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## Seismic reconstruction using FWI with dual-sensors data.

Maarten V. de Hoop, Florian Faucher, Giovanni Alessandrini, Romina Gaburro, Eva Sincich, H el ene Barucq

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# Seismic reconstruction using FWI with dual-sensors data.

Florian Faucher<sup>1</sup>,

Giovanni Alessandrini<sup>2</sup>, H el ene Barucq<sup>1</sup>, Maarten V. de Hoop<sup>3</sup>,  
Romina Gaburro<sup>4</sup> and Eva Sincich<sup>2</sup>.

**GdR MecaWave, Fr ejus, France**

November 5<sup>th</sup>–9<sup>th</sup>, 2018



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<sup>1</sup>Project-Team Magique-3D, Inria Bordeaux Sud-Ouest, France.

<sup>2</sup>Dipartimento di Matematica e Geoscienze, Universit a di Trieste, Italy.

<sup>3</sup>Department of Computational and Applied Mathematics and Earth Science, Rice University, Houston, USA

<sup>4</sup>Department of Mathematics and Statistics, Health Research Institute (HRI), University of Limerick, Ireland.

# Overview

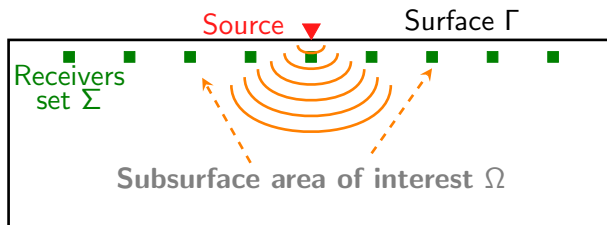
- 1 Introduction
- 2 Time-Harmonic Inverse Problem, FWI
- 3 Reconstruction procedure using dual-sensors data
- 4 Numerical experiments
  - Comparison of misfit functions
  - Changing the numerical acquisition with  $\mathcal{J}_{\mathcal{G}}$
- 5 Conclusion

# Plan

## 1 Introduction

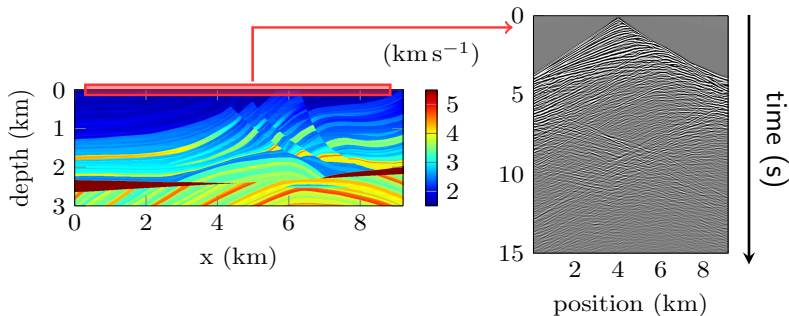
# Seismic inverse problem

Reconstruction of subsurface Earth properties from seismic campaign: collection of **wave** propagation data at the surface.



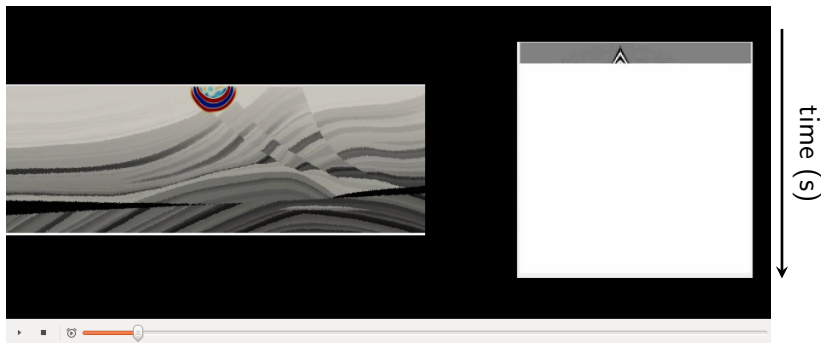
# Seismic data

We work with **back-scattered partial data** from **one-side illumination** on large domain.



