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Continuous and multidimensional assessment of resilience based on functionality analysis for interconnected systems

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**ABSTRACT**

The increasing number of disruptions to critical infrastructure, like natural disasters, terrorist attacks or internal failure is today a major problem of society. Concern is even greater when considering the interconnected nature of critical infrastructure, which might lead to failure propagation, causing domino and cascade effects. To mitigate such outcomes, critical infrastructure must recover its capacity to function with regard to several criteria. Stakeholders must therefore analyze and improve the resilience of critical infrastructure before any disruption occurs, and base this analysis on different models so as to guarantee society’s vital needs. Current resilience assessment methods are mainly oriented toward the context of a single system, thus narrowing their criteria metrics, limiting flexibility and adaptation to other contexts and overlooking the interconnected nature of systems. This article introduces a new tool-equipped approach that makes it possible to define a model to evaluate the functionalities of interconnected systems. The model is then used to assess the resilience of these systems based on simple and generic criteria that can be extended and adapted. Several assertions related to the concept of resilience and some resilience indicators are also introduced. A case study provides the validation performed by experts from several domains.

**KEYWORDS**

Resilience; functionality evaluation; resilience assessment; risk; decision criteria; simulations; critical infrastructure; engineering systems

1. Introduction

Modern society relies on the functioning and mutual exchange of services of various interconnected and independent infrastructures, i.e. systems or systems of systems (healthcare, energy, transport, manufacturing, financial, etc.) (Maier, 2009; Mattsson & Jenelius, 2015). The interdependencies between and within systems make them less resilient to disruptions (Johanson, 2010). Specifically, a disruption in an interconnected system can lead to domino and cascade effect that impacts on the other systems related to the initially affected one (Ouyang, Dueñas-Osorio, & Min, 2012). This becomes a problem when considering the currently increasing number of natural disasters. For instance, in July 2012, the largest blackout in history affected more than 600 million people in India. Through a cascade effect, several other systems (transport, telecommunication, finance,…) also failed (Lai, Zhang, Lai, Xu, & Mishra, 2013), (Romero, 2012). In 2011, the flooding in South-East Asia led to a lack of hard drives and to an increase in the price of these devices all over the world (Booth, 2014). The growing number of hurricanes – Sandy, Isabel, Harvey and Irma – has provoked not only human and material damage, but also economic and production/service capacity failures (Saleh et al., 2017).

The notion of resilience is related to the functioning of critical infrastructures or systems and is here defined as ‘the capacity of a system to recover, in a minimum time, with minimum costs (financial, human, workload, etc.) a certain functioning capacity on all dimensions of its performances’. Some aspects of the resilience of a system can be assessed by analyzing its functionalities in several situations: (a) before a disruptive event, (b) during a disruptive event and (c) after a disruptive event. During each situation, it is important to (i) be able to assess the resilience at a given timestamp and/or period, including during disruptive events, and (ii) to identify preventive actions for different scenarios, to improve the results of the resilience assessment.

This work focuses on situation (a) and proposes a methodology for continuous and multidimensional resilience assessment based on analysis of the functionalities of interconnected systems. Current resilience assessment approaches are oriented towards individual systems (Pursiainen, 2018), whether they be financial, healthcare or transport systems. These approaches are therefore inflexible (difficult to adapt to other domains), with fixed criteria that generally concern performance (other criteria that might be important in the assessment of resilience are overlooked) and not applicable in the context of interconnected systems.

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This article defines a new, tool-equipped approach that makes it possible to define a model to evaluate the functionalities of interconnected systems that are linked to some aspects of resilience. The model is further used to assess resilience, based on several criteria. The proposed generic criteria can easily be extended and adapted depending on the context and needs. The originality of the approach lies in: (1) the combination of functionality-analysis models and continuous resilience assessment following several dimensions of systems, (2) flexibility of criteria metrics for easy adaptation in different contexts and (3) the possibility to aggregate the results of several functionality-analysis models with continuous assessment of the resilience of interconnected systems.

The article is organised as follows: The next section reviews the literature on the definition of resilience and resilience assessment. The contribution is presented in the third section and illustrated in the fourth section by a case study, before concluding in the last section by an evaluation of the limits and perspectives of the research work.

2. Literature review

This section reviews the literature on the definition of resilience and resilience assessment.

2.1. Resilience definition

The need for resilience has deep roots in many civilisations (Harrison & Williams, 2016). The original meaning of the word resilience comes from the Latin word ‘resilire’, which means ‘bounce back’ (Lenort, Grakova, Karkula, Wicher, & Staš, 2014). The common and probably initial acceptance of this term comes from the domain of metallurgy and describes the ability of a piece of metal to return to its initial shape after a thermic or physical deformation (Anaut, 2005). The definition of resilience varies in different domains (Hosseini, Barker, & Ramirez-Marquez, 2016).

Table 1 provides an overview of several definitions. Each one of these definitions highlights three common properties: (i) the term resilience supposes a ‘disruption’, (ii) it refers to the ‘performance of a system’ before, during and after a disruption and (iii) it is relevant to evaluate it at different moments for various ‘decision-making processes’ that help to maintain a required level of performance. In a decision-making process it is necessary to compare the resilience after the disruption to its initial value (before the disruption). Some of these definitions ignore the dependent nature of systems (to other systems or to their environment and its constraints). There are several types of dependencies described by Kamissoko (2013) and Rinaldi, Peerenboom, and Kelly (2001).

