	3998.41	0.00	1.60	3998.41	0.00	3.66	0	1	3998.41	0.00	66.47
he9	3998.41	0.00	0.70	3998.41	0.00	1.84	0	1	3998.41	0.00	62.89
Avg.		0.00	0.83		0.00	1.85	0.00	1.22	4134.17	0.00	53.56
r105c5											
he1	4653.45	0.00	0.23	4653.45	0.00	0.91	0	2	4653.45	0.00	21.44
he2	4005.06	0.00	0.05	4005.06	0.00	0.58	0	1	4005.06	0.00	4.75
he3	4005.06	0.00	0.09	4005.06	0.00	0.64	0	1	4005.06	0.00	5.89
he4	4005.06	0.00	0.09	4005.06	0.00	0.53	0	1	4005.06	0.00	8.84
he5	4653.45	0.00	0.59	4653.45	0.00	2.31	0	2	4653.45	0.00	98.47
he6	4005.06	0.00	0.12	4005.06	0.00	0.77	0	1	4005.06	0.00	11.58
he7	4005.06	0.00	1.42	4005.06	0.00	3.25	0	1	4005.06	0.00	16.89
he8	4005.06	0.00	0.14	4005.06	0.00	0.75	0	1	4005.06	0.00	17.44
he9	4005.06	0.00	0.16	4005.06	0.00	0.83	0	1	4005.06	0.00	18.63
Avg.		0.00	0.33		0.00	1.17	0.00	1.22	4149.15	0.00	22.66
r202c5											
he1	4623.09	0.00	0.26	4623.09	0.00	0.86	0	2	4623.09	0.00	12.97
he2	3979.55	0.00	0.05	3979.55	0.00	0.56	0	1	3979.55	0.00	2.52
he3	3979.55	0.00	0.08	3979.55	0.00	0.42	0	1	3979.55	0.00	3.56
he4	3979.55	0.00	0.22	3979.55	0.00	0.87	0	1	3979.55	0.00	8.97
he5	3628.73	0.00	0.66	3628.73	0.00	1.55	0	2	3628.73	0.00	16.47
he6	3979.55	0.00	0.31	3979.55	0.00	1.22	0	1	3979.55	0.00	6.58
he7	3628.73	0.00	1.76	3628.73	0.00	4.68	0	2	3628.73	0.00	7.70
he8	3979.55	0.00	0.45	3979.55	0.00	1.38	0	1	3979.55	0.00	13.13
he9	3979.55	0.00	1.10	3979.55	0.00	2.07	0	1	3979.55	0.00	13.52
Avg.		0.00	0.47		0.00	1.51	0.00	1.33	3973.09	0.00	9.49
r203c5											
he1	18639.54	0.00	0.66	18639.54	0.00	1.75	1	2	18639.54	0.00	115.75
he2	5779.71	0.00	0.34	5779.71	0.00	1.03	0	2	5779.71	0.00	44.64
he3	6778.67	0.00	0.47	6778.67	0.00	1.70	0	2	6778.67	0.00	756.47
he4	5779.71	0.00	1.86	5779.71	0.00	4.64	0	2	5779.71	0.00	79.66
he5	6468.59	0.00	1.39	6468.59	0.00	4.86	0	3	6468.59	0.00	123.73
he6	5778.67	0.00	2.09	5778.67	0.00	4.83	0	2	5778.67	0.00	230.63
he7	5779.71	0.00	2.14	5779.71	0.00	6.01	0	2	5779.71	0.00	332.19
he8	5778.67	0.00	1.59	5778.67	0.00	6.33	0	2	5778.67	0.00	422.89
he9	5779.71	0.00	2.49	5779.71	0.00	6.18	0	2	5779.71	0.00	582.22
Avg.		0.00	1.45		0.00	4.15	0.11	2.11	7395.89	0.00	298.69

Table 7: Results obtained from BDA on the instances in rc105c5, rc108c5, rc204c5, and rc208c5