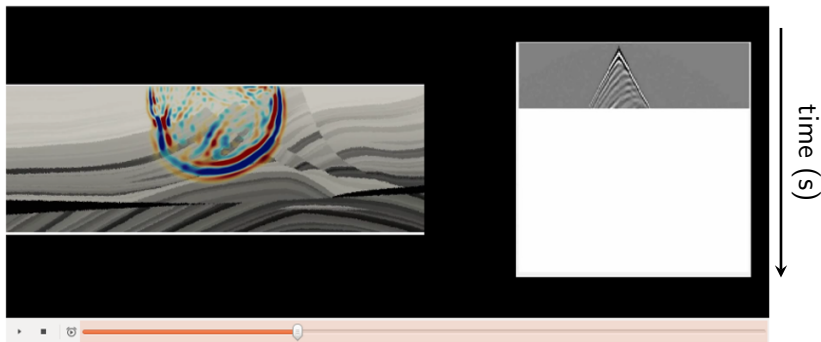
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# Seismic data

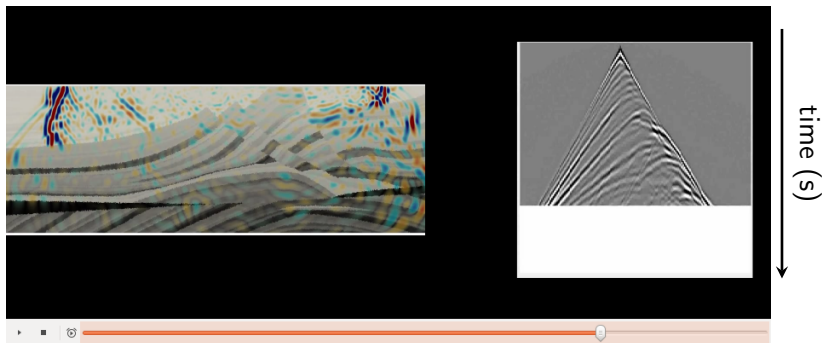
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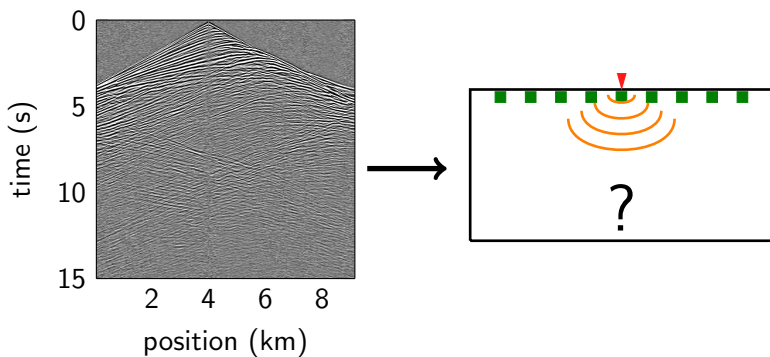
# Seismic data

We work with **back-scattered partial data** from **one-side** illumination on large domain.



# Seismic data

**Inverse problem:** from seismic traces to subsurface?



**nonlinear, ill-posed inverse problem.**

# Plan

## 2 Time-Harmonic Inverse Problem, FWI

# Time-harmonic wave equation

We consider propagation in acoustic media, given by the Euler's equations, for the recovery of the medium parameters  $\kappa$  and  $\rho$ :

$$\begin{cases} -i\omega\rho\mathbf{v} = -\nabla p, \\ -i\omega p = -\kappa\nabla\cdot\mathbf{v} + f. \end{cases}$$

$p$ : scalar pressure field,  
 $\mathbf{v}$ : vectorial velocity field,  
 $f$ : source term,

$\kappa$ : bulk modulus,  
 $\rho$ : density,  
 $\omega$ : angular frequency.

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$\omega$ : angular frequency.