According to Kamissoko (2013), the categorisation of dependencies depends on the circulation of flow. Hence, the term dependence induces the circulation of flow between the systems and their components. The term influence is used in situations where there is no flow circulation. Flow can be material or immaterial (information). In the context of this article, considering the circulation of flow, dependencies are defined as the exchange of services and information. They refer to the term dependence defined in (Kamissoko, 2013). Whenever the performance of one system decreases, all systems that are dependent on this system can be directly impacted. The performance of the system is then likely to decrease further, impacting other systems and consequently forming a domino effect that eventually reaches all systems as well as the environment they belong to. To know what must be done and prevent such situations, the systems must be sufficiently resilient. For this purpose, an assessment of their resilience is needed.

Further analysis of the definitions reveals the following:

- Functionality analysis: a functionality analysis binds together a temporal dimension with the behaviour of the system to express the change in state (e.g., performance, stability or integrity) of systems throughout the whole process (i.e., before, during and after a disturbance). For instance, Barker et al. (2017) and Haines (2009) argue that a time dimension must be defined to measure the system recovery after a perturbation. The idea of recovery time reduction is also defended by Vugrin, Warren, Ehlen, and Camphouse (2010) as the ‘duration of deviation’. For these reasons, we argue that resilience assessment is based on a functionality analysis model (FAM) that is a time-dependent function.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Definition</th>
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<tbody>
<tr>
<td>(Hollnagel, 2011)</td>
<td>‘the intrinsic ability of a system to adjust its functioning prior to, during, or following changes and disturbances, so that it can sustain required operations under both expected and unexpected condition’</td>
</tr>
<tr>
<td>(Cimellaro, Reinhorn, &amp; Bruneau, 2010)</td>
<td>‘the ability to anticipate, prepare for, respond to, adapt to disruptions and to mitigate the consequences as well as to recover in timely and efficient manner including preservation restoration of services’</td>
</tr>
<tr>
<td>(Cutter et al., 2013)</td>
<td>‘the ability to adapt to changing conditions and withstand and rapidly recover from disruption’</td>
</tr>
<tr>
<td>(Barker et al., 2017)</td>
<td>‘the ability of system to withstand a major disruption within acceptable degradation parameters and to recover with a suitable time and reasonable costs and risks’</td>
</tr>
<tr>
<td>(Allenby &amp; Fink, 2005)</td>
<td>‘the ability of an engineered system to autonomously sense and respond to adverse changes in health conditions, to withstand failure events, and to recover from the effects of these unpredicted events’</td>
</tr>
<tr>
<td>(Haines, 2009)</td>
<td>‘given the occurrence of a disruptive event (or set of events), the resilience of a system to that event (or events) is that system’s ability to reduce efficiently both the magnitude and duration of deviations from targeted system performance levels’</td>
</tr>
<tr>
<td>(Yodo &amp; Wang, 2016)</td>
<td>‘the capability of a system to maintain its functions and structure while facing internal and external changes and to degrade gracefully when it is obliged to’</td>
</tr>
<tr>
<td>(Vugrin et al., 2010)</td>
<td>‘the capability to sustain functionality and recover from losses generated by extreme events’</td>
</tr>
</tbody>
</table>
Multi-dimensional nature: Several authors argue that there are many dimensions involved in assessing the resilience of a system. For instance, in Allenby and Fink (2005) resilience is related to the maintenance of 'functions and structure', while in (Haimes, 2009) the focus is on the 'degradation parameters'. Vugrin et al. (2010) elaborate some 'performance levels' as dimensions of the resilience. Considering simultaneously multiple and relevant dimensions might improve the resilience of a system even further. Due to these multi-dimension aspects, it is mandatory to study and include all aspects through different criteria in the assessment of resilience.

In the context of this article, to consider the temporal, interdependent and multi-dimensional nature of a system, resilience is defined as 'the aptitude of a system to successfully interact (i.e. comply with constraints, provide the required services) with other systems and its environment and fulfil the assigned objectives while facing various disruptions'. This definition takes account of the multi-dimensional nature of resilience. The objectives and constraints refer to several dimensions that may be technical, economic, environmental, etc.

The objectives and the interactions of a system are defined through several independent criteria. A criterion is a tool allowing comparison of alternatives according to a point of view (Bouyssou, 1990). For example, a criterion is considered in a functionality analysis to allow the comparison of system states as a function of the change in state over time. Each criterion can be represented by a surrogate measure of performance, represented by some measurable attribute (Ramanathan, Mathirajan, & Ravindran, 2017).

2.2. Resilience assessment

Resilience assessment is a process across a lifespan (Liu, Reed, & Girard, 2017). Its assessment methodologies are classified into a quantitative and a qualitative category (Hosseini et al., 2016). The quantitative category relies on formal and numerical methods for criteria evaluation and aggregation to provide a quantitative end-value. The qualitative category relies on more abstract methods that provide qualitative end-results.

Figure 1 presents the subcategories of this classification. This classification is controversial for two reasons: the first reason is that simulation models and optimisation models do not need to be separate. They can be mixed together. The second reason is that several simulation models are based on optimisation algorithms. To complete this classification, (Hosseini et al., 2016) introduce two complementary subcategories of the semi-quantitative assessment approach. These two approaches are: Data analysis and Expertise.

2.2.1. Data analysis

New emerging methods for resilience assessment through big data have been proposed in the domain of smart cities. For instance, during Hurricane Sandy, over 20 million tweets were posted (Barker et al., 2017), embedding information that could be extracted and exploited. Due to this quantity of data, the social, economic and political processes that provide structure to society can be better characterised (Meier, 2013). These methods introduce analytics sensors on several sources to capture relevant data streams. Sensors may be traditional media providing data and information (Newspapers, Television, etc.), or open data (weather, traffic, air/water etc.), or they might be Mobile Devices, Social Media, Audio/Video feeds, Satellite images, etc. The innovation in these methods is the combination of structured data with unstructured data.

