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The electric location-routing problem: Formulation and Benders decomposition approach

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Abstract

In this paper, we focus on a problem that requires location of recharging stations and routing of electric vehicles in a goods distribution system. The goods are disseminated from a depot and distributed to the customers via electric vehicles with limited capacity. Differently from the classical vehicle routing problem, the vehicles have battery restrictions that need to be recharged at some stations if a trip is longer than their range. The problem reduces to finding the optimal location of the recharging stations and their number to minimize the total cost, which includes the routing cost, the recharging cost, and the fixed costs of opening stations and operating vehicles. We propose a novel mathematical formulation and an efficient Benders decomposition algorithm to solve this environmental logistics problem. Our methods solve the problem in a general setting with non-identical stations and vehicles by allowing multiple visits to the stations and partial recharging.

Keywords: Location, Electric Vehicle Routing, Benders Decomposition, Integer Linear Programming, Environmental Logistics

1 Introduction

The transport sector is responsible, to a large extent, for energy consumption and greenhouse gas emissions. According to the European Environment Agency (2017), energy consumption increases by 25% from 1990 to 2015 in the EU-28. To tackle environmental and energy challenges in this sector, several countries around the world are considering the prospect of carbon neutrality over the next 30 years, with the objective of stopping the sale of vehicles emitting greenhouse gases. This objective has already begun with the introduction of low-emission zones (LEZ), where the

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most polluting vehicles are regulated. In such zones vehicles with higher emissions either cannot enter the area or have to pay more if they enter the low emission zone. For instance, the traffic pollution charge in London LEZ is £100 per day for larger vans and minibuses and rises to double this amount for lorries, buses, and coaches etc. Vehicles with alternative fuels such as electric vehicles and hydrogen vehicles, are credible solutions for achieving the carbon neutrality target.

Unlike the hydrogen vehicle, which is currently at the experimental stage, and consequently having an exorbitant cost, the electric vehicle has reached an industrial maturity that makes it competitive compared to the combustion vehicle. However, electric vehicle (EV) is still facing weaknesses related to their availability and their battery management.

From a logistics point of view, the first weakness concerns the limited choice of light duty electric vehicles in terms of capacity and volume offered by car industry that are mainly needed in the last mile logistics. The second weakness is the limited electric vehicle driving range. For instance, for light duty electric vehicles, the range is between 120km and 180km. Note that the range can depend on topology of the road, weather and driving conditions. The third weakness is related to the long charging time of electric vehicles where the time to fully charge a vehicle can take up to 8 hours depending on the capacity of the battery pack and chargers level. The last weakness concerns the lack of availability of charging infrastructures in existing road networks where the establishment of new facilities is crucial.

Although all these weaknesses are manageable in practice, the cost of electric vehicle presents a barrier to their extensive use. An opportunity to reduce vehicle price is focusing on the developments on those markets that are ready to adopt such a green based strategy. Such markets allow a large-scale production of electric vehicles which can consequently lead to the reduction of vehicle costs. Last miles logistic transportation provides this opportunity to speed up the market penetration of electric vehicles. In such market, an electric vehicle has the advantage meeting the requirement of low-emission zones that are mainly located in city centers as the distances covered in last mile logistic are either within its limit range or requires one charging session along the route only. Furthermore, even though the acquisition cost for electric vehicles is usually higher than the combustion engine vehicles, this difference can be offset at the operational cost of usage of electric vehicle. This is because a high utilization of electric vehicle favors their TCO (Total Cost Ownership) since their operating costs (maintenance, tax, fuel, and depreciation) are low compared to those of their counterparts.

In this paper, we consider a goods distribution system that utilizes electric vehicles. We assume there is a sufficient number of charging stations and electrical grid capacity so that all vehicles are fully charged before their departure from the depot. However, we may need to recharge them during their trips if the total energy consumption to visit certain customers is larger than the battery capacity. Once a station is open, it might be visited multiple times by any vehicle and the vehicles do not need to be fully recharged as we allow partial recharging.

Besides, we do not impose any restrictions on the types of stations or vehicles. In other words, we allow the utilization of slow or fast charging stations as well as the use of heterogeneous vehicles.

The problem is to decide on the number and location of stations, the number of vehicles needed, the amount of recharging needed for each vehicle, and the route(s) for visiting all the customers. The objective is to minimize the total cost including the variable cost of routing and recharging as well as the fixed costs of opening stations and operating vehicles.

Our contributions are twofold:

- to propose a new mixed integer programming formulation for this strategic electric location-

routing problem and

- to develop a Benders decomposition algorithm based on our formulation to solve the problem optimally.

The rest of the paper is organized as follows: Section 2 gives an informative review on the related works. In Section 3, we provide the notation used throughout the paper and present our mathematical formulation. In Section 4, we propose our Benders decomposition algorithm followed by Sections 5 and 6 describing the implementation and the intermediate processing respectively. In Section 7, we provide the setting and present the results of our computational study. We conclude in Section 8 with a summary of our findings and a highlight of some future research directions.

2 Related work

Location of recharging stations can be seen as a facility location problem. The purpose is then to decide on the optimal number and location of facilities given the position of customers to serve. In this vein, He et al. (2016) present a study case in Beijing, China. Their objectives are to incorporate the local constraints of supply and demand on public electric vehicle charging stations into facility location models, and to compare the optimal locations from three different location models: the set covering model, the maximal covering location model, and the p-median model. Liu and Wang (2017) address the optimal location of multiple types of charging facilities, including dynamic wireless charging facilities and different levels of plug-in charging stations. Their tri-level programming first treats the model as a black-box optimization, which is then solved by an efficient surface response approximation model based solution algorithm.

However, as raised in Salhi and Rand (1989), facility location and routing decisions are interdependent and should be tackled simultaneously. In the general case where both vehicles and depots are capacitated, the problem is known as the capacitated location routing problem (CLRP). The aim here is to i) define which depots must be opened, ii) assign each serviced node (customer) to one and only one depot and, iii) route the vehicle to serve the nodes, in such a way that the sum of the depot cost and the total routing cost is minimized. Many papers appeared in the subject and more particularly during the last decade, as shown in surveys by Nagy and Salhi (2007); Prodhon and Prins (2014), and Schneider and Drexler (2017). To solve this NP-hard problem, exact methodologies such as branch-and-cut algorithm (Belenguer et al., 2011) and set partitioning based exact methods (Akca et al., 2009; Contardo et al., 2013), are limited to medium-scale instances. To solve larger instances, new efficient metaheuristics have been proposed. These include a cooperative Lagrangean relaxation-granular tabu search heuristic by Prins et al. (2007), an adaptive large-neighborhood search (ALNS) by Hemmelmayr et al. (2012), and a three-phase matheuristic by Contardo et al. (2014). Other studies cover a multiple ant colony optimization algorithm (Ting and Chen, 2013) and a two-phase hybrid heuristic (Escobar et al., 2013). Very recently, a tree-based search algorithm by Schneider and Löffler (2017) and a Genetic Algorithm by Lopes et al. (2016) are proposed.

Despite the interest of LRP, integrating issues from electric vehicles is scarce and worth exploring. A first reason may come from the fact that in LRP, a route should end at the same depot as its departure. However, this kind of constraint is not pertinent when the location stands on recharging stations since they become an intermediate stop in the route. Similar models that use such satellite depots can be found in the Truck and Trailer problem (Villegas et al., 2013; Parragh and Cordeau, 2017). In this particular problem, when a vehicle leave a satellite, it should go back

to it before continuing its route, leading to a long trip including sub-tours. A closer model is the vehicle routing problem (VRP) with intermediate depots as described by Schneider et al. (2015) but the location aspect is not considered as part of the decisions.

The integration of the location of recharging stations with the routing decision, also called electric location-routing problem (ELRP), is relatively recent though it can lead to a massive environmental impact. To the best of our knowledge, the first study of simultaneous vehicle routing and charging station siting for commercial electric vehicles is presented in a conference paper in 2012 by Worley et al. (2012). Then, Yang and Sun (2015) introduce the interesting battery swap station location-routing problem, where the charge is completely fulfilled at each stop. The authors develop two heuristic approaches. The problem is revisited by Hof et al. (2017) who adapt an AVNS originally dedicated to the VRP with intermediate depots.