The system reduces to the Helmholtz equation when  $\rho$  is constant,

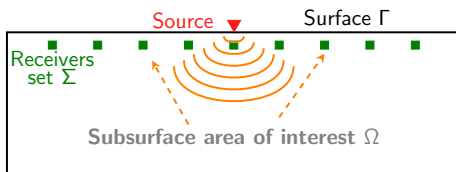
$$(-\omega^2 c^{-2} - \Delta)p = 0,$$

$$\text{with } c = \sqrt{\kappa\rho^{-1}}.$$

# Dual-sensors devices

The inverse problem aims the recovery of the subsurface medium parameters from surface measurements of pressure **and** normal (vertical) velocity:

$$\mathcal{F} : m = (\kappa, \rho) \rightarrow \{\mathcal{F}_p ; \mathcal{F}_v\} = \{p(\mathbf{x}_1), p(\mathbf{x}_2), \dots, p(\mathbf{x}_{n_{rcv}}); v_n(\mathbf{x}_1), v_n(\mathbf{x}_2), \dots, v_n(\mathbf{x}_{n_{rcv}})\}.$$



D. Carlson, N. D. Whitmore *et al.*

Increased resolution of seismic data from a dual-sensor streamer cable – Imaging of primaries and multiples using a dual-sensor towed streamer

SEG, 2007 – 2010



CGG & Lundun Norway (2017–2018)

TopSeis acquisition ([www.cgg.com/en/What-We-Do/Offshore/Products-and-Solutions/TopSeis](http://www.cgg.com/en/What-We-Do/Offshore/Products-and-Solutions/TopSeis))

# Full Waveform Inversion (FWI)

FWI provides a **quantitative reconstruction** of the subsurface parameters by solving a minimization problem,

$$\min_{m \in \mathcal{M}} \mathcal{J}(m) = \frac{1}{2} \|\mathcal{F}(m) - d\|^2.$$

- ▶  $d$  are the observed data,
- ▶  $\mathcal{F}(m)$  represents the simulation using an initial model  $m$ :



P. Lailly

The seismic inverse problem as a sequence of before stack migrations  
Conference on Inverse Scattering: Theory and Application, SIAM, 1983



A. Tarantola

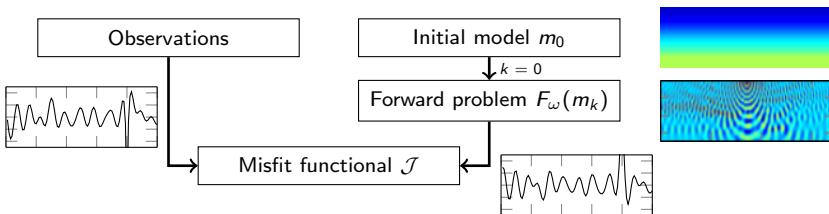
Inversion of seismic reflection data in the acoustic approximation  
Geophysics, 1984



A. Tarantola

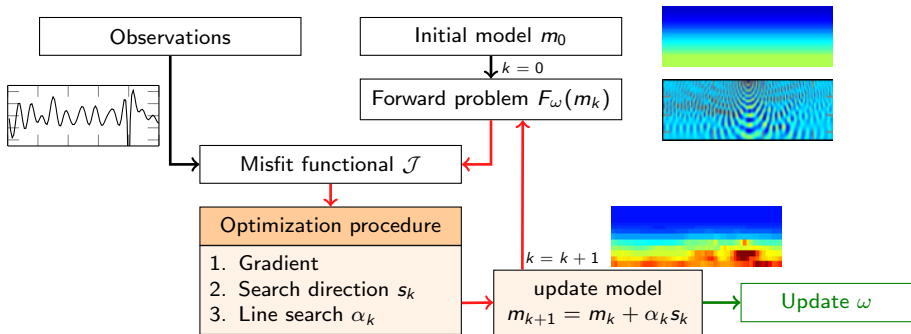
Inversion of travel times and seismic waveforms  
Seismic tomography, 1987