	BDA Phase I			BDA Phase II			$N^S$	$N^V$	PF		
	Obj	$g_1$	$t(s)$	Obj	$g_2$	$t_2(s)$			Obj	$g_1$	$t(s)$
rc105c5											
he1	30285.61	0.00	0.52	30285.61	0.00	1.92	1	2	30285.61	0.00	281.55
he2	5859.27	0.00	0.20	5859.27	0.00	0.78	0	2	5859.27	0.00	116.05
he3	6829.32	0.00	0.41	6829.32	0.00	1.98	0	2	6829.32	0.00	304.55
he4	5859.27	0.00	1.36	5859.27	0.00	2.91	0	2	5859.27	0.00	412.72
he5	6513.08	0.00	0.97	6513.08	0.00	3.03	0	3	6513.08	0.00	128.28
he6	5829.32	0.00	2.89	5829.32	0.00	5.28	0	2	5829.32	0.00	355.00
he7	5859.27	0.00	1.19	5859.27	0.00	3.37	0	2	5859.27	0.00	452.09
he8	5829.32	0.00	1.96	5829.32	0.00	5.32	0	2	5829.32	0.00	564.47
he9	5859.27	0.00	2.35	5859.27	0.00	5.38	0	2	5859.27	0.00	419.58
Avg.		0.00	1.42		0.00	3.33	0.11	2.11	8747.08	0.00	337.14
rc108c5											
he1	56887.93	0.00	0.36	56887.93	0.00	1.74	2	2	56887.93	0.00	27.30
he2	32384.41	0.00	0.27	32384.41	0.00	0.90	1	2	32384.41	0.00	109.75
he3	6937.62	0.00	0.38	6937.62	0.00	1.39	0	2	6937.62	0.00	15.55
he4	6937.62	0.00	0.76	6937.62	0.00	1.96	0	2	6937.62	0.00	30.13
he5	48999.08	0.00	1.42	48999.08	0.00	4.12	2	3	48999.08	0.00	42.97
he6	6937.62	0.00	0.62	6937.62	0.00	2.45	0	2	6937.62	0.00	31.52
he7	6937.62	0.00	1.34	6937.62	0.00	2.98	0	2	6937.62	0.00	45.28
he8	6937.62	0.00	2.44	6937.62	0.00	5.39	0	2	6937.62	0.00	48.78
he9	6937.62	0.00	2.58	6937.62	0.00	5.90	0	2	6937.62	0.00	118.97
Avg.		0.00	1.13		0.00	2.98	0.56	2.11	19988.57	0.00	52.25
rc204c5											
he1	4752.39	0.00	0.34	4752.39	0.00	0.89	0	2	4752.39	0.00	7.48
he2	5751.17	0.00	0.25	5751.17	0.00	0.84	0	2	5751.17	0.00	359.70
he3	6727.99	0.00	0.50	6727.99	0.00	1.61	0	2	6727.99	0.00	2393.78
he4	4752.39	0.00	1.95	4752.39	0.00	3.21	0	2	4752.39	0.00	287.72
he5	4752.39	0.00	2.13	4752.39	0.00	4.73	0	2	4752.39	0.00	22.08
he6	5727.99	0.00	2.83	5727.99	0.00	5.56	0	2	5727.99	0.00	719.70
he7	4752.39	0.00	1.92	4752.39	0.00	4.27	0	2	4752.39	0.00	536.75
he8	4752.39	0.00	1.77	4752.39	0.00	3.92	0	2	4752.39	0.00	325.55
he9	4752.39	0.00	1.75	4752.39	0.00	4.34	0	2	4752.39	0.00	328.89
Avg.		0.00	1.49		0.00	3.26	0.00	2.00		0.00	553.52
rc208c5											
he1	12430.21	0.00	0.28	12430.21	0.00	1.05	1	2	12430.21	0.00	102.72
he2	4088.00	0.00	0.05	4088.00	0.00	0.38	0	1	4088.00	0.00	5.83
he3	4088.00	0.00	0.08	4088.00	0.00	0.58	0	1	4088.00	0.00	8.28
he4	4088.00	0.00	0.08	4088.00	0.00	0.73	0	1	4088.00	0.00	61.75
he5	12430.21	0.00	0.80	12430.21	0.00	2.30	1	2	12430.21	0.45	3600.00
he6	4088.00	0.00	0.22	4088.00	0.00	1.03	0	1	4088.00	0.00	41.13
he7	4088.00	0.00	0.16	4088.00	0.00	0.80	0	1	4088.00	0.00	143.44
he8	4088.00	0.00	0.12	4088.00	0.00	1.03	0	1	4088.00	0.00	71.19
he9	4088.00	0.00	0.12	4088.00	0.00	0.89	0	1	4088.00	0.00	42.69
Avg.		0.00	0.21		0.00	0.98	0.22	1.22		0.05	453.00