The first paper dealing with partial recharge may come from Felipe et al. (2014), and is dedicated to a Green Vehicle Routing Problem (G-VRP). In G-VRP the fleet is composed of Alternative Fuel Vehicles (AFV) where, in addition to the routing of each electric vehicle, the amount of energy recharged and the technology used must also be determined. However, the location aspect is not considered. Constructive and improving heuristics are embedded in a Simulated Annealing framework. The partial recharging policies are then reused showing that they may considerably improve the routing decisions as noted by Keskin and Çatay (2016). Thus, Schiffer and Walther (2017a) extend the problem by including siting charging stations which leads to the electric location routing problem with time windows and partial recharging (ELRP-TWPR). The authors focus on a problem with a single type of vehicle and single type of station with time windows. They also consider partial recharging and multiple visits to the stations. They propose a mathematical formulation based on Miller-Tucker-Zemlin type constraints, supported by several preprocessing steps to eliminate the arcs that violate time windows, capacity, and battery restriction constraints. They compare the formulation with five different objective functions minimizing (i) the total driven distance, (ii) the number of vehicles used, (iii) the number stations sited (iv), a convex combination of the number of vehicles used and the number of stations sited, and (v) the total cost. The latter includes the vehicle costs, station costs, and the routing cost. Regarding the problem with the objective of minimizing the total cost, the authors are able to solve most problems with 5 nodes to optimality within two hours of time limit. Among the test instances of 10 and 15 nodes, their model provides a solution with proven optimality for only two of the instances within the time limit. However, the optimality gap at the end of two hours is found to be less than 0.1% for any problem instance tested. Later, the authors extend their work to consider a robust location-routing problem with strategic planning of electric logistics (Schiffer and Walther, 2018). The Location Routing Problem with Intraroute Facilities which is a generalization of the ELRP-TWPR is explored by Schiffer and Walther (2017b) where large instances are solved using an ALNS which is enhanced by local search and dynamic programming components.

Our problem can be considered as an electric vehicle routing problem with location decisions or an electric location-routing problem (ELRP) with a heterogeneous fleet, multi-type stations, multi-visit, and partial recharging. In the next section we provide the notation and a mathematical formulation of the problem.

3 Notation and Problem Formulation (PF)

We are given a network $G = (N, A)$ with arc set A and node set $N = I \cup J \cup \{0\}$ where I is the set of customer locations, J is the set of potential locations for charging stations, and '0' is a depot node. We are required to select a subset of J to locate recharging stations. Each customer should be served by a vehicle originating from the depot and each vehicle can perform a single trip. The vehicles have a battery restriction and they have to visit charging stations before the battery is depleted if a trip longer than their range is to be traversed. In addition, we consider the vehicles to have restricted capacities.

Below we list the following parameters:

K : the set of vehicles.

d_i : the demand of client $i \in I$.

c_{ij} : the routing cost of traversing arc $(i, j) \in A$.

e_{ij} : the energy consumption on arc $(i, j) \in A$.

f_j : the fixed cost of opening a charging station at node $j \in J$.

r_k : the unit cost of recharging for vehicle $k \in K$.

v_k : the fixed cost of operating vehicle $k \in K$.

Q^k : the capacity of vehicle $k \in K$.

β^k : the battery restriction of vehicle $k \in K$.

We further define the following decision variables:

$y_j = 1$ if station $j \in J$ is open, 0 otherwise.

$x_{ij}^k = 1$ if arc (i, j) is traversed by vehicle $k \in K$, 0 otherwise.

z_j^k is the amount of recharging units at station $j \in J$ for vehicle $k \in K$.

b_{ij}^k is the battery level of vehicle $k \in K$ at node $i \in I$ before leaving for node $j \in I$.

l_{ij}^k is the load of vehicle $k \in K$ at node $i \in N$ before leaving for node $j \in N$.

3.1 Catering for multiple visits

In order to allow multiple visits to a station, we perform the following modification on our input network:

Step 1: Let m be the number of demand nodes.

Step 2: Create m copies of each station,

Step 3: Form set $J_j^A = \{j_1, j_2, \dots, j_m\}$ for each $j \in J$ and $J^A = \bigcup_{j \in J} J_j^A$.

Step 4: For each $j \in J$, set $f_{j_1} = f_j$; $d_{j_1} = d_j$ and $f_{j_i} = d_{j_i} = 0, i = 2, \dots, m$ where $j_1, \dots, j_m \in J_j^A$.

Step 5: Let $N^E = I \cup J^A \cup \{0\}$ and $A^E = A \cup (i, j) : i, j \in N^E; i \neq j; \neg(i, j \in J_l^A \text{ for some } l \in J)$.

Step 6: Define $A^k = \{(i, j) \in A^E : e_{ij} \leq \beta^k; d_i + d_j \leq Q^k\}$ for $k \in K$.

Our formulation PF is given in Section 3.2.

3.2 The PF Formulation

$$\min \sum_{k \in K} \sum_{(i,j) \in A^k} c_{ij} x_{ij}^k + \sum_{k \in K} \sum_{j \geq 1} r_k z_j^k + \sum_{j \geq 1} f_j y_j + \sum_{k \in K} \sum_{(0,i) \in A^k} v_k x_{0i}^k \quad (1)$$

$$\text{s.t. } y_i \leq y_j, \quad i \in J_j^A : i \neq j \quad (2)$$

$$\sum_{i \geq 1} x_{0i}^k \leq 1, \quad \forall k \in K \quad (3)$$

$$\sum_{k \in K} \sum_{(j,i) \in A^k} x_{ji}^k = 1, \quad i \geq 1 : d_i > 0 \quad (4)$$

$$\sum_{(j,i) \in A^k} x_{ji}^k \leq y_i, \quad i \geq 2 : d_i = 0, \forall k \in K \quad (5)$$

$$\sum_{(i,j) \in A^k} x_{ij}^k - \sum_{(j,i) \in A^k} x_{ji}^k = 0, \quad i \geq 1, \forall k \in K \quad (6)$$

$$\sum_{(i,j) \in A^k} (l_{ij}^k - d_i x_{ij}^k) = \sum_{(j,i) \in A^k} l_{ji}^k, \quad i \geq 1, \forall k \in K \quad (7)$$

$$\sum_{j \geq 1} l_{0j}^k = 0, \quad \forall k \in K \quad (8)$$

$$l_{ij}^k \leq Q^k x_{ij}^k, \quad \forall k \in K, (i, j) \in A^k \quad (9)$$

$$\sum_{(i,j) \in A^k} e_{ij} x_{ij}^k - \sum_{j \geq 1} z_j^k \leq \beta^k, \quad \forall k \in K \quad (10)$$

$$\sum_{(i,j) \in A^k} b_{ij}^k = \sum_{(j,i) \in A^k} (b_{ji}^k - e_{ji} x_{ji}^k) + z_i^k, \quad \forall i \geq 1, k \in K \quad (11)$$

$$z_j^k \leq \beta^k y_j, \quad j \geq 1, \forall k \in K \quad (12)$$

$$z_j^k \leq \beta^k \sum_{(i,j) \in A^k} x_{ij}^k, \quad j \geq 1, \forall k \in K \quad (13)$$

$$b_{0j}^k = \beta^k x_{0j}^k, \quad k \in K, (0, j) \in A^k \quad (14)$$

$$b_{ij}^k \leq \beta^k x_{ij}^k, \quad k \in K, (i, j) \in A^k \quad (15)$$

$$b_{ij}^k \geq e_{ij} x_{ij}^k, \quad k \in K, (i, j) \in A^k \quad (16)$$

$$y_j \in \{0, 1\}, \quad j \geq 1 \quad (17)$$

$$x_{ij}^k \in \{0, 1\}, \quad k \in K, (i, j) \in A^k \quad (18)$$

$$l_{ij}^k \geq 0, \quad k \in K, (i, j) \in A^k \quad (19)$$

$$b_{ij}^k \geq 0, \quad k \in K, (i, j) \in A^k \quad (20)$$

$$z_j^k \geq 0, \quad j \geq 1, k \in K. \quad (21)$$

The objective function (1) minimizes the total sum of routing costs, charging costs, fixed costs of opening stations, and fixed cost of using vehicles. If a zero-demand copy of a station is opened, Constraints (2) force the original copy of this node to be opened and therefore, ensure that the costs of the stations are counted in the objective function. By Constraints (3), we restrict the number of trips by each vehicle to at most one. Constraints (3)-(6) together ensure that each client is served by a unique vehicle trip that starts at the depot and the capacities of vehicles are respected. Constraints (5) ensure that a zero-demand copy of any station is visited only if that station is open. We ensure the elimination of sub-tours for each vehicle trip via the load balance constraints (7)-(9). Battery restriction on the vehicles are imposed by Constraints (10) and (11). Constraints (12) and (13) avoid recharging of a vehicle at a node that has no station and that is not visited by that vehicle, respectively.