Resilience is assessed by means of data visualisation (Khan, Anjum, & Kiani, 2013), data monitoring (Garcia, Lin, & Meerkov, 2012), statistical models (Campbell-Sills, Cohan, & Stein, 2006), data mining (Chan & Wong, 2007), machine learning (Zhang, Burton, Sun, & Shokrabadi, 2018), natural language and crowdsourcing (Mejri, Menoni, Matias, & Aminoltaheri, 2017). Data analytics are hence centred around three perspectives: descriptive, predictive and prescriptive (Barker et al., 2017). The descriptive perspective focuses on the problem of description and visualisation during and after a disruptive event. The quantification of future behaviour and, often, risk is the aim of the predictive perspective. Prescriptive analysis provides guidance on how to achieve desirable outcomes.

2.2.2. Expert analysis

Assessing resilience through expertise relies on sending surveys to identified experts (Taysom & Crilly, 2017). Experts determine different criteria and alternatives to quantify the situation and assess each decision for each criterion. Criteria identification could be done through a methodology called 3S indicator validation. This methodology is proposed by

Figure 1. Classification of resilience assessment methodologies by Hosseini et al. (2016).
Cloquell-Ballester, Cloquell-Ballester, Monterde-Díaz, and Santamarina-Siurana (2006). It is also used by Grafakos and Flamos (2017) and consists of Desk and internal peer review, Internal validation, Expert validation and Stakeholder validation.

Desk and internal review validation identify criteria under some conditions of operationality, value relevance, reliability, measurability, non-redundancy, decomposability, preferential independence, intelligibility, comprehensiveness, completeness and data availability. Multiple-source studies allow the impact of decisions on criteria assessment to be evaluated (Roth et al., 2009). Cutter et al. (2008) use this approach to assess community resilience in its ecological, social, economic, institutional and infrastructure dimensions.

2.2.3. Simulation

Simulation-based approaches can be used to define functionality models of systems by plotting and analysing different criteria related to their functioning (e.g. a performance curve) in various situations (i.e. before, during and after a disruption), as illustrated in Figure 2. Depending on the disruption, the capabilities of a system vary (see \( f_0, f_w, f_d \) and \( f_e \)). Resilience is assessed by comparing the state of the system according to a specific performance parameter before and after a disruption (Hosseini et al., 2016). In Figure 2, \( t_0 \) is the occurrence time, \( t_1 \) corresponds to the time of the worst performance, \( t_2 \) marks the beginning of performance improvement, \( t_3 \) is the time of new stable state and \( t_4 \) is the time considered for resilience assessment.

Other examples are detailed in Bruneau et al. (2003), Gama Dessavre, Ramirez-Marquez, and Barker (2016), Henry and Emmanuel Ramirez-Marquez (2012), Tran, Balchans, Domerçant, and Mavris (2017) and Zobel, (2011). Guidotti et al. (2016) use a Monte Carlo Simulation approach to assess the percentages of demand nodes, while Ash and Newth (2007) use a simulation model based on an evolutionary algorithm to analyse the resilience through it topology.

Table 2 shows an overview of the most relevant formulas to assess some aspects of resilience based on functionality analysis. Several common properties of these formulas are synthesised hereafter:

1. **Functionality model and resilience assessment**: Even though the functionality analysis can be defined based on several criteria (e.g. performance and safety), these authors use a single criterion (e.g. performance). This further limits the assessment of resilience, because only the aspect of resilience related to the given criterion can be assessed (Francis & Bekera, 2014; Henry & Emmanuel Ramirez-Marquez, 2012).

2. **Linear behaviour**: Functionality analysis is assumed to be linear (Hosseini et al., 2016). This is contradictory to the non-linear behaviour of complex critical infrastructures and systems.

3. **Need for a disruption**: Current approaches are unable to assess resilience during a normal functioning time period, because their core formulas rely on a disruption hypothesis (i.e. a comparison of performance before and after a disruption), (Henry & Emmanuel Ramirez-Marquez, 2012; Zobel, 2011). Indeed, when considering a FAM based on several criteria, the ‘dropdown/overload’ of a criterion does not necessarily mean that a disruption has occurred. ‘Dropdown/overload’ refers to a situation when the value of a criterion exceeds an authorised maximal (for overload) or minimal (for dropdown) value.

4. **Recovery of new stable state**: The system is supposed to recover into a new stable state (Enjalbert, Vanderhaegen, Pichon, Ouedraogo, & Millot, 2011). It is therefore not possible to assess the functionalities if the system stays in a state of disruption. Furthermore, it is not possible to assess resilience in the current state if the previous state is unknown.

5. **Lack of consideration of criteria for all resilience capabilities**: Absorptive, adaptive, restorative (Francis & Bekera, 2014; Nan & Sansavini, 2017).

3. Proposal for resilience assessment methodology

This section first introduces a model to evaluate the functionality of interconnected systems (Functionality Evaluation...
Model - FEM) for the purpose of resilience assessment. Secondly, several formulas for resilience assessment are defined as assertions. The Functionality Evaluation Model is defined through three types of object (Components, Systems and Territories) and several types of criteria that can be assigned to the objects, depending on the context:

- **Component** is a part of the system. The criteria assigned to a Component characterise the Component at a low level and depend on its type and functions within the Territory. For instance, the pressure at the outlet can characterise a pump.