Table 8: Results obtained from BDA on the instances in c101c10, c104c10, c202c10, and c205c10

	BDA Phase I			BDA Phase II			$N^S$	$N^V$
	Obj	$g_1$	$t(s)$	Obj	$g_2$	$t_2(s)$		
c101c10								
he1				INFEASIBLE				
he2	40941.67	0.00	2.67	40941.67	0.00	8.23	1	2
he3	26734.90	0.00	54.56	26734.90	0.00	119.38	2	2
he4	8722.39	0.00	35.20	8722.39	0.00	81.23	0	3
he5	54369.66	0.00	44.41	54369.66	0.00	82.00	3	3
he6	9722.39	0.00	149.34	9722.39	0.00	362.69	0	3
he7	8722.39	0.00	74.21	8722.39	0.00	167.81	0	3
he8	8722.39	0.00	80.41	8722.39	0.00	235.86	0	3
he9	8029.19	0.00	31.71	8029.19	0.00	130.22	0	2
Avg.		0.00	59.06		0.00	148.43	0.75	2.63
c104c10								
he1	97436.74	0.00	10.68	97436.74	0.00	236.67	2	2
he2	38005.50	0.00	6.43	38005.50	0.00	10.16	1	2
he3	16067.65	0.00	19.44	16067.65	0.00	45.50	1	2
he4	8684.17	0.00	54.07	8684.17	0.00	92.60	0	3
he5	46212.79	0.00	9.78	46212.79	0.00	30.99	2	3
he6	9684.17	0.00	767.48	9684.17	0.00	1380.00	0	3
he7	8684.17	0.00	137.41	8684.17	0.00	258.47	0	3
he8	8684.17	0.00	368.60	8684.17	0.00	973.49	0	3
he9	8014.08	0.00	77.37	8014.08	0.00	217.72	0	2
Avg.		0.00	161.25		0.00	360.62	0.67	2.56
c202c10								
he1				INFEASIBLE				
he2	7500.43	0.00	3.61	7500.43	0.00	9.84	1	2
he3	6908.44	0.00	0.59	6908.44	0.00	3.19	0	2
he4	6908.44	0.00	28.12	6908.44	0.00	48.98	0	2
he5	19412.86	0.00	87.14	19412.86	0.00	193.21	3	3
he6	6908.44	0.00	1.31	6908.44	0.00	6.18	0	2
he7	6908.44	0.00	36.91	6908.44	0.00	94.29	0	2
he8	6908.44	0.00	31.25	6908.44	0.00	64.41	0	2
he9	6908.44	0.00	38.63	6908.44	0.00	64.00	0	2
Avg.		0.00	28.45		0.00	60.51	0.50	2.13
c205c10								
he1	74514.96	0.00	8.65	74514.96	0.00	334.92	2	2
he2	19463.88	0.00	9.43	19463.88	0.00	21.18	2	2
he3	6872.57	0.00	5.57	6872.57	0.00	13.28	0	2
he4	6872.57	0.00	8.92	6872.57	0.00	23.75	0	2
he5	17362.48	0.00	9.18	17362.48	0.00	73.07	1	3
he6	6872.57	0.00	21.55	6872.57	0.00	50.01	0	2
he7	6872.57	0.00	15.95	6872.57	0.00	37.25	0	2
he8	6872.57	0.00	65.64	6872.57	0.00	114.23	0	2
he9	6872.57	0.00	38.54	6872.57	0.00	68.58	0	2
Avg.		0.00	20.38		0.00	81.81	0.56	2.11

Table 9: Results obtained from BDA on the instances in r102c10, r103c10, r203c10, and r203c10