We initialize the battery level for each vehicle to 100% by Constraints (14). For each arc-vehicle pair, Constraints (15) restrict the amount of battery level with full battery level if the arc is traversed by the vehicle and set it to zero otherwise; Constraints (16) make sure that the battery level is larger than the energy consumption on the arc that will be traversed by the vehicle. Finally, Constraints (17)-(21) represent the binary and non-negativity restrictions on the decision variables.

4 Benders Decomposition Algorithm (BDA)

Our mathematical formulation can be solved by using a Benders decomposition (Benders, 1962) framework that we briefly describe here before presenting the details of our algorithm. The classical Benders decomposition method aims to solve a mixed integer program (MIP) with a group of integer variables and a group of continuous variables by decomposing the MIP into a master problem (MP) with all integer variables and a series of subproblems (SP) of continuous variables. For each feasible solution of MP, we construct a subproblem which is equivalent to the MIP where all the integer variables are fixed to the value obtained from the master problem. Then, from each extreme ray and extreme point of the dual of this SP, we obtain a so called feasibility and an optimality cut, respectively, for the MP. Since enumeration of the extreme points and extreme rays might be impractical, the cutting plane procedures are usually employed for the generation and the addition of these cuts.

The classical Benders decomposition method might suffer from slow convergence especially if the subproblem is difficult to solve. On the other hand, the method might perform very efficiently if the subproblem can be decomposed further into smaller and easy-to-solve problems as in multi-commodity, multi-period, or multi-scenario problems (Birge and Louveaux, 2011). Motivated by this fact, we aim to further decompose our subproblem into $|K|$ smaller problems, each one corresponding to a single vehicle trip. For this purpose, we decide to keep \mathbf{y} , \mathbf{x} , \mathbf{l} variables in the master problem and \mathbf{z} , \mathbf{b} variables in the subproblems. In our implementation, we remove the optimality cuts, which are known to slow down the convergence of the Benders methods. In order to achieve this, we introduce an additional non-negative decision variable $w^k, \forall k \in K$ and make a slight modification to our model to ensure that w^k takes value $\sum_{j \geq 1} z_j^k, \forall k \in K$. The modified Benders formulation (BF), as given below, is defined by Constraints (2)-(21) and (22)-(24):

$$(BF) \quad \min \sum_{k \in K} \sum_{i \geq 0} \sum_{j \geq 0: i \neq j} c_{ij} x_{ij}^k + \sum_{k \in K} r_k w^k + \sum_{j \geq 1} f_j y_j + \sum_{k \in K} \sum_{i \geq 1} v_k \bar{x}_{0i}^k \quad (22)$$

$$\text{s.t. } w^k \geq \sum_{j \geq 1} z_j^k, \quad \forall k \in K \quad (23)$$

$$w^k \geq 0, \quad \forall k \in K \quad (24)$$

$$(2) - (21).$$

4.1 The Master Problem (MP)

We solve the master problem (MP) in a branch-and-cut framework. We use the dual of the subproblems described in Section 4.2 as the separation problems to cut a solution of MP violated by any subproblem.

$$(MP) \quad \min (22) \\ \text{s.t. } (2) - (10), (17) - (19), (24) \\ w^k \geq \sum_{i \geq 0} \sum_{j \geq 0: j \neq i} e_{ij} x_{ij}^k - \beta^k, \quad \forall k \in K. \quad (25)$$

4.2 The subproblem and its dual

Once we obtain a vector of feasible solutions $(\bar{y}, \bar{x}, \bar{\mathbf{l}}, \bar{w})$ from the master problem, for each $k \in K$ with $\bar{w}^k > 0$, we construct and solve the dual of the subproblem $SP_k(\bar{y}, \bar{x}, \bar{\mathbf{l}}, \bar{w})$.

$$SP_k(\bar{y}, \bar{x}, \bar{\mathbf{l}}, \bar{w}) \quad \min 0 \quad (26)$$

$$\text{s.t. } \sum_{j \geq 1} z_j^k \geq \bar{w}^k, \quad (27)$$

$$z_j^k \leq \beta^k \bar{y}_j, \quad \forall j \geq 1 \quad (28)$$

$$z_j^k \leq \beta^k \sum_{i \geq 0: i \neq j} \bar{x}_{ij}^k, \quad \forall j \geq 1 \quad (29)$$

$$\sum_{j \geq 0: j \neq 1} b_{ij}^k = \sum_{j \geq 0: j \neq 1} (b_{ji}^k - e_{ji} \bar{x}_{ji}^k) + z_i^k, \quad \forall i \geq 1 \quad (30)$$

$$\sum_{j \geq 1} b_{0j}^k = \beta^k \bar{x}_{0j}^k, \quad (31)$$

$$b_{ij}^k \leq \beta^k \bar{x}_{ij}^k, \quad i, j \geq 0: i \neq j \quad (32)$$

$$b_{ij}^k \geq \bar{e}_{ij} x_{ij}^k, \quad i, j \geq 0: i \neq j \quad (33)$$

$$b_{ij}^k \geq 0, \quad i, j \geq 0: i \neq j \quad (34)$$

$$z_j^k \geq 0, \quad \forall j \in J \quad (35)$$

Here, one can easily observe that if $\bar{w}^k = 0$, then, there will be no violation from the subproblem of the corresponding vehicle trip. After elimination of equality constraints and necessary rearrangements on the remaining subproblem, we obtain the following SP_k in canonical maximization form for each $k \in K$:

$$\max 0 \tag{36}$$

$$\text{s.t. } -\sum_{j \in J} z_j^k \leq -\bar{w}^k, \tag{37}$$

$$z_j^k \leq \beta^k \bar{y}_j, \quad \forall j \geq 1 \tag{38}$$

$$z_j^k \leq \beta^k \sum_{i \geq 1: i \neq j} \bar{x}_{ij}^k, \quad \forall j \geq 1 \tag{39}$$

$$z_j^k + \sum_{i \geq 1: i \neq j} b_{ij}^k - \sum_{i \geq 1: i \neq j} b_{ji}^k \leq \beta^k \bar{x}_{j0}^k - \beta^k \bar{x}_{0j}^k + \sum_{i \geq 0: i \neq j} e_{ij} \bar{x}_{ij}^k, \quad \forall j \geq 1 \tag{40}$$

$$-z_j^k - \sum_{i \geq 1: i \neq j} b_{ij}^k + \sum_{i \geq 1: i \neq j} b_{ji}^k \leq \beta^k \bar{x}_{0j}^k - e_{j0} \bar{x}_{j0}^k - \sum_{i \geq 0: i \neq j} e_{ij} \bar{x}_{ij}^k, \quad \forall j \geq 1 \tag{41}$$

$$b_{ij}^k \leq \beta^k \bar{x}_{ij}^k, \quad \forall i, j \geq 1 : i \neq j \tag{42}$$

$$-b_{ij}^k \leq -e_{ij} \bar{x}_{ij}^k, \quad \forall i, j \geq 1 : i \neq j \tag{43}$$

$$b_{ij}^k \geq 0, \quad \forall i, j \geq 1 : i \neq j \tag{44}$$

$$z_j^k \geq 0, \quad \forall j \geq 1 \tag{45}$$

Let $\alpha, \delta_j, \gamma_j, \pi_j, \rho_j, \phi_{ij}, \epsilon_{ij}$ be the dual variables associated with constraints (37)-(43), respectively. Then, we can write the equivalent dual problem $D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$ for each $k \in K$ as follows:

$$\begin{aligned} D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}}) \quad \min \quad & -\bar{w}^k \alpha + \sum_{j \geq 1} \beta^k \bar{y}_j \delta_j + \sum_{i \geq 0} \sum_{j \geq 1: i \neq j} \beta^k \bar{x}_{ij}^k \pi_j \\ & + \sum_{j \geq 1} \beta^k (\bar{x}_{j0}^k - \bar{x}_{0j}^k) \gamma_j + \sum_{i \geq 0} \sum_{j \geq 1: i \neq j} e_{ij} \bar{x}_{ij}^k \gamma_j \\ & + \sum_{j \geq 1} \beta^k \bar{x}_{0j}^k \rho_j - \sum_{j \geq 1} e_{j0} \bar{x}_{j0}^k \rho_j - \sum_{i \geq 0} \sum_{j \geq 1: i \neq j} e_{ij} \bar{x}_{ij}^k \rho_j \\ & + \sum_{i \geq 1} \sum_{j \geq 1: i \neq j} \beta^k \bar{x}_{ij}^k \phi_{ij} - \sum_{i \geq 1} \sum_{j \geq 1: i \neq j} e_{ij} \bar{x}_{ij}^k \epsilon_{ij} \end{aligned} \tag{46}$$

$$\text{s.t. } -\alpha + \delta_j + \pi_j + \gamma_j - \rho_j \geq 0, \quad \forall j \geq 1 \tag{47}$$

$$-\gamma_i + \gamma_j + \rho_i - \rho_j + \phi_{ij} - \epsilon_{ij} \geq 0, \quad \forall i, j \geq 1 : i \neq j \tag{48}$$

$$\alpha \geq 0, \tag{49}$$

$$\delta_j, \gamma_j, \pi_j, \rho_j \geq 0, \quad \forall j \geq 1 \tag{50}$$

$$\phi_{ij}, \epsilon_{ij} \geq 0, \quad \forall i, j \geq 1 : i \neq j \tag{51}$$

In order to ensure that the dual problem is bounded, we further bound variables $\alpha, \gamma_j, \rho_j, \forall j \geq 1$, and $\epsilon_{ij}, \forall i, j \geq 1 : i \neq j$ by 1 from above. If the optimal value of $D_k(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$ is negative valued, we add the feasibility cut (52) to MP to cut the current solution $(\bar{\mathbf{y}}, \bar{\mathbf{x}}, \bar{\mathbf{l}}, \bar{\mathbf{w}})$.

$$\begin{aligned}
& -\bar{\alpha}w + \sum_{j \geq 1} \beta^k \bar{\delta}_j y_j + \sum_{i \geq 0} \sum_{j \geq 1: i \neq j} \beta^k \bar{\pi}_j x_{ij}^k + \sum_{j \geq 1} \beta^k \bar{\gamma}_j (x_{j0}^k - x_{0j}^k) + \sum_{i \geq 0} \sum_{j \geq 1: i \neq j} e_{ij} \bar{\gamma}_j x_{ij}^k \\
& + \sum_{j \geq 1} \beta^k \bar{\rho}_j x_{0j}^k - \sum_{j \geq 1} e_{j0} \bar{\rho}_j x_{j0}^k - \sum_{i \geq 0} \sum_{j \geq 1: i \neq j} e_{ij} \bar{\rho}_j x_{ij}^k + \sum_{i \geq 1} \sum_{j \geq 1: i \neq j} \beta^k \bar{\phi}_{ij} x_{ij}^k - \sum_{i \geq 1} \sum_{j \geq 1: i \neq j} e_{ij} \bar{\epsilon}_{ij} x_{ij}^k \geq 0
\end{aligned} \tag{52}$$

5 Implementation Details - General Framework

Our decomposition algorithm mainly consists of two phases. In Phase I, we aim to solve the problem with at most one visit to each station. In the second phase, we solve the general problem that allows multiple visits to stations. Between the two phases, we perform an intermediate procedure (See Section 6) to decrease the size of the problem in Phase II as much as possible without eliminating any potential solution better than the Phase I solution.

Through our preliminary experiments, we observe that our algorithm has a better convergence behavior if we are able to introduce a high quality initial feasible solution to our master problem. In order to achieve this, we perform a ‘Step 0’ process where we solve our BDA formulation without any valid inequalities and by restricting the number of feasible solutions to one. We also introduce a partial *warm start* solution to CPLEX by opening all potential stations. In our experiments, CPLEX usually finds a solution with all stations opened. We then try to improve this solution by closing some of the stations. This removal process is based on checking the energy consumption between three consecutive stations and then closing the intermediate one if the battery level is sufficient to go from the first one to the third one.

Finally, we introduce the set of open stations of this improved solution as a partial *warm start* solution for our Phase II model.

5.1 Valid Inequalities for Phase I

Let N_{min}^V and N_{min}^S be lower bounds on the number of vehicles and the number of stations needed for any feasible solution, respectively. We can obtain N_{min}^V by solving a bin packing problem (BPP) as follows:

$$(BPP) \quad N_{min}^V = \min \sum_{k \in K} v_k \tag{53}$$

$$\text{s.t.} \quad \sum_{k \in K} a_{ik} = 1, \quad \forall i \in I \tag{54}$$

$$\sum_{i \in I} d_i a_{ik} \leq Q^k v_k, \quad \forall k \in K \tag{55}$$

$$v_k \in \{0, 1\}, \quad \forall k \in K \tag{56}$$

$$a_{ij} \in \{0, 1\}, \quad \forall i \in N, k \in K. \tag{57}$$

Moreover, we can obtain N_{min}^S by solving the following minimal covering problem (CP):

$$(CP) \quad N_{min}^S = \min \sum_{j \in J} y_j \quad (58)$$

$$\text{s.t.} \quad \sum_{j \in J: e_{ij} \leq \beta^{max}} y_j \geq 1, \quad \forall i \in I : (e_{0i} + e_{i0} > \beta^{max}) \quad (59)$$

$$y_j \in \{0, 1\}, \quad \forall j \in J \quad (60)$$

where $\beta^{max} = \max_{k \in K} \beta^k$.

Our preliminary experiments revealed that introducing Constraint (61) usually reduces the solving time. This observation has led us to include this constraint in our computations for every model of Phase I and Phase II. On the other hand, Constraint (62) did not contribute a significant improvement and hence this constraint is not considered in any subsequent experiments.

$$\sum_{k \in K} \sum_{j \geq 1} x_{0j}^k \geq N_{min}^V \quad (61)$$

$$\sum_{j \geq 1} y_j \geq N_{min}^S \quad (62)$$

When we solve BDA to optimality with at most one visit to each station, we include the following sets of valid inequalities to our master problem:

$$\sum_{j: (i,j) \in A^k} x_{ij}^k - \sum_{j: (j,0) \in A^k} x_{j0}^k \leq 0, \quad i \geq 0, \forall k \in K \quad (63)$$

$$\sum_{j: (j,i) \in A^k} x_{ji}^k \leq y_i, \quad i \geq 2 : d_i = 0, \forall k \in K \quad (64)$$

$$\sum_{k \in K} \sum_{j: (j,i) \in A^k} x_{ji}^k \leq 1, \quad i \geq 1 : d_i > 0 \quad (65)$$

$$\sum_{i \geq 1} x_{i0}^k \leq 1, \quad \forall k \in K \quad (66)$$

$$y_i \leq \sum_{k \in K} \sum_{j \geq 0} x_{ij}, \quad \forall i \geq 1 : d_i = 0 \quad (67)$$

$$y_j \leq \sum_{k \in K} \sum_{i \geq 0} x_{ij}, \quad \forall j \geq 1 : d_j = 0 \quad (68)$$

$$w^k \leq \sum_{j \geq 1} \beta^k y_j \quad \forall k \in K. \quad (69)$$

Even though most of these constraints are implied by the original constraints, their inclusion improves the solving time performance of our algorithm considerably.

5.2 Valid Inequalities for Phase II

When we solve BDA for the last time with all possible copies of potential stations, in addition to the valid inequalities (63)-(69), we also introduce the following set of valid inequalities to break the symmetry between the copies of stations:

$$\sum_{i:(i,j) \in A^k} x_{ij}^k \leq \sum_{i:((j-1),i) \in A^k} x_{(j-1)i}^k, \quad \forall k \in K, j \text{ is the } m^{\text{th}} \text{ copy of some } j_1 : d_{j_1} > 0, m \geq 3 \quad (70)$$

$$\sum_{i:(i,j) \in A^k} x_{ij}^k \leq \sum_{i:((j-1),i) \in A^k} x_{(j-1)i}^k, \quad \forall k \in K, j \text{ is the } m^{\text{th}} \text{ copy of some } j_1 : d_{j_1} = 0, m \geq 2. \quad (71)$$

Constraints (70) and (71) make sure that an additional copy of any station is visited by a vehicle only if the preceding copy is visited by the same vehicle. Exceptionally, the second copy (the first non-original copy), might be visited by a vehicle not serving the original copy if it is a demand node.