# FWI, iterative minimization

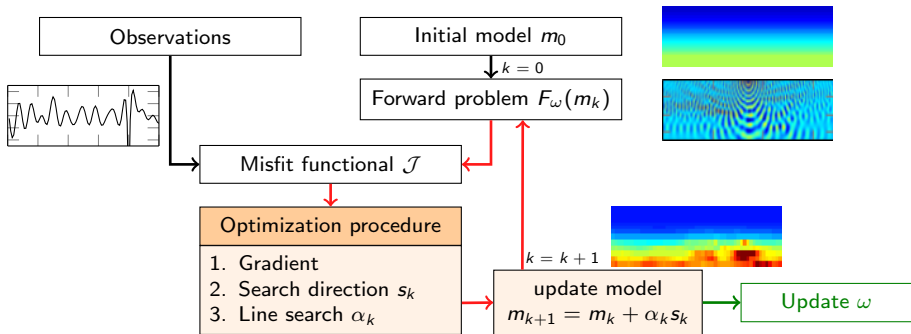




# FWI, iterative minimization



# FWI, iterative minimization



## Numerical methods

- ▶ Adjoint-method for the gradient computation,
- ▶ forward problem resolution with Discontinuous Galerkin methods,
- ▶ parallel computation, HPC, large-scale optimization,
- ▶ Rk: the code also works for **elastic** anisotropy and **viscous** media.

# Plan

## 3 Reconstruction procedure using dual-sensors data

# Minimization of the cost function

The appropriate misfit functional to minimize with pressure and vertical velocity measurements.

- ▶ Compare the pressure and velocity fields separately:

$$\mathcal{J}_{L2} = \sum_{source} \frac{1}{2} \|\mathcal{F}_p^{(s)} - d_p^{(s)}\|^2 + \frac{1}{2} \|\mathcal{F}_v^{(s)} - d_v^{(s)}\|^2.$$

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- ▶ Compare the fields multiplication for all combinations:

$$\mathcal{J}_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization  
[preprint](#)

# Minimization of the cost function

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

From Euler's equation,  $v_n(\mathbf{x}_i) = -i(\omega\rho)^{-1} \partial_n p(\mathbf{x}_i)$ .

# Minimization of the cost function

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

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- ▶ **Cauchy data:** the cost function follows **Green's identity**.

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From Euler's equation,  $v_n(\mathbf{x}_i) = -i(\omega\rho)^{-1} \partial_n p(\mathbf{x}_i)$ .

- ▶ **Cauchy data**: the cost function follows **Green's identity**.
- ▶ **Reciprocity gap functional** in inverse scattering.



D. Colton and H. Haddar

An application of the reciprocity gap functional to inverse scattering theory  
*Inverse Problems* 21 (1) (2005), 383398.



G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich

Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data  
[arXiv:1702.04222](https://arxiv.org/abs/1702.04222), 2017



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization  
[preprint](#)



# Stability results

**Lipschitz type stability** is obtained for the Helmholtz equation with **partial Cauchy data**.

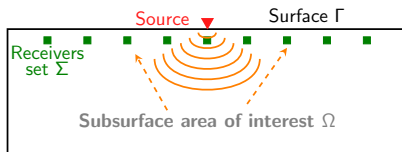
$$\|c_1 - c_2\| \leq \mathcal{C}(\mathcal{J}_{\mathcal{G}}(c_1, c_2))^{1/2}$$

# Stability results

Lipschitz type stability is obtained for the Helmholtz equation with **partial Cauchy data**.

$$\|c_1 - c_2\| \leq \mathcal{C}(\mathcal{J}_{\mathcal{G}}(c_1, c_2))^{1/2}$$

- ▶ Using back-scattered data from one side in a domain with free surface and absorbing conditions,



- ▶ for piecewise linear parameters.



G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich

Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data  
arXiv:1702.04222, 2017



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## Additional possibilities

It allows the non-collocation of numerical and observational sources:

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

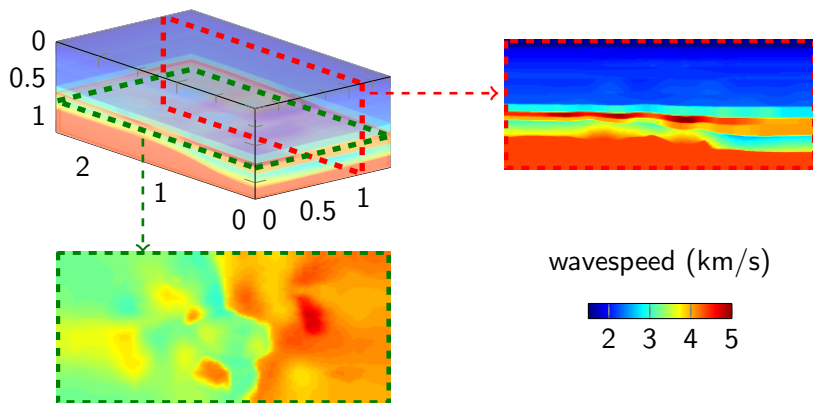
- ▶  $s_1$  is fixed by the observational setup,
- ▶  $s_2$  is chosen for the numerical comparisons.