	BDA Phase I			BDA Phase II			$N^S$	$N^V$
	Obj	$g_1$	$t(s)$	Obj	$g_2$	$t_2(s)$		
r102c10								
he1	22490.81	0.00	9.62	22490.81	0.00	13.40	1	2
he2	5805.69	0.00	4.28	5805.69	0.00	10.70	0	2
he3	6805.69	0.00	21.03	6805.69	0.00	29.08	0	2
he4	5805.69	0.00	5.93	5805.69	0.00	14.73	0	2
he5	6527.95	0.00	25.67	6527.95	0.00	43.71	0	3
he6	5827.42	0.00	6.91	5827.42	0.00	17.65	0	2
he7	5805.69	0.00	15.70	5805.69	0.00	30.22	0	2
he8	5805.69	0.00	16.18	5805.69	0.00	35.20	0	2
he9	5805.69	0.00	27.35	5805.69	0.00	46.29	0	2
Avg.		0.00	14.74		0.00	26.78	0.11	2.11
r103c10								
he1	4646.67	0.00	0.80	4646.67	0.00	2.62	0	2
he2	4029.23	0.00	0.77	4029.23	0.00	2.93	0	1
he3	4029.23	0.00	0.28	4029.23	0.00	1.75	0	1
he4	4029.23	0.00	3.26	4029.23	0.00	9.27	0	1
he5	4646.67	0.00	6.60	4646.67	0.00	10.99	0	2
he6	4029.23	0.00	1.88	4029.23	0.00	5.38	0	1
he7	4029.23	0.00	2.90	4029.23	0.00	14.21	0	1
he8	4029.23	0.00	4.53	4029.23	0.00	12.32	0	1
he9	4029.23	0.00	3.68	4029.23	0.00	10.48	0	1
Avg.		0.00	2.74		0.00	7.77	0.00	1.22
r201c10								
he1	INFEASIBLE							
he2	5746.83	0.00	7.69	5746.83	0.00	14.02	0	2
he3	6746.83	0.00	13.08	6746.83	0.00	27.62	0	2
he4	5746.83	0.00	21.96	5746.83	0.00	48.73	0	2
he5	6397.82	0.00	471.62	6397.82	0.00	762.24	0	3
he6	5765.39	0.00	37.05	5765.39	0.00	69.45	0	2
he7	5746.83	0.00	167.91	5746.83	0.00	330.79	0	2
he8	5746.83	0.00	159.66	5746.83	0.00	293.70	0	2
he9	5746.83	0.00	99.04	5746.83	0.00	243.71	0	2
Avg.		0.00	122.25		0.00	223.78	0.00	2.13
r203c10								
he1	41633.03	0.00	8.23	41633.03	0.00	31.42	2	2
he2	11554.98	0.00	8.75	11554.98	0.00	16.15	1	2
he3	6889.35	0.00	8.83	6889.35	0.00	16.64	0	2
he4	6889.35	0.00	32.80	6889.35	0.00	55.27	0	2
he5	10279.95	0.00	94.86	10279.95	0.00	154.09	1	3
he6	6889.35	0.00	16.25	6889.35	0.00	59.59	0	2
he7	6889.35	0.00	88.40	6889.35	0.00	166.85	0	2
he8	6889.35	0.00	108.97	6889.35	0.00	266.08	0	2
he9	6889.35	0.00	97.37	6889.35	0.00	180.74	0	2
Avg.		0.00	51.61		0.00	105.20	0.44	2.11

Table 10: Results obtained from BDA on the instances in rc102c10, rc108c10, rc201c10, and rc205c10