6 Intermediate Processing

Creating multiple copies of stations leads to a large-size formulation and excessive solving times. We develop a two phase method that solves our Benders formulation initially for a single copy of each station. Based on the value Z^1 of the solution obtained at this stage, we apply an intermediate processing that checks the availability of a solution with multiple copies of stations that has a smaller objective value than Z^1 . This is an iterative procedure that proceeds by increasing the number of copies considered, say m , one by one and applies lower bound checking steps.

The aim of this procedure is to check whether there exists a solution of BDA with exactly m copies for some station j whose cost is lower than Z^1 .

Lemma 6.1. *Let $Z_{(m,j,k)}^{LB}$ be a lower bound on the cost when exactly m copies of station j is visited by vehicle k . If $Z_{(m,j,k)}^{LB} \geq Z^1, \forall j, k$, then, there exists no solution with m copies of any station whose value is less than Z^1 .*

Lemma 6.2. *If there exists some lower bound $Z_{(m,j,k)}^{LB}$ such that $Z_{(m',j,k)}^{LB} \geq Z_{(m,j,k)}^{LB}, \forall m' \geq m$ and if Lemma 6.1 holds for such $Z_{(m,j,k)}^{LB}$, then, there exist no solution with more than or equal m copies of any stations whose value is less than Z^1 .*

Proof. If $Z_{(m',j,k)}^{LB} \geq Z_{(m,j,k)}^{LB}, \forall m' \geq m$ for given j, k, m , then, $Z_{(m',j,k)}^{LB} \geq Z_{(m,j,k)}^{LB} \geq Z^1, \forall j, k, m'$ such that $m' \geq m$ by Lemma 6.1. Then, there exists no solution with m' copies of any station whose value is less than Z^1 . \square

Below we give the details on how we obtain a lower bound that satisfies Lemma 6.2.

Let us consider a potential station j . If we use exactly m copies of this station, it means that we visit at least $m - 1$ different customers with some vehicle k . This leads to a partial network structure as $0 \dots j \dots i_1 \dots j \dots i_2 \dots \dots i_{m-1} \dots j \dots 0$.

Let E^m and R^m be the amount of energy consumption and the amount of recharging needed, respectively, when we visit station j exactly m times by some vehicle k_1 . Now, we can consider two cases:

Case 1: all customers are visited by k_1 .

Case 2: some customers are visited by other vehicle(s).

For any of Case 1 or Case 2, the following observation holds:

Observation:

- (i) $E^m > E_m^{base} = (m - 1)\beta^{k_1} + e_{j_0}$ and $R^m > R_m^{base} = (m - 2)\beta^{k_1} + e_{j_0}$ if m is even.
- (ii) $E^m > E_m^{base} = (m - 1)\beta^{k_1}$ and $R^m > R_m^{base} = (m - 2)\beta^{k_1}$ if m is odd.

In Figure 1, we illustrate this observation for $m = 2$ and $m = 3$. In this figure, if $E^m \leq \beta^{k_1} + e_{j_0}$, we would not need to visit j twice. Similarly, if $E^m \leq 2\beta^{k_1}$, it would be redundant to visit j three times.

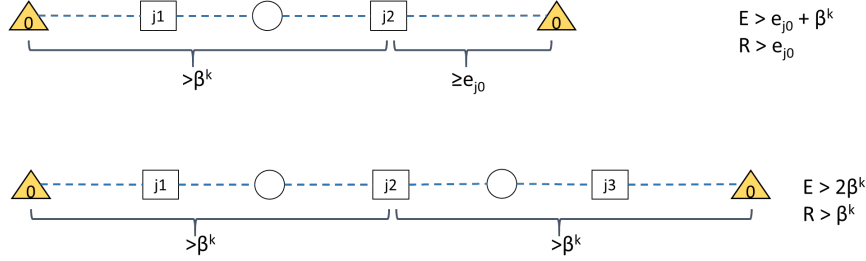


Figure 1: Illustration of E_m^{base} and R_m^{base} for visiting m copies of j with vehicle k for $m = 2, 3$.

We further check whether E^m and R^m values are much larger than E^{base} and R^{base} , respectively. This is performed as follows:

In Case 1, we calculate the minimal possible energy consumption on such a partial network. More explicitly, we define $E^1 = e_{0j} + e_{ji_1} + e_{i_1j} + e_{ji_2} + e_{i_2j} + \dots + e_{ji_{m-1}} + e_{j_0}$ where i_1, \dots, i_{m-1} are $m - 1$ closest customers to j (e.g. see Figure 2). For this particular case, we can further obtain a lower bound on the total energy consumption by constructing a 1-tree obtained via a minimum spanning tree which spans the union set of all customers and m copies of j and that is connected to the depot node with two minimal edges. Let the energy consumption on this 1-tree be E^T and $E = \max\{E^1, E^T, E^m\}$. We obtain a lower bound C on the total routing cost similarly. Then, $R = \max\{E - \beta^{k_1}, 0, R^m\}$ gives us the amount of recharging needed for this partial network and $Z^{LB} = R \times r_{k_1} + C + f_j + v_{k_1}$ gives us a lower bound on the cost of routing all customers by vehicle k via visiting j m times or more.

When we look at Case 2, we investigate all possible vehicle combinations that need to be considered by iteratively increasing the number of additional vehicles. If we find a combination with k vehicles whose lower bound is less than z^1 , we do not check the combinations with more than k vehicles. Let us assume that in addition to k_1 , we use $K^* = \{k_2, \dots, k_l\}$. This means that we are visiting a different customer by each additional vehicle. Therefore, we add $0 \dots i_{k_h} \dots 0$ as a connected component to our partial network for each vehicle $k_h \in K^*$ where i_{k_h} is the closest customer to depot which is not served by preceding vehicles. In a similar fashion to that of

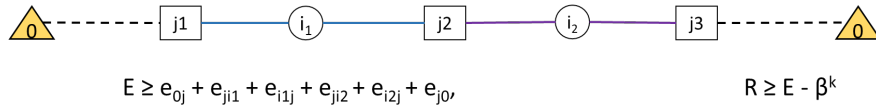


Figure 2: Calculation of E^1 for visiting $m = 3$ copies of j with vehicle k .

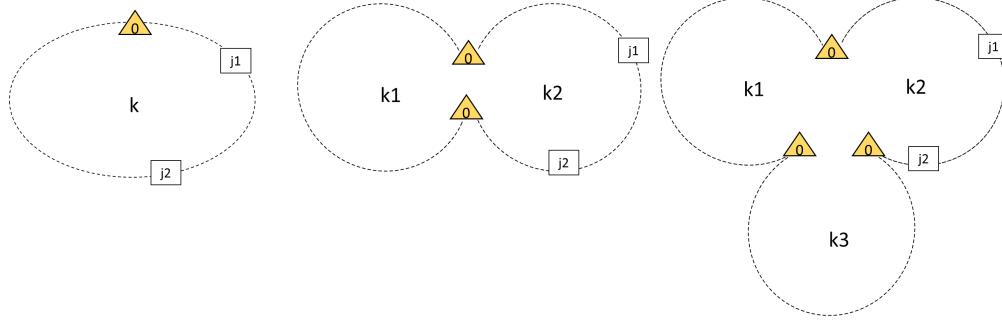


Figure 3: LB2 on the cost of visiting $m = 2$ copies of j with 1,2, or 3 vehicles.

Case 1, we calculate the total energy consumption E^h , the amount of recharging needed R^h , the total routing cost and hence a lower bound $Z_{K^*}^{LB1}$ on the total cost with the corresponding vehicle combination. In order to obtain another lower bound $Z_{K^*}^{LB2}$ from the 1-tree constructed in Figure 3, this time, we use $\beta = \sum_{k \in K^* \cup k_1} \beta^k$ as the total battery available in our calculation for the amount of recharging needed, that is, $R^2 = \max\{\max\{E^2, E^T\} - \beta, 0\}$ and $r = \min_{k \in K^* \cup k_1} r_k$ as the unit recharging cost. Our bound $Z_{K^*}^{LB}$ for the corresponding combination is defined as $Z_{K^*}^{LB} = \max\{Z_{K^*}^{LB1}, Z_{K^*}^{LB2}\}$.