# Plan

- 4 Numerical experiments
  - Comparison of misfit functions
  - Changing the numerical acquisition with  $\mathcal{I}_{\mathcal{G}}$

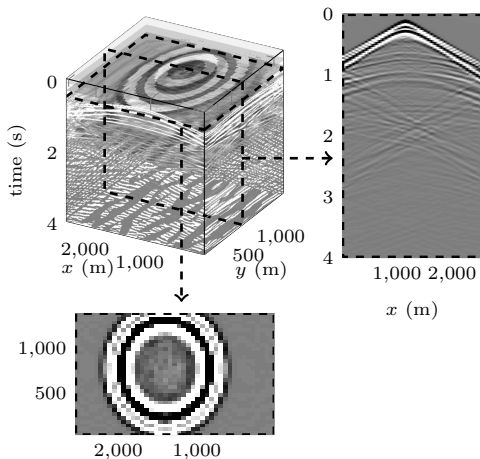
# Experiment setup

3D velocity model  $2.5 \times 1.5 \times 1.2\text{km}$  using dual-sensors data.



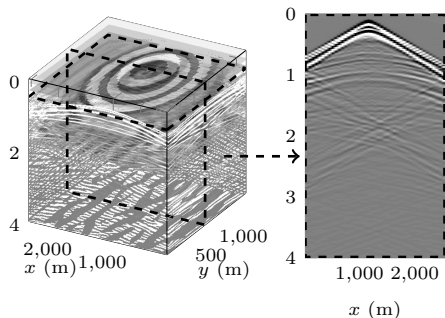
# Experiment setup

We work with time-domain data acquisition.



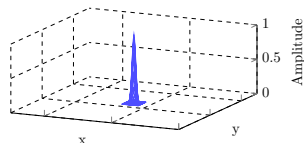
# Experiment setup

We work with time-domain data (pressure and velocity).



Acquisition for the measures

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,



For the reconstruction, we apply a Fourier transform of the time data.

# Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of  $\mathcal{J}_{L2}$  or  $\mathcal{J}_{\mathcal{G}}$ , frequency from 3 to 15Hz.

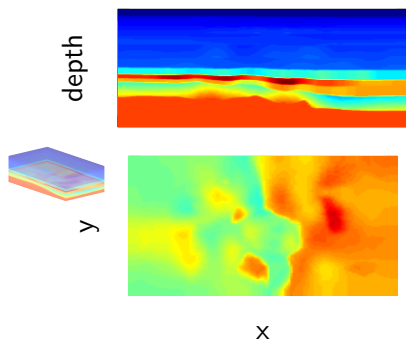
$$\mathcal{J}_{L2} = \sum_{source} \frac{1}{2} \|\mathcal{F}_p^{(s)} - d_p^{(s)}\|^2 + \frac{1}{2} \|\mathcal{F}_v^{(s)} - d_v^{(s)}\|^2.$$

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{source} \sum_{source} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