	BDA Phase I			BDA Phase II			$N^S$	$N^V$
	Obj	$g_1$	$t(s)$	Obj	$g_2$	$t_2(s)$		
rc102c10								
he1	INFEASIBLE							
he2	76690.66	0.00	5.17	76690.66	0.00	14.13	3	2
he3	37865.41	0.00	2.88	37865.41	0.00	6.18	1	2
he4	8828.98	0.00	1.19	8828.98	0.00	5.70	0	3
he5	86013.10	0.00	5.67	86013.10	0.00	14.81	3	3
he6	9828.98	0.00	29.86	9828.98	0.00	76.39	0	3
he7	8828.98	0.00	2.89	8828.98	0.00	7.95	0	3
he8	8828.98	0.00	7.58	8828.98	0.00	15.26	0	3
he9	8828.98	0.00	26.74	8828.98	0.00	47.55	0	3
Avg.		0.00	10.25	30714.26	0.00	23.50	0.88	2.75
rc108c10								
he1	124573.00	0.00	10.99	124573.00	0.00	104.82	2	2
he2	82773.00	0.00	11.09	82773.00	0.00	42.82	1	2
he3	42320.90	0.00	15.98	42320.90	0.00	63.07	2	2
he4	12013.54	0.00	50.80	12013.54	0.00	114.87	1	3
he5	86279.65	0.00	36.33	86279.65	0.00	3593.30	2	3
he6	9872.52	0.00	78.42	9872.52	0.00	208.28	0	3
he7	10613.42	0.00	100.28	10613.42	0.00	262.38	0	4
he8	9872.52	0.00	89.67	9872.52	0.00	195.21	0	3
he9	8167.99	0.00	3.97	8167.99	0.00	10.44	0	2
Avg.		0.00	44.17	42942.95	0.00	510.58	0.89	2.67
rc201c10								
he1	67397.81	0.00	7.92	67397.81	0.00	14.57	2	2
he2	29172.61	0.00	7.32	29172.61	0.00	22.68	1	2
he3	6970.88	0.00	4.83	6970.88	0.00	11.10	0	2
he4	6970.88	0.00	2.84	6970.88	0.00	9.11	0	2
he5	32631.14	0.00	23.79	32631.14	0.00	124.47	2	3
he6	6970.88	0.00	5.39	6970.88	0.00	17.00	0	2
he7	6970.88	0.00	3.81	6970.88	0.00	12.93	0	2
he8	6970.88	0.00	6.01	6970.88	0.00	17.10	0	2
he9	6970.88	0.00	13.50	6970.88	0.00	26.70	0	2
Avg.		0.00	8.38	19002.98	0.00	28.41	0.56	2.11
rc205c10								
he1	114637.14	0.00	3.42	114637.14	0.00	54.62	2	2
he2	73637.14	0.00	8.80	73637.14	0.00	38.19	2	2
he3	32637.14	0.00	7.61	32637.14	0.00	15.59	2	2
he4	11302.91	0.00	37.63	11302.91	0.00	63.07	1	3
he5	74017.00	0.00	14.75	74017.00	0.00	271.92	3	3
he6	9805.75	0.00	102.60	9805.75	0.00	198.58	0	3
he7	11302.91	0.00	167.22	11302.91	0.00	346.33	1	3
he8	9805.75	0.00	245.76	9805.75	0.00	486.49	0	3
he9	8123.37	0.00	3.33	8123.37	0.00	9.35	0	2
Avg.		0.00	65.68	38363.23	0.00	164.90	1.22	2.56

Table 11: Results obtained from BDA on the instances in c103c15, c106c15, c202c15, and c208c15

	BDA Phase I			BDA Phase II			$N^S$	$N^V$
	Obj	$g_1$	$t(s)$	Obj	$g_2$	$t_2(s)$		
c103c15								
he1	INFEASIBLE							
he2	35972.13	0.00	61.99	35972.13	0.00	135.28	2	2
he3	15627.39	0.00	304.52	15627.39	0.00	730.97	1	2
he4	8753.00	0.00	614.77	8753.00	0.00	1807.29	0	3
he5	42487.92	0.31	10800.00	Memory Limit				
he6	9705.97	0.20	10800.15	9705.97	0.16	21609.34	0	3
he7	8753.00	0.00	2614.33	8753.00	0.00	7028.82	0	3
he8	8753.00	0.09	10800.23	8753.00	0.07	10800.52	0	3
he9	8033.82	0.00	2396.53	8033.82	0.00	4213.18	0	2
Avg.		0.07	4799.07		0.03	6617.91	0.43	2.57
c106c15								
he1	40236.32	0.00	29.72	40236.32	0.00	97.25	1	2
he2	5857.67	0.00	6.23	5857.67	0.00	15.85	0	2
he3	6857.67	0.00	175.43	6857.67	0.00	338.04	0	2
he4	5857.67	0.00	46.15	5857.67	0.00	113.44	0	2
he5	6546.65	0.00	45.72	6546.65	0.00	241.86	0	3
he6	6857.67	0.00	4675.78	6857.67	0.00	5472.12	0	2
he7	5857.67	0.00	79.12	5857.67	0.00	253.03	0	2
he8	5857.67	0.00	97.94	5857.67	0.00	279.20	0	2
he9	5857.67	0.00	137.21	5857.67	0.00	338.73	0	2
Avg.		0.00	588.14		0.00	794.39	0.11	2.11
c202c15								
he1	INFEASIBLE							
he2	78027.86	0.00	76.13	78027.86	0.00	209.37	2	2
he3	46465.93	0.00	64.59	46465.93	0.00	333.44	2	2
he4	11821.63	0.00	1108.28	11821.63	0.00	1716.57	1	3
he5	101057.82	0.00	812.10	101057.82	0.00	10029.75	3	3
he6	9811.94	0.00	7931.55	9811.94	0.00	13665.04	0	3
he7	10530.88	0.00	9859.18	10530.88	0.00	15680.36	0	4
he8	9811.94	0.00	7968.24	9811.94	0.00	13957.72	0	3
he9	8153.33	0.00	14.70	8153.33	0.00	91.24	0	2
Avg.		0.00	3479.35		0.00	6960.44	1.00	2.75
c208c15								
he1	INFEASIBLE							
he2	34116.90	0.00	3.82	34116.90	0.00	15.60	1	2
he3	11927.76	0.00	312.81	11927.76	0.00	505.10	1	2
he4	8683.64	0.00	1528.66	8683.64	0.00	5650.08	0	3
he5	66401.92	0.00	426.32	66401.92	0.00	1044.02	3	3
he6	9683.64	0.17	10800.16	9683.64	0.18	21944.58	0	3
he7	8683.64	0.00	4758.60	8683.64	0.00	13249.26	0	3
he8	8683.64	0.05	10800.20	8683.64	0.00	19530.75	0	3
he9	8012.56	0.00	4773.94	8012.56	0.00	8767.81	0	2
Avg.		0.03	4175.56		0.02	8838.40	0.63	2.63