If $\min\{Z_{K^*}^{LB}, Z_{K^*}^{LB}\} \geq Z^{m-1}$ for every (k_1, K^*) combination, then, the value of any solution visiting j no less than m times will be no better than Z^{m-1} . So, in further iterations, we do not need more than $m - 1$ copies of j . If this holds for all stations, we can terminate the iterative checking procedure and solve our algorithm *BDA* with at most $m - 1$ copies for each station.

Additional speed-up mechanism:

When solving *BDA* for this last time, we further apply variable fixing by using the information we obtained from this iterative procedure. More explicitly, if it is decided that we do not need more than l copies at a given station j , we fix all y values to zero for all those copies of j . Similarly, if we decided that visiting more than l copies of station j with vehicle k^1 is not optimal, then we set all x variables of those copies to zero for vehicle k^1 .

7 Computational Study

In order to test our methods, we generated problem instances based on the data set provided by Schneider et al. (2014). This data set has 36 different instances with 5, 10, and 15 customers (12 instances for each customer size). From these instances, we retrieved the demand and network information (node coordinates). The vehicle freight capacities are equal to 200 in the original data. We introduced additional levels of capacities (80, 100), especially, to test relatively smaller instances. Similarly, for the battery capacities, we conducted tests for low, medium, and high capacities (10, 16, 22 Kwh) to avoid extremely loose values on the tests of small problems.

In our experiments, we use IBM ILOG CPLEX 12.8.1 in a Java environment. We run our tests on a PC with Intel(R) Core(TM) i7-7920HQ CPU at 3.10 GHz processor and 32 GB RAM. For each experiment, we use we a memory limit of 16 GB and set a time limit of 3600, 5400, and 10800 seconds for instances with 5, 10, and 15 customers, respectively.

We assume that the system will be equipped with fast charging facilities. The lifetime of a charging facility is estimated to be 3 years and it is 5 years for the vehicles. When we calculate the

fixed costs of opening stations and purchasing/leasing vehicles, we divide their costs by the number of days within their lifetime. We approximately obtain $f_j = 8$ as the fixed cost of opening stations and $v_k = 16, 26, 36$ as the fixed cost of low, medium, and high capacity vehicles, respectively. Let l_{ij} be the distance between nodes $i, j \in N$; then, we set $c_{ij} = l_{ij} \times 0.03$ (cents/km), $r_k = r = 0.07$ (cents/km) for $k \in K$, $e_{ij} = l_{ij} \times 135$ (Wh/km). In order to better tackle the precision issues of CPLEX, we multiplied all the cost values by 100.

We conduct experiments on instances with 2, 3, and 4 vehicles of three different types, see Table 1.

Table 1: Information about vehicle data.

Vehicle type	1	2	3
Q^k	80	100	200
β^k	10	16	22
v^k	16	26	36

For each instance of Schneider et al. (2014), we test the problem with the vehicle combinations shown in Table 2.

Table 2: Vehicle combinations.

instance	K	Vehicle types available
he1	2	1,2
he2	2	1,3
he3	2	2,3
he4	3	1,2,3
he5	3	1,1,2
he6	3	2,2,3
he7	4	1,1,2,3
he8	4	1,2,2,3
he9	4	1,2,3,3

We provide average results for each group of instances in Tables 3 and 4. The detailed results are not given here for clarity but can be collected from the authors. For illustration, we also display the results in Figures 4-9. In these figures and tables, N^S and N^V show the number of stations opened and the number of vehicles used, respectively. The value ‘ g_1 ’ represents the gap provided by CPLEX at the end of the time limit (0.00 if the problem is solved optimally) for the corresponding model solved. Similarly, ‘ $t(s)$ ’ represents the total time spent in seconds for the corresponding model or algorithm if it includes additional processes. Let ‘Obj’ be the value of the best solution obtained from a model. Then, the ‘ g_2 ’ values are the gap values between the objective value of the best solution found and the best dual bound ($DObj$) of the second phase model of BDA, more explicitly, $g_2 = (Obj - DObj)/Obj$. Finally, $t_2(s)$ is the total time (including $t(s)$ of Phase I) spent by BDA. We use ‘NA’ to refer to the fact that no primal or dual bound is obtained within the time limit.

Our formulation PF can solve all but one instance of $|I| = 5$ to optimality within one hour. The average time spent by PF on these instances is 178.68 seconds, which is much larger compared to that of BDA. On the other hand, we observe that our algorithm BDA can solve all the instances

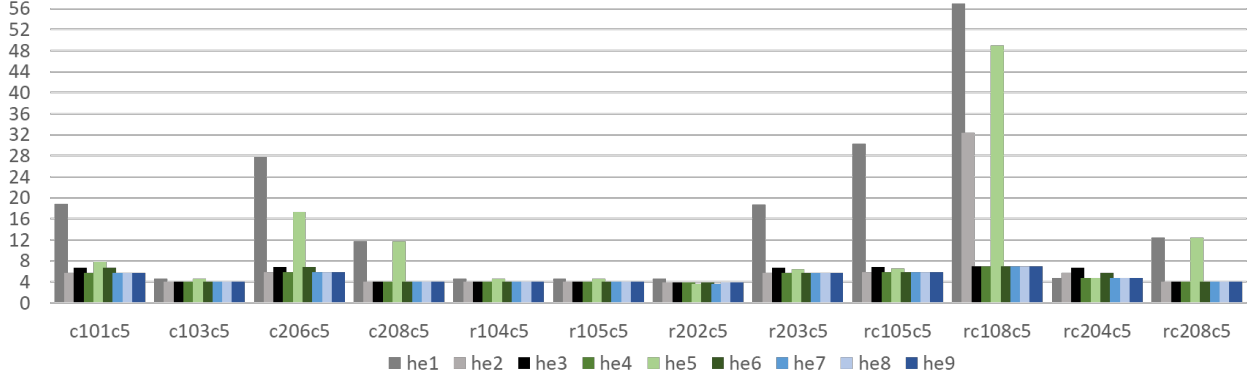


Figure 4: The optimal values (in thousands) for $|I| = 5$. The horizontal axis indicate the instance code given in Schneider et al. (2014).

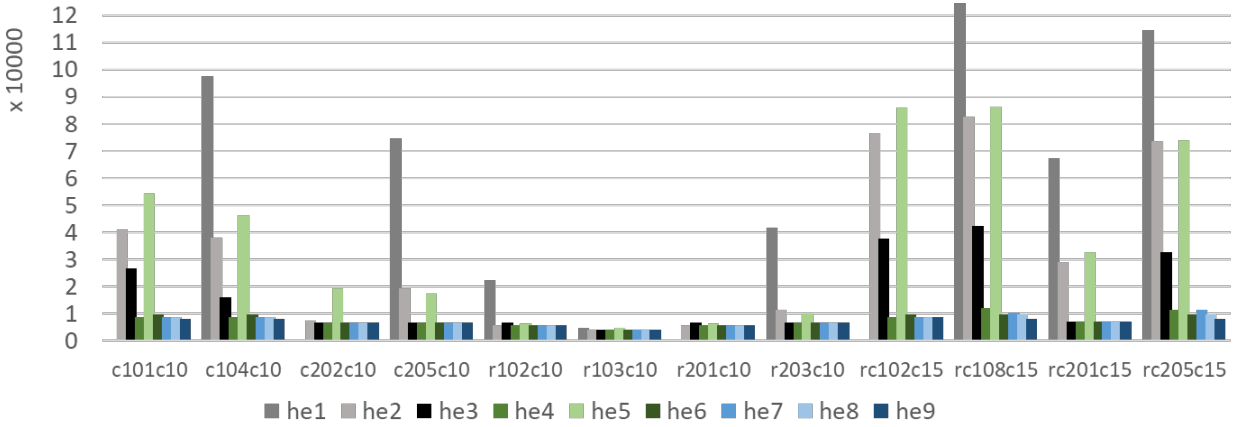


Figure 5: The optimal values for $|I| = 10$. The horizontal axis indicate the instance code given in Schneider et al. (2014).

with $|I| = 5, 10$ to optimality. In Figures 4 and 5, we show the optimal values of the instances of $|I| = 5$ and $|I| = 10$, respectively. The average time spent by BDA for solving the instances with $|I| = 5$ is 2.25 seconds and it is 146.39 seconds for the ones with $|I| = 10$. In Figure 6, we compare the average solving time of PF and BDA over 12 instances of Schneider et al. (2014) with $|I| = 5$. We can easily see that the performance of BDA is much better compared to PF. In fact, BDA is faster than PF in every single instance.