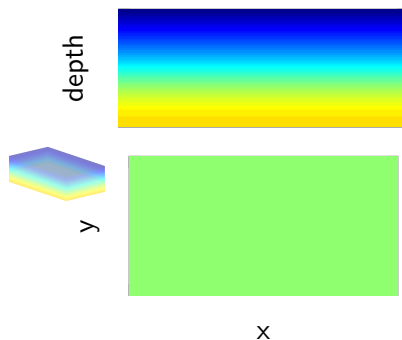


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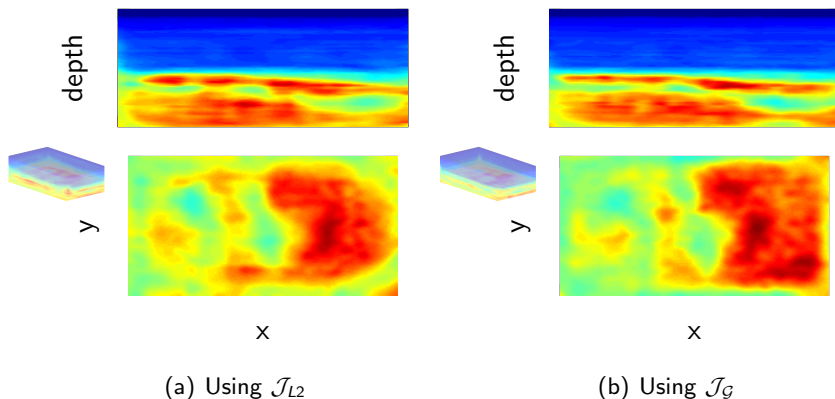
(a) True velocity



(b) Starting velocity

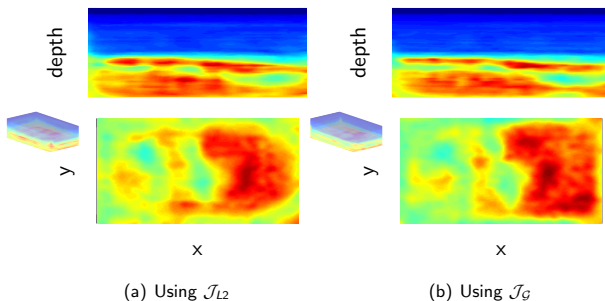
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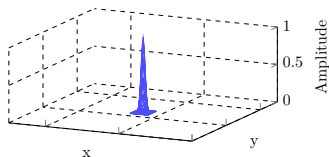
**But the major advantage of  $\mathcal{J}_G$  is the possibility to consider alternative acquisition setup.**

# Experiment with different obs. and sim. acquisition

$$\min \mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

Acquisition for the measures  $s_1$

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,

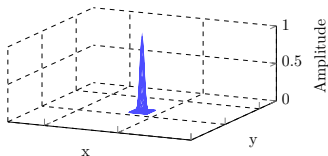


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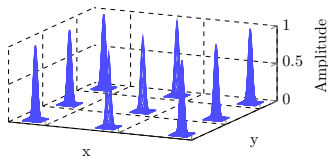
Acquisition for the measures  $s_1$

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,



Arbitrary numerical acquisition  $s_2$

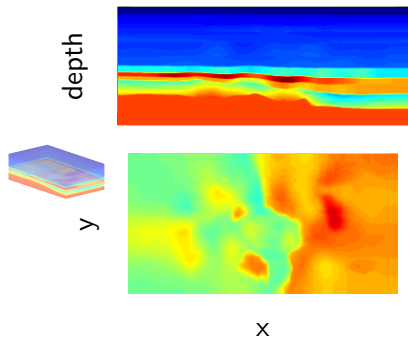
- ▶ **5 sources,**
- ▶ **80m depth,**
- ▶ **multi-point sources,**



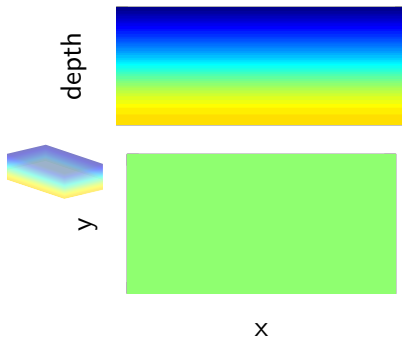
- ▶ No need to know observational source wavelet.
- ▶ Differentiation impossible with least squares types misfit.

# Experiment with different obs. and sim. acquisition

Data from frequency between 3 to 15 Hz, domain size  $2.5 \times 1.5 \times 1.2$  km, **Simulation using 5 sources only.**