Table 12: Results obtained from BDA on the instances in r102c15, r105c15, r202c15, and r209c15

	BDA Phase I			BDA Phase II			$N^S$	$N^V$
	Obj	$g_1$	$t(s)$	Obj	$g_2$	$t_2(s)$		
r102c15								
he1	INFEASIBLE							
he2	19772.60	0.00	185.56	19772.60	0.00	382.04	2	2
he3	6956.29	0.00	25.19	6956.29	0.00	62.01	0	2
he4	6956.29	0.00	108.55	6956.29	0.00	189.70	0	2
he5	24632.65	0.97	9128.06	Memory limit				
he6	6956.29	0.00	136.97	6956.29	0.00	366.41	0	2
he7	6956.29	0.00	354.79	6956.29	0.00	637.32	0	2
he8	6956.29	0.00	787.49	6956.29	0.00	1048.69	0	2
he9	6956.29	0.00	906.36	6956.29	0.00	2164.50	0	2
Avg.		0.12	1454.12		0.00	692.95	0.29	2.00
r105c15								
he1	INFEASIBLE							
he2	21864.09	0.00	164.84	21864.09	0.00	377.97	1	2
he3	6968.14	0.00	45.07	6968.14	0.00	84.90	0	2
he4	6968.14	0.00	176.11	6968.14	0.00	462.12	0	2
he5	22441.59	0.00	861.12	22441.59	0.00	1845.61	2	3
he6	6968.14	0.00	236.61	6968.14	0.00	659.36	0	2
he7	6968.14	0.00	621.14	6968.14	0.00	1626.27	0	2
he8	6968.14	0.00	1326.38	6968.14	0.00	3560.06	0	2
he9	6968.14	0.00	743.83	6968.14	0.00	1693.28	0	2
Avg.		0.00	521.89		0.00	1288.70	0.38	2.13
r202c15								
he1	INFEASIBLE							
he2	54739.58	0.00	70.94	54739.58	0.00	168.63	2	2
he3	9126.41	0.00	80.11	9126.41	0.00	171.37	2	2
he4	8766.77	0.00	775.92	8766.77	—	21600.00	0	3
he5	INFEASIBLE							
he6	9126.41	0.07	10800.17	9126.41	0.04	21631.12	2	2
he7	8766.77	0.00	2045.60	8766.77	0.00	4351.67	0	3
he8	8766.77	0.07	10800.58	8766.77	0.06	22225.61	0	3
he9	8076.85	0.00	901.42	8076.85	0.00	1592.38	0	2
Avg.		0.02	3639.25		0.02	10248.68	0.86	2.43
r209c15								
he1	INFEASIBLE							
he2	22048.46	0.00	56.94	22048.46	0.00	125.10	2	2
he3	6967.49	0.00	2.98	6967.49	0.00	13.33	0	2
he4	6967.49	0.00	76.41	6967.49	0.00	124.19	0	2
he5	31800.31	0.00	99.90	31800.31	0.00	267.06	2	3
he6	6967.49	0.00	6.41	6967.49	0.00	19.14	0	2
he7	6967.49	0.00	144.45	6967.49	0.00	318.28	0	2
he8	6967.49	0.00	152.79	6967.49	0.00	290.81	0	2
he9	6967.49	0.00	131.37	6967.49	0.00	223.50	0	2
Avg.		0.00	83.91		0.00	172.68	0.50	2.13