When we try to solve larger problems ($|I| = 10, 15$) with PF, optimality could not be guaranteed within the time limit and for many instances even finding a feasible solution is not possible. Based on these observations, for larger problems, we present the results for BDA only.

In Figure 7, we see the average amount of time spent for Phase I ($t_1(s)$) and the total time of BDA ($t_2(s)$) for $|I| = 10$ instances. As before, these average values are calculated over 12 instances of Schneider et al. (2014). Phase I consumes 48.85 seconds on average only with the worst time being 767.48 seconds.

Moreover, BDA can solve 80% of the instances with $|I| = 15$ and the average time spent for all instances with $|I| = 15$ is just below 6500 seconds. In Figure 8, we show the values of the best

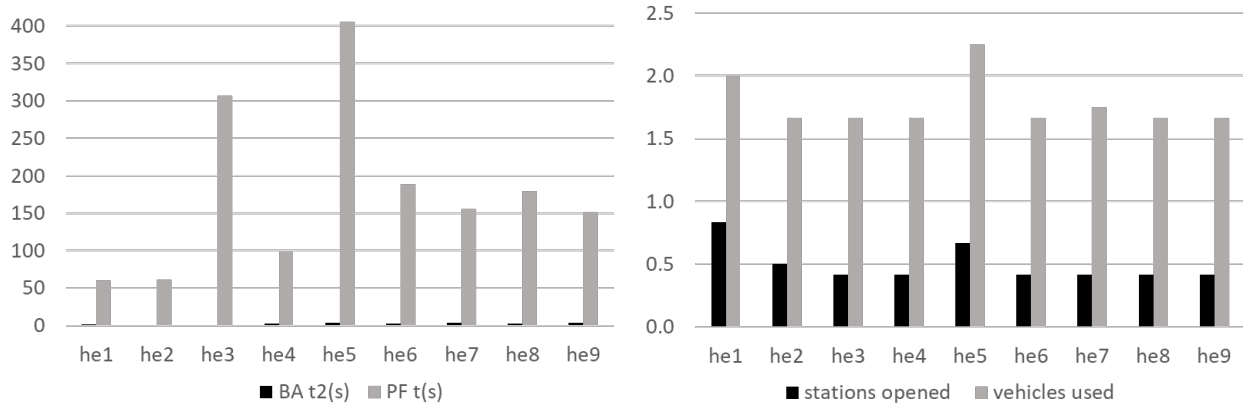


Figure 6: Average results for PF and BDA on instances with $|I| = 5$.

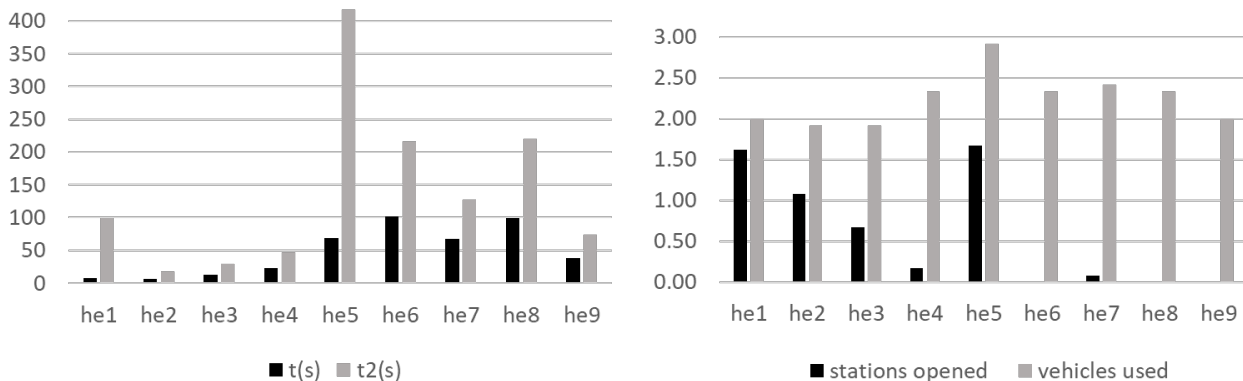


Figure 7: Average results for BDA on instances with $|I| = 10$.

solutions found for these instances. We further present the average values of $t(s)$, $t_2(s)$, g_1 , g_2 , N^S , and N^V over the 12 instances of Schneider et al. (2014) with $|I| = 15$ in Figure 9. For these instances, we observe that BDA requires more time to reach optimality when K is larger.

It is interesting to note that small vehicles result in higher costs, an observation which is not that obvious. We can also note that the instances of ‘he1’ combination is frequently infeasible due to insufficient freight capacity because the total capacity of the small and medium size vehicles is less than the total demand of those instances provided by Schneider et al. (2014). The ‘he5’ combination is also infeasible for some instances and for some other ones, the algorithm reaches the time or memory limit before producing optimal solutions or light duality gaps.

8 Conclusion

In this paper, we introduce an electric location-routing problem with heterogeneous fleet and partial recharging. We initially propose a new mixed integer programming formulation for this problem. This is a flow-based formulation with three-index binary routing variables. The sub-tour elimination is enhanced via a group of load balancing constraints using these decision variables. We further

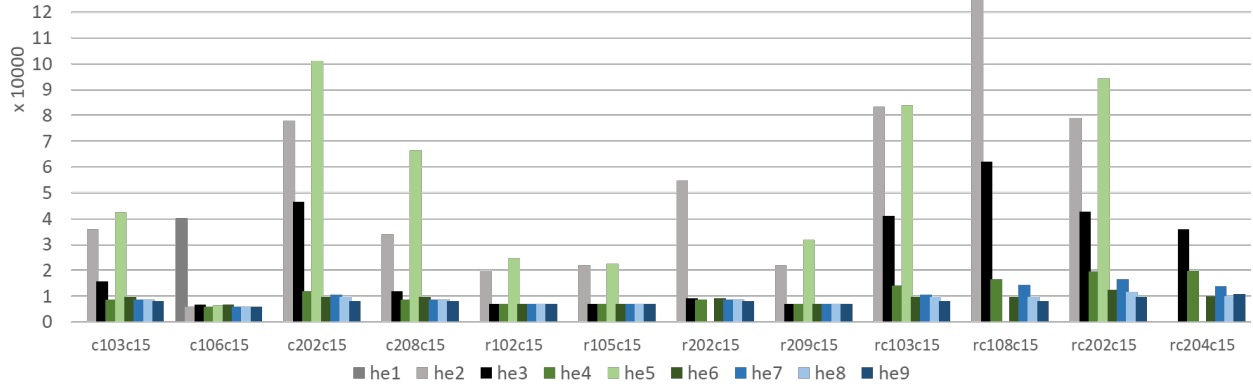


Figure 8: Best values for $|I| = 15$. The horizontal axis indicate the instance code given in Schneider et al. (2014).

utilize additional non-negative continuous variables to satisfy battery restrictions and energy-related constraints.

We test our formulation on small problem instances from the literature. Although the formulation is able to solve instances with 5 customers to optimality, we observe that its performance is not satisfactory for solving larger problems.

As we aim to solve this problem optimally, we further develop a two-phase algorithm based on the Benders decomposition of our formulation. The first phase aims to solve a restricted version of the problem that allows at most one visit to each station. By using the information obtained, the second phase problem, which is the generalized problem allowing multiple visits to any station, is reduced in size, making it relatively easier compared to the case with no a priori processing. This enhancement allows us to solve all instances of 5 and 10 customers, and 80% of the instances with 15 customers to optimality. The average optimality gap over all other instances is just 0.02. In summary, our approach obtains very high quality solutions within the time limit.