- **System** is a critical infrastructure that operates in the territory. For instance, a System may refer to a power station, a production system, a supply chain, a transport infrastructure, or a healthcare system (Yodo & Wang, 2016). The criteria of several Components are aggregated at a higher level to assess those of the System. For instance, within the field of systems engineering, quality attributes as non-functional requirements are used to evaluate the performance of a system (Weck, Roos, & Magee, 2011). Notable quality attributes include redundancy, efficiency, diversity (Molyneaux, Wagner, Froome, & Foster, 2012), stability, technological maturity, innovative ability (Grafakos & Flamos, 2017) reliability, severity, time to recovery (Guidotti et al., 2016), robustness, fault tolerance, survivability, agility (Hosseini et al., 2016), vulnerability, recoverability (Henry & Emmanuel Ramirez-Marquez, 2012), rapidity (Zhang, Wang, & Nicholson, 2017) and resourcefulness (Cimellaro, Reinhorn, & Bruneau, 2010).

- **Territory** is a portion of geographical space that coincides with the spatial extension of a government’s jurisdiction (Gottmann, 1975). The higher-level criteria of several systems are aggregated to assess those of the Territory. For instance, a territory is characterised by diversity, ecological variability, modularity, acknowledged slow variables, tight feedback, social capital, innovation, overlap in government, ecosystem governance (Gaudreau & Gibson, 2010) and climate resilience (Grafakos & Flamos, 2017).

The main concepts of the proposal are illustrated in Figure 3. A system can be composed of several components. A territory can hosts several systems. The relations between systems might be functional or non-functional. Functional relations allow the circulation of flows. Non-functional relations refer to influence relations described by Kamissoko (2013). A component is characterised by several criteria. A criterion is defined by a value and a unit. To consider uncertainty, in normal functioning, a criterion value is comprised between a minimum value $f_{\text{min}}$ and a maximum one $f_{\text{max}}$. The value of a criterion must not be outside the limits $f_{\text{min}}$ and $f_{\text{max}}$.

Criteria and Systems are composable (i.e. contain other criteria or systems). An evolution function defines a behaviour to simulate the evolution of a criterion and to change its value over time. The aggregation function, as its name suggests, aggregates the values of at least two criteria. The influence function determines how a given criterion/system changes another one under specified conditions. Note that feared events and flows are also instances of the concept System (see Figure 3).

A functionality evaluation process defines a Functionality Evaluation Model that is further used for resilience assessment. This process is based on three steps: (1) modelling, (2) model transformation and (3) simulation:

- **Step 1: modelling** consists of identifying systems, criteria, evolution, aggregation and influence functions. After that, relevant criteria characterising the system are also identified. Then, a function that manages every criterion is defined if needed. From a technical point of view, the modelling of these concepts is done in the Obeo Designer environment (https://www.obeodesigner.com/en).

### Table 2. Approaches to resilience assessment based on a functionality analysis.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Authors</th>
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<tbody>
<tr>
<td>$R(t) = \int_0^t (f(t) - f_0(t)),dt$</td>
<td>(Bruneau et al., 2003)</td>
</tr>
<tr>
<td>$R(t) = \int_0^t \frac{1}{1 - \exp(-t)},dt$</td>
<td>(Zobel, 2011)</td>
</tr>
<tr>
<td>$R(t) = \frac{F(t)}{\exp(-t)}$</td>
<td>(Rose, 2007)</td>
</tr>
<tr>
<td>$R(t) = \frac{f(t) - f_0(t)}{F(t)}$</td>
<td>(Henry &amp; Emmanuel Ramirez-Marquez, 2012)</td>
</tr>
<tr>
<td>$R(t) = \frac{f(t) - f_0(t)}{F(t)}$</td>
<td>(Enjalbert, Vanderhaegen, Pichon, Ouedraogo, &amp; Millot, 2011)</td>
</tr>
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</table>

Figure 3. Conceptual model.
Step 2: model transformation is an automated process that transforms designed models into a simulation environment. Technically, a model-to-text transformation script is written in ObeoAceleo to perform the transformation from Obeo Designer to the GAMA platform. This transformation is based on Nastov, Chapurlat, Dony, and Pfister (2016a, 2016b).

Step 3: simulation consists of simulating the results of the model transformation and plotting the results (i.e. a functionality curve). This is technically provided through the GAMA multi-agent simulation platform (http://gama-platform.org/).

This proposal is included in the framework of the MAIIEUTIC project (http://maiieutic.mines-ales.fr/) founded by the CARNOT M.I.N.E.S. institute. The validation process includes several meetings with experts from System Engineering, Crisis Management, Risk Management and Multi-criteria Decision-Making approaches.

Throughout this article, assertions 1 to 10 below define some aspects of the Functionality Evaluation Model related to resilience assessment:

- **Assertion (1):** Several elementary independent criteria that characterise a system can be used to define a Functionality Evaluation Model as a parameter to assess a particular aspect of the resilience of this system.

- **Assertion (2):** A Functionality Evaluation Model must consider the objectives and constraints of the Territory. Several authors ignore the objectives of the system and the constraints of the Territory (Bruneau et al., 2003; Rose, 2007). For a given system, at a given time, the FAM must change according to the objectives and the constraints of the Territory. For a single system characterised by a set of criteria \( C = \{ C_1, C_2, \ldots, C_c \} \), Figure 4 describes the evolution of a system’s criterion over time by a function \( f_n(t) \). All the values in Figure 4 are for a single criterion \( C \), while \( f_n(t) \) models the behaviour of \( C \). It changes between a...
values cannot be above (respectively below) a certain value \( f_{\text{max}}(t) \) (respectively \( f_{\text{min}}(t) \)). Above them, the system is considered as non-functional.