— Not available. The model hits the time limit before finding a feasible solution.

Table 13: Results obtained from BDA on the instances in rc103c15, rc108c15, rc202c15, and rc204c15

	BDA Phase I			BDA Phase II			$N^S$	$N^V$
	Obj	$g_1$	$t_1(s)$	Obj	$g_2$	$t_2(s)$		
rc103c15								
he1	INFEASIBLE							
he2	83276.41	0.00	111.75	83276.41	0.53	10921.09	2	2
he3	41149.97	0.00	359.31	41149.97	0.00	1591.95	2	2
he4	14115.34	0.00	5613.98	14115.34	0.00	13561.48	1	3
he5	83831.10	0.00	845.83	83831.10	0.00	3035.31	3	3
he6	9836.81	0.57	10800.00	9836.81	0.66	21636.20	0	3
he7	10583.00	0.14	10800.00	10583.00	0.29	18699.63	0	4
he8	9836.81	0.54	10800.00	9836.81	0.94	22227.84	0	3
he9	8132.34	0.00	75.90	8132.34	0.00	286.11	0	2
Avg.		0.16	4925.85		0.30	11494.95	1.00	2.75
rc108c15								
he1	INFEASIBLE							
he2	124758.58	0.00	746.79	124758.58	0.00	11555.48	2	2
he3	62132.20	0.00	180.29	62132.20	0.00	11028.02	2	2
he4	16428.49	0.00	376.09	16428.49	0.00	1251.25	2	3
he5	INFEASIBLE							
he6	9857.04	0.00	1466.78	9857.04	0.00	1980.25	0	3
he7	14281.31	0.13	10800.26	14281.31	0.00	18935.20	1	4
he8	9857.04	0.00	1166.45	9857.04	0.00	2254.59	0	3
he9	8143.23	0.00	27.66	8143.23	0.00	54.27	0	2
Avg.		0.02	2109.19		0.00	6722.72	1.00	2.71
rc202c15								
he1	INFEASIBLE							
he2	79004.57	0.00	5.16	79004.57	0.00	3058.47	1	2
he3	42795.42	0.00	162.46	42795.42	0.00	761.17	2	2
he4	19597.07	0.00	1400.59	19597.07	0.00	4199.88	1	3
he5	94305.77	0.14	Memory limit					
he6	12557.00	0.24	10800.24	12557.00	0.25	21615.56	1	3
he7	16636.76	0.43	10800.33	16636.76	0.42	21961.32	1	4
he8	11554.38	0.20	10800.20	11554.38	0.25	23013.04	0	4
he9	9840.74	0.06	10800.48	9840.74	0.08	21789.51	0	3
Avg.		0.13	6395.64		0.14	13771.28	0.86	3.00
rc204c15								
he1	INFEASIBLE							
he2	INFEASIBLE							
he3	35895.95	0.00	12.45	35895.95	0.00	94.25	1	2
he4	19828.42	0.00	1617.84	19828.42	—	14614.88	1	3
he5	INFEASIBLE							
he6	9890.67	0.00	7576.35	9890.67	0.00	11903.85	0	3
he7	13888.20	0.26	10800.33	13888.20	0.30	21841.39	1	4
he8	9890.67	0.04	10800.74	9890.67	0.00	16407.53	0	3
he9	10853.21	0.19	10800.40	10853.21	0.20	22006.29	0	3
Avg.		0.08	6934.69		0.10	14450.66	0.50	3.00

— Not available. The model hits the time limit before finding a feasible solution.