We observe through our experimental study that the problem is usually harder to solve when the vehicle capacities are smaller. Also we found that using small vehicles results in higher costs. Moreover, in our experiments, the system tends to avoid recharging as much as possible because the sum of fixed station costs and recharging costs on the intermediate stations (other than the depot) dominates the routing cost plus vehicle costs. As a result, often we obtain tours without any recharging if the customers are within the range and we have sufficient number of vehicles for distinct tours.

The main focus of this work is on exact methods where their performance is limited to small sized instances. However, our methods can be easily modified or combined with additional procedures such as metaheuristics leading to powerful matheuristics to obtain near optimal solutions for larger problems, see Salhi (2017).

Finally, several deterministic and stochastic extensions of this problem can be worth exploring. These include consideration of time windows, multiple depots and/or additional location decisions for the selection of depots, as well as periodicity or uncertainty in the customer demand.

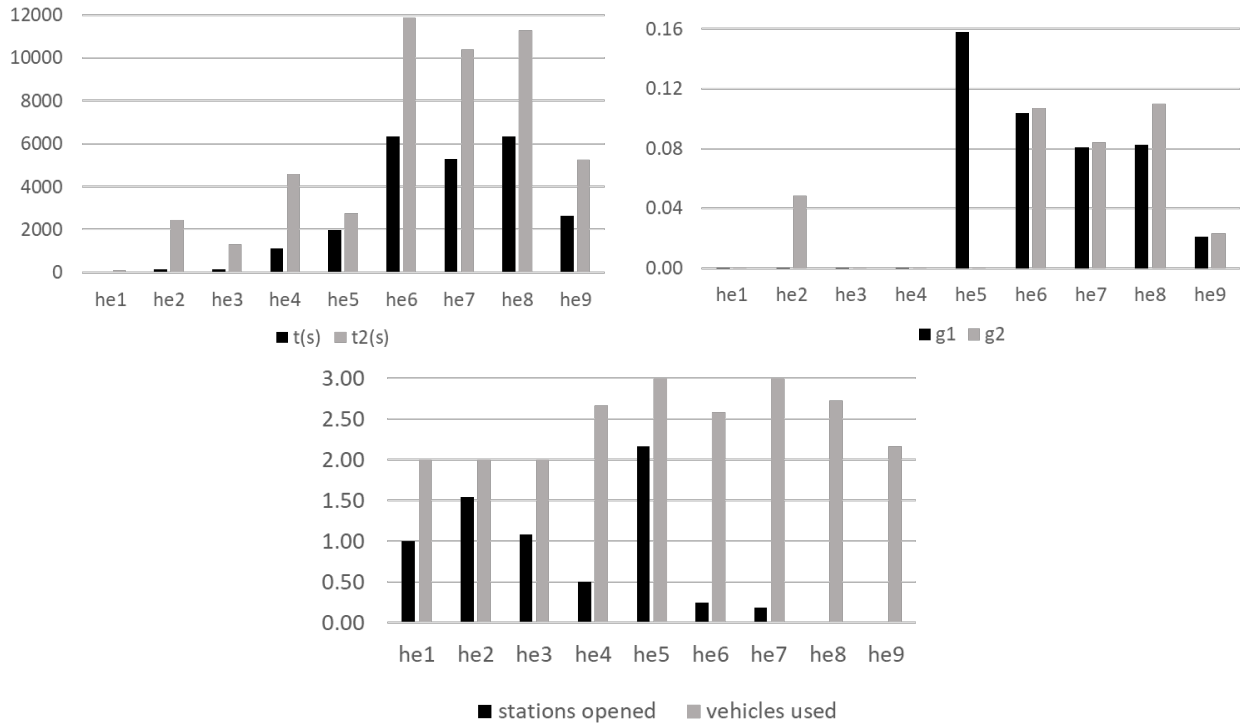


Figure 9: Average results for BDA on instances with $|I| = 15$.

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Table 3: Average results for c, r, and rc instances.

	BDA Phase I			BDA Final			PF	
	g_1	$t(s)$	g_2	$t_2(s)$	N^S	N^V	g_1	$t(s)$
c5	0.00	0.65	0.00	1.95	0.25	1.67	0.00	90.95
r5	0.00	0.77	0.00	2.17	0.03	1.47	0.00	96.10
rc5	0.00	1.06	0.00	2.64	0.22	1.86	0.01	348.98
c10	0.00	67.29	0.00	162.84	0.62	2.35		
r10	0.00	47.84	0.00	90.88	0.14	1.89		
rc10	0.00	32.12	0.00	181.85	0.89	2.52		
c15	0.03	3260.53	0.01	5802.79	0.54	2.51		
r15	0.04	1150.72	0.00	3100.75	0.50	2.17		
rc15	0.10	5091.34	0.14	11609.90	0.84	2.80		
Avg:	0.02	1072.48	0.02	2328.42	0.45	2.14	0.00	178.68

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Table 4: Average results for each group of instances.

	$ I $	$ J $	BDA Phase I			BDA Final			PF	
			g_1	$t(s)$	g_2	$t_2(s)$	N^S	N^V	g_1	$t(s)$
c101c5		7	0.00	0.46	0.00	1.51	0.33	2.11	0.00	62.03
c103c5		7	0.00	0.45	0.00	1.82	0.00	1.22	0.00	25.59
c206c5		8	0.00	1.45	0.00	3.45	0.44	2.11	0.00	255.39
c208c5		7	0.00	0.26	0.00	1.02	0.22	1.22	0.00	20.80
r104c5		7	0.00	0.83	0.00	1.85	0.00	1.22	0.00	53.56
r105c5	5	7	0.00	0.33	0.00	1.17	0.00	1.22	0.00	22.66
r202c5		7	0.00	0.47	0.00	1.51	0.00	1.33	0.00	9.49
r203c5		8	0.00	1.45	0.00	4.15	0.11	2.11	0.00	298.69
rc105c5		8	0.00	1.42	0.00	3.33	0.11	2.11	0.00	337.14
rc108c5		8	0.00	1.13	0.00	2.98	0.56	2.11	0.00	52.25
rc204c5		8	0.00	1.49	0.00	3.26	0.00	2.00	0.00	553.52
rc208c5		7	0.00	0.21	0.00	0.98	0.22	1.22	0.05	453.00
Avg:			0.00	0.83	0.00	2.25	0.17	1.67	0.00	178.68
c101c10		14	0.00	59.06	0.00	148.43	0.75	2.63		
c104c10		13	0.00	161.25	0.00	360.62	0.67	2.56		
c202c10		14	0.00	28.45	0.00	60.51	0.50	2.13		
c205c10		12	0.00	20.38	0.00	81.81	0.56	2.11		
r102c10		13	0.00	14.74	0.00	26.78	0.11	2.11		
r103c10	10	12	0.00	2.74	0.00	7.77	0.00	1.22		
r201c10		13	0.00	122.25	0.00	223.78	0.00	2.13		
r203c10		14	0.00	51.61	0.00	105.20	0.44	2.11		
rc102c10		13	0.00	10.25	0.00	23.50	0.88	2.75		
rc108c10		13	0.00	44.17	0.00	510.58	0.89	2.67		
rc201c10		13	0.00	8.38	0.00	28.41	0.56	2.11		
rc205c10		13	0.00	65.68	0.00	164.90	1.22	2.56		
Avg:			0.00	48.85	0.00	146.39	0.55	2.25		
c103c15		19	0.07	4799.07	0.03	6617.91	0.43	2.57		
c106c15		17	0.00	588.14	0.00	794.39	0.11	2.11		
c202c15		19	0.00	3479.35	0.00	6960.44	1.00	2.75		
c208c15		18	0.03	4175.56	0.02	8838.40	0.63	2.63		
r102c15		22	0.12	357.84	0.00	692.95	0.29	2.00		
r105c15	15	20	0.00	521.89	0.00	1288.70	0.38	2.13		
r202c15		20	0.02	3639.25	0.02	10248.68	0.86	2.43		
r209c15		19	0.00	83.91	0.00	172.68	0.50	2.13		
rc103c15		19	0.16	4925.85	0.30	11494.95	1.00	2.75		
rc108c15		19	0.02	2109.19	0.00	6722.72	1.00	2.71		
rc202c15		19	0.13	6395.64	0.14	13771.28	0.86	3.00		
rc204c15		21	0.08	6934.69	0.10	14450.66	0.50	2.75		
Avg:			0.05	3058.24	0.05	6469.88	0.63	2.48		

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