In Figure 4, \( G = \{ G_1, G_2, \ldots, G_g \} \) are the good functioning periods. \( B = \{ U_1, U_2, \ldots, U_n \} \) are the bad functioning periods when the value is greater than the normal maximum. \( B' = \{ D_1, D_2, \ldots, D_j \} \) are bad functioning periods when the value is lower than the normal minimum. The number of good functioning periods is \( g \); the number of bad functioning periods when the value is greater than the normal maximum is \( u \); the number of bad functioning periods when the value is lower than the normal minimum is \( d \). Note that the values of \( f_{\text{max}} \) and \( f_{\text{min}} \) (or \( f_{\text{max}} \) and \( f_{\text{min}} \)) do not necessarily have to be constant and depend on the nature of the system of interest or the context. They could be a curve. For instance, in the use case section, the evolution functions for \( f_{\text{max}} \), \( f_{\text{min}} \), \( f_{\text{max}} \), \( f_{\text{min}} \) are defined. Their value changes with respect to time.

**Assertion (3):** The value of the assessed resilience is between 0 (not resilient at all) and 1 (fully resilient).

\[
0 \leq R_i \leq 1
\]

where \( R_i \) is the value of assessed resilience of a system for a given FAM. Assertion (3) highlights the necessity to have bounded resilience so as to allow comparison.

**Assertion (4):** The value of assessed resilience of a system depends on the value of each criterion aggregated into the Functionality Evaluation Model.

\[
R_i = \prod_{i=1}^{c} R_{i_j}
\]

where \( R_i \) is the value of assessed resilience for the criterion \( i \) and \( c \) is the number of criteria. Assessed resilience for a criterion must respect Assertions 1 to 3.

Considering Equation (2), if the value of assessed resilience of all criteria is equal to one, then that of the system is also one. On the other hand, if that of one criterion is zero, that of the system will also be zero. The more the value of assessed resilience of a criterion tends towards 0, the more that of the system tends towards 0 (and inversely). It is possible to have a value of assessed resilience of a system that is very low if that of several criteria is inferior to 1. For this reason and according to the context, a more complex aggregation function can be defined, instead of using the product of various \( R_i \). In a general way, the resulting value of assessed resilience from several criteria is:

\[
R_i = a(R_{i_j})
\]

where \( a \) is an aggregation function respecting Assertion (4). Resilience assessment is based on a Functionality Evaluation Model. The value of assessed resilience based on a FAM is calculated using seven ‘resilience factors’ \( (R_{C_0} \text{ to } R_{C_6}) \) that are defined hereafter.

- **Assertion (5):** Based on the Functionality Evaluation Model, the value of a given criterion must not be greater than certain values.

Assertion (5) considers the constraints on a criterion \( C \). The value of assessed resilience of a system becomes null if, at any given time-stamp, the value of a criterion is greater than the authorised maximum value or lower than the authorised minimum value. The time-stamp refers to an instant in the simulation. Due to the behaviour of the criterion \( C \), the value of assessed resilience for \( C \) is the product of two values: \( R_{C_0}^P \) and \( R_{C_0}^D \). The first indicates the exceeding of \( f_{\text{max}} \) while the second indicates that of \( f_{\text{min}} \):

\[
R_{C_0}^P = \begin{cases} 0 & \text{if } \exists f(t) \mid f(t) \geq f_{\text{max}} \\ 1 & \text{otherwise} \end{cases}
\]

\[
R_{C_0}^D = \begin{cases} 0 & \text{if } \exists f(t) \mid f(t) \leq f_{\text{min}} \\ 1 & \text{otherwise} \end{cases}
\]

\[
R_{C_0} = R_{C_0}^P \times R_{C_0}^D
\]

\( R_{C_0} \) defines the value of assessed resilience for the criterion \( C \) induced by its behaviour considering the two limits \( f_{\text{max}} \) and \( f_{\text{min}} \).

- **Assertion (6):** Based on the Functionality Evaluation Model, the value of a given criterion must be between certain values.

Assertion (6) considers the objective on a criterion. For a given criterion, there is a normal functioning zone limited by two values \( f_{\text{min}} \) and \( f_{\text{max}} \). The value of assessed resilience of the system remains at 1 if the value of the criterion, calculated by the Functionality Evaluation Model, remains in this zone.

- **Assertion (7):** There is a decrease in the value of assessed resilience due to the value of the criteria dropping out of the limits \( f_{\text{min}} \) and \( f_{\text{max}} \).

Assertion (7) addresses the situations where the value of the criterion is between \( [f_{\text{min}}, f_{\text{max}}] \) or between \( [f_{\text{max}}, f_{\text{max}}] \). If the limit \( f_{\text{min}} = f_{\text{min}} \), or \( f_{\text{max}} = f_{\text{max}} \), the value of assessed resilience is calculated using Assertion (5).

Based on the FAM (e.g. see Figure 4), the more the value of criteria stays in the interval \( B \cup B' \), the lower the value of assessed resilience becomes. The value of assessed resilience induced by this behaviour is calculated by the resilience factor \( R_{C_1} \). It is assessed for every time-stamp.
R_{C1}^{up} = \prod_{i=1}^{u} \left( 1 - \frac{S_i}{S_i'} \right) \quad (7) \\
R_{C1}^{down} = \prod_{i=1}^{v} \left( 1 - \frac{S_i}{S_i'} \right) \quad (8) \\
R_{C1} = R_{C1}^{up} \times R_{C1}^{down} \quad (9)

R_{C1}^{up}$ is the factor when the value is greater than the normal maximum and $R_{C1}^{down}$ is the factor when the value is lower than the normal minimum. $S_i$ is the area of the profile in bad functioning and $S_i'$ is the total area of bad functioning at the $i^{th}$ bad functioning period, as shown in Figure 4. The two areas are calculated every time-stamp based on the Newton-Cotes quadrature rule (Dehghan, Masjed-Jamei, & Eslahchi, 2005).

Based on Assertion (7), at a given time-stamp $t$, if $S_i = 0$ then the system remains in the normal functioning interval $[f_{n \text{min}}, f_{n \text{max}}]$. So its value of assessed resilience is equal to 1 (fully resilient at $t$). In other words, the situation $S_i = S_i'$ describes the case where the system remains in the good functioning situation described by Assertion (5). The equality $S_i = S_i'$ describes the situation where the system reaches the maximum limits. In such a situation its value of assessed resilience is equal to 0.

- **Assertion (8): Based on a Functionality Evaluation Model, the closer the value of criteria is to the limit $f_{n \text{max}}$ or $f_{n \text{min}}$, the lower the value of assessed resilience is.

Figure 5(a) illustrates two FAMs, curves P1 and P2. The surface of $S_i$ and $S_i'$ are equal for both, but, P2 (in blue) has been closer to the limit $f_{n \text{max}}$ than P1 (in red). Resilience factor $R_{C2}$ is defined to consider such situations of proximity of criteria value to the limits.

$$R_{C2}^{up} = \prod_{t=0}^{\tau} \left( 1 - \frac{f_n(t)-f_{n \text{max}}(t)}{f_{n \text{max}}(t)-f_{n \text{min}}(t)} \right) \quad (10)$$

where $\tau$ is the duration of the simulation and $t$ is a given time-stamp. $R_{C2}^{up}$ is computed at every time-stamp of the simulation. So, the more $f_n(t)$ is closer to $f_{n \text{max}}(t)$, the lower the value of assessed resilience becomes. It is assumed that $f_{n \text{max}}$ and $f_n$ are different during $\tau$. In case of equality, Assertion (5) predominates. If $f_n$ and $f_{n \text{max}}$ are equal, the part induced by Assertion (8) is 1. If $f_n$ is inferior to $f_{n \text{max}}$ then Assertion (6) predominates.

Similarly, the value of assessed resilience related to the proximity of the value of a criterion to $f_{n \text{min}}$ is assessed by the following factors:

$$R_{C2}^{down} = \prod_{t=0}^{\tau} \left( 1 - \frac{f_{n \text{min}}(t)-f_n(t)}{f_{n \text{max}}(t)-f_{n \text{min}}(t)} \right) \quad (11)$$

$$R_{C2} = R_{C2}^{up} \times R_{C2}^{down} \quad (12)$$

- **Assertion (9): Based on a Functionality Evaluation Model, if the value of a criterion is between $f_{n \text{min}}$ and $f_{n \text{max}}$ or $f_{n \text{max}}$ and $f_{n \text{max}}$ (this situation is denoted as 'the system stays in a bad functioning period') for more than a given period, then the value of assessed resilience decreases.

As shown in Figure 5(b), the two profiles P2 and P3 have the same resilience factors $R_{C2}$ and $R_{C2}$. But P2 stays in bad functioning longer than P3. To consider this situation, the three proposals are defined hereafter:

**Proposal 1:** A social acceptance time $t_u$ is used when the value is greater than $f_{n \text{max}}$ and $t_d$ or lower than $f_{n \text{min}}$, $t_u$ and $t_d$ are defined considering one period of bad functioning. The value of assessed resilience becomes null if the system stays longer than $t_u$ or $t_d$. The limits $t_u$, $t_d$ depend on the context, the criterion, the system and the territory.

$$R_{C3}^{up} = \begin{cases} 0 & \text{if } \forall i \in [1 \ldots u], \exists U_i \in B[U_i \geq t_u] \\ \prod_{i=1}^{c} \left( 1 - \frac{U_i}{t_u} \right) & \text{otherwise} \end{cases} \quad (13)$$

$$R_{C3}^{down} = \begin{cases} 0 & \text{if } \forall i \in [1 \ldots d], \exists D_i \in B[D_i \geq t_d] \\ \prod_{i=1}^{d} \left( 1 - \frac{D_i}{t_d} \right) & \text{otherwise} \end{cases} \quad (14)$$

$$R_{C3} = R_{C3}^{up} \times R_{C3}^{down} \quad (15)$$

**Proposal 2:** There are situations where the system stays in a bad functioning state less than $t_u$ or $t_d$ but several times. This situation is managed by the social acceptance times $T_u$ and $T_d$ (related to the frequency of disruption) for all bad functioning periods of the simulation. $T_u$, $T_d$ depend on the context, the criterion, the system and the territory. The value of assessed resilience become null if:

- The duration of all bad functioning periods defined by $B$ is greater than $T_u$.

$$R_{C4}^{up} = \begin{cases} 0 & \text{if } \sum_{i=1}^{u} U_i \geq T_u \\ \sum_{i=1}^{u} \frac{U_i}{T_u} & \text{otherwise} \end{cases} \quad (16)$$

- The duration of all bad functioning periods defined by $B$ is greater than $T_d$. 

"Figure 7. A use case based on a network of infrastructures."
Proposal 3: In some situations, the constraint concerns the overall bad functioning time whether it is spent in minimal \((B_0)\) or maximal \((B)\) bad functioning. In such a situation, there is a global time \(T\) that cannot be exceeded whatever the disruption. The next factor considers this situation:

\[
R_{CS} = \begin{cases} 
0 & \text{if } \sum_{i=1}^d D_i + \sum_{i=1}^u U_i \geq T \\
1 - \frac{\sum_{i=1}^d D_i + \sum_{i=1}^u U_i}{T} & \text{otherwise}
\end{cases}
\]

\[ (19) \]

**Assertion (10):** The more often the value of a criterion goes outside of \(f_{min}\) and \(f_{max}\), the lower the value of the assessed resilience is.

Assertion (10) considers the behaviour of a system that goes into bad functioning periods more than once. Previous assertions consider some aspects of this assertion, but not all of them. Let us consider Figure 6. It shows two situations \(a'\) and \(b'\) with two profiles \(P1'\) and \(P2'\). The areas of \(P1'\) and \(P2'\) are the same. Nonetheless, \(P2'\) has two bad functioning periods. This assertion takes such a situation into account:

\[
R_{CS}^{up} = \prod_{i=1}^u \frac{1}{i} (i \geq 1)
\]

\[ (20) \]

\[
R_{CS}^{down} = \prod_{i=1}^d \frac{1}{i} (i \geq 1)
\]

\[ (21) \]
In some situations, Assertion (10) might not be mandatory. In fact, previous assertions already consider some of its aspects.

According to these assertions, it is now possible to assess the resilience, whether or not there is a disturbance. Actually, according to the objectives and constraints $\ell_{\text{max}}, \ell_{\text{max}}, \ell_{\text{min}}, \ell_{\text{min}}$, the system might be non-functional even though there is no disturbance. The changing of objective could change the FAM and consequently the value of assessed resilience. The identification of relevant criteria that characterise the system is the main challenge of this model. Criteria such as the number of times the system does not fulfil the objective on a criterion must not be considered. They are integrated in the previous assertions.

$$R_{C6} = R_{C6}^{\text{up}} \times R_{C6}^{\text{down}}$$  \hspace{1cm} (22)$$

The resulting value of the resilience for criterion $C$ is then:

$$R_C = R_{C0} \times R_{C1} \times R_{C2} \times R_{C3} \times R_{C4} \times R_{C5} \times R_{C6}$$ (23)$$

The resulting value of the resilience in Equation (23) uses a product model to aggregate resilience factors. According to the context, the resilience factors could be aggregated differently. Hence, instead of using a simple product in the assessment, other aggregation functions such as the mean and the median could be used. This is important for the adaptation of this research to other contexts. However, the study of these aggregation functions is outside the scope of this article.

4. Case study

The proposed methodology is applied to a simple case study. The aim here is to demonstrate the claims of the
The system considered here is a network of infrastructures composed of: (1) a coal power plant, (2) a road (transport system) and (3) a signalling system. The power plant produces a certain quantity of electricity and needs trucks to deliver coal. The amount of electricity depends on the overall production capacity and the quantity of coal in incoming trucks (i.e. the quantity of transported coal). The performance of the signalling system depends on the electricity used.

A bad performance impacts the safety of the road that furthermore influences the number of trucks in circulation (i.e. the quantity of transported coal to produce electricity). Therefore, a decreasing number of trucks decreases the electricity production of the power plant. Figure 7 depicts the relationship between the criteria of these systems, where the arrows represent the dependence relationship.

This example is modelled using the Obeo Designer platform, transformed based on an Acceleo transformation script and simulated based on the multi-agent platform GAMA. The focus here is on the simulation results. There are six criteria: The Quantity of Produced Electricity (QPE), the Number of Incoming Trucks (NIT), the Quantity of Required Electricity (QRE), the Performance (PER), the Safety (SAF) and the Number of Circulating Trucks (NCT). These criteria characterise the global system, as shown in the Figure 7. The proposal complies with Assertions 1 to 4; 6, defined in the previous section.

For the sake of space and simplicity, only the Quantity of Produced Electricity (QPE) criterion is discussed hereafter. The objective of the power plant, assumed for this case study, is the production of 5.0 units (e.g. gigawatts); authorised minimum and maximum are assumed to be
respectively 0.1 and 10. Note that Figure 3 defines the meta-model for these parameters. In normal functioning, the fluctuation might be between 3.0 and 7.0. Five units are tolerated if the objective is not fulfilled. The evolution function for the QPE, i.e. $f_n(t)$ is defined:

$$f_n(0) = 5.0$$
$$f_n(t + 1) = f_n(t) \pm \rho(0.5)$$

where $\rho(x)$ is a random function that for a given number $x$ returns a random number $n$ such that $0 \leq n \leq x$.

In addition, the value of $f_{\min}^n$, $f_{\max}^n$, $f_{\min}$ and $f_{\max}$ should change with respect to time. For instance, the power consumption of a city depends on seasons (e.g. electricity consumption increases during winter) and thus the minimal production of the power plant should be greater than 3.0. Consequently, the evolution functions defined by Equations (25)–(28) are assigned to $f_{\min}^n$, $f_{\max}^n$, $f_{\min}$ and $f_{\max}$. Note that for the sake of simplicity, these functions randomly change the value of their parameters:

$$f_{\min}(0) = 0.1$$
$$f_{\min}(t + 1) = f_{\min}(t) \pm \rho(0.1)$$

$$f_{\max}(0) = 10$$
$$f_{\max}(t + 1) = f_{\max}(t) \pm \rho(0.1)$$

$$f_{\min}^n(0) = 3.0$$
$$f_{\min}^n(t + 1) = f_{\min}^n(t) \pm \rho(0.1)$$

$$f_{\max}^n(0) = 7.0$$
$$f_{\max}^n(t + 1) = f_{\max}^n(t) \pm \rho(0.1)$$

For all situations the value of assessed resilience is between 0 and 1 and depends on all six criteria. However, there is no constraint on the duration of the bad
functioning periods, and consequently, Assertions 9 and 10 are not tested in this use case. For the other assertions, i.e. Assertions 5, 7 and 8, two simulations are proposed: one for the up value, and one for the down value. For each assertion, the simulation result shown here is for 12 timestamps. Figures 8–13 illustrate the evolution of $f_n$, $f_{\text{up}}$, $f_{\text{down}}$, $f_{\text{max}}(t)$ and the related resilience factor through 6 examples – Figures 8 and 9 present $R_{C0}$, Figures 10 and 11 illustrate $R_{C1}$ and Figures 12 and 13 illustrate $R_{C2}$.

Example 1 - $R_{C0}$: Figure 8 illustrates a disruptive event that causes a drop-up (overload) of the QPE that cannot be mitigated and causes a system crash. Resilience factor $R_{C0}$ remains 1 until the disruptive event occurs at timestamp 9 (see Figure 8(a)). After timestamp 9, a drop-up of the QPE causes the system to enter a bad functioning period. Shortly after, i.e. just before timestamp 10, the system enters an unacceptable state because the QPE becomes superior to $f_{\text{max}}$ and $R_{C0}$ becomes 0 (see Figure 7(b)).

Example 2 - $R_{C0}$: Figure 9 presents a disruptive event that causes a drop-down of the QPE, followed by a domino effect that causes a system crash. $R_{C0}$ remains 1 after the first disruptive event occurs at timestamp 2 because QPE is superior to $f_{\text{min}}$ (see Figure 8(a)). The disruption is managed at timestamp 5.6. However, the decrease in QPE provokes a domino effect (not enough coil) that causes another drop down of QPE at timestamp 6.2. This causes the system to enter an emergency state and $R_{C0}$ until timestamp 8.6 when QPE becomes inferior to $f_{\text{min}}$ and $R_{C0}$ reaches 0 (see Figure 8(b)).

Example 3 - $R_{C1}$: Figure 10 illustrates a disruptive event at timestamp 3.8 that causes a drop-down of the QPE. The drop-down of the QPE $<f_{\text{min}}$ causes the system to enter a bad functioning period. $R_{C1}$ is used to consider such a
situation. It starts decreasing after a certain period (see Figure 10(b)). When the system re-enters normal functioning (time-stamp 6.15), $R_{C_1}^{down}$ stops decreasing.

**Example 4 - $R_{C_1}^{up}$**: Similarly, Figure 11 depicts a drop-up of QPE from timestamp 3.4 to timestamp 5.8. The system enters a bad functioning period and thus the value of $R_{C_1}^{up}$ starts decreasing after timestamp 3 until timestamp 6, when the system re-enter normal functioning.

**Example 5 - $R_{C_2}^{down}$**: The proximity to the inferior limit induces a decrease in the value of assessed resilience, as shown in Figure 12. The value of QPE is outside the normal functioning at timestamp 2.6, reaching a minimal value at time-stamp 5, and re-enters normal functioning after time-stamp 6.7. The value of the resilience factor $R_{C_2}^{down}$ starts decreasing after time-stamp 2, because the value of QPE decreases during this period. $R_{C_2}^{down}$ reaches a minimal value at time-stamp 5 when the value of QPE reaches the lowest value.

**Example 6 - $R_{C_2}^{down}$**: With regard to the behaviour concerning proximity to the maximum limit, Figure 13 plots the evolution of the resilience factor in Assertion (8). For the time-stamps where the value of the criteria is outside normal functioning, the induced resilience factor is 0.72.

5. Conclusions

Infrastructures are nowadays facing an increasing number of disruptions, from natural disasters to terrorist attacks and internal failures. Mitigating these negative effects means that the infrastructures must recover their initial functioning capacity in minimum time, with minimum costs, i.e. they must be resilient. Thus, stakeholders must analyse the resilience of infrastructures before any disruption to anticipate the right decisions at the right time.
This article introduces a methodology for the evaluation of the functionality of interconnected systems for the purpose of resilience assessment. The methodology is designed to prepare stakeholders for different disruptions before they happen, allowing them to make the right decisions at the right time. The major contributions of the methodology are:

- It provides a relevant tool for the combination of FAMs and continuous resilience assessment following several dimensions of systems.
- It proposes a way to connect rough data with functional analysis and resilience assessment.
- It proposes an agile, evolutive and continuous resilience assessment paradigm (it can be extended with other criteria, other data sources to define criteria, other aggregation formulas, etc.).

On the other hand, the main limitation of the approach is its dependence on relevant input data. In other words, an incorrect domain knowledge as input will surely provide an incorrect evaluation. In this sense, it is unable to warn users about the inconsistency of their input data.

The first perspective of this study concerns Big-data and the way to exploit incoming data continuously from heterogeneous, various and non-dedicated data sources. The second perspective involves a better use of the timeframe and the way to manage the history of data (what timeframe should be considered to include data in the criterion calculation). Depending on the criterion and on the data sources, the volatility of data is very different and this aspect should be considered to refine and improve criteria evaluation. The third perspective concerns the aggregation functions. The product in the resilience assessment has been considered, however, other resilience functions could be used. The last perspective concerns guiding users to provide relevant data without errors and mistakes.

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