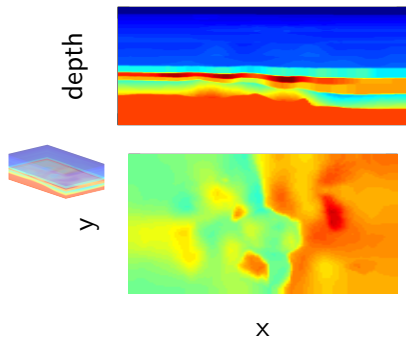
(a) True velocity



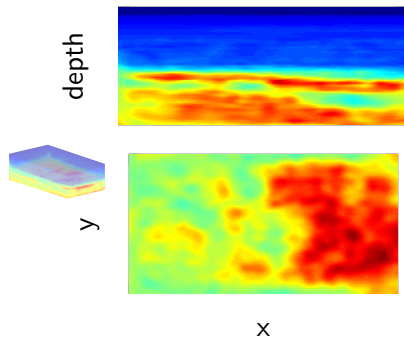
(b) Starting velocity

# Experiment with different obs. and sim. acquisition

Frequency from 3 to 15 Hz,  $2.5 \times 1.5 \times 1.2$  km,  
Simulation using 5 sources only. -33% computational time.



(a) True velocity



(b) 15 Hz reconstruction

# Plan

## 5 Conclusion



# Conclusion

Seismic inverse problem using pressure and vertical velocity data:

- ▶ appropriate cost function to minimize,
- ▶ allow minimal information on the acquisition setup,
- ▶ other applications,
- ▶ **perspective**: design the most efficient numerical setup,
- ▶ Rk: possible for elastic media with measures of traction.

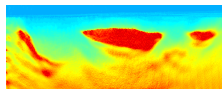
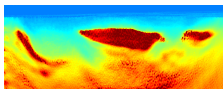
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Quantitative reconstruction algorithm toolbox for time-harmonic wave,

- ▶ **Discontinuous Galerkin** discretization in HPC framework,
- ▶ large scale optimization scheme using back-scattered data,
- ▶ **acoustic, elastic, anisotropy, 2D, 3D, attenuation.**



P- and S-wavespeed reconstructions

# Conclusion

Seismic inverse problem using pressure and vertical velocity data:

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Quantitative reconstruction algorithm toolbox for time-harmonic wave,

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THANK YOU

# APPENDIX

# Stability of the Helmholtz Inverse Problem

$$\|c_1^{-2} - c_2^{-2}\| \leq C(\|F(c_1^{-2}) - F(c_2^{-2})\|)$$



G. Alessandrini

Stable determination of conductivity by boundary measurement  
[Applicable Analysis 1988](#)



N. Mandache

Exponential instability in an inverse problem for Schrödinger equation  
[Inverse Problems 2001](#)

# Stability of the Helmholtz Inverse Problem

$$\|c_1^{-2} - c_2^{-2}\| \leq C(\|F(c_1^{-2}) - F(c_2^{-2})\|)$$

target      initial model      observation      simulation

$\delta$

- ▶ Stability associate data and model correspondence
- ▶ Reconstruction is based on the iterative minimization of the difference between observation and simulation using an initial model.



G. Alessandrini

Stable determination of conductivity by boundary measurement  
[Applicable Analysis 1988](#)



N. Mandache

Exponential instability in an inverse problem for Schrödinger equation  
[Inverse Problems 2001](#)

# Stability of the Helmholtz Inverse Problem

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target      initial model      observation      simulation

$\delta$

- ▶ Stability associate data and model correspondence
- ▶  $C(\delta) \leq C(\log(1 + \delta^{-1}))^{-\alpha}$



G. Alessandrini

Stable determination of conductivity by boundary measurement  
Applicable Analysis 1988



N. Mandache

Exponential instability in an inverse problem for Schrödinger equation  
Inverse Problems 2001

## Conditional Lipschitz stability: assumptions

- ▶  $c(x)$  is bounded  $B_1 \leq c^{-2}(x) \leq B_2$  in  $\Omega$
- ▶  $c(x)$  has a **piecewise constant** representation of size  $N$

$$c(x)^{-2} = \sum_{k=1}^N c_k \chi_k(x)$$

- ▶  $\Omega$  has Lipschitz boundary

$$\|c_1^{-2} - c_2^{-2}\|_{L^2(\Omega)} \leq \mathcal{C} \|F(c_1^{-2}) - F(c_2^{-2})\| \quad (1)$$



G. Alessandrini and S. Vessella

Lipschitz stability for the inverse conductivity problem  
[Advances in Applied Mathematics 2005](#)



E. Beretta, M. V. de Hoop, F. and O. Scherzer

Inverse boundary value problem for the Helmholtz equation: quantitative conditional Lipschitz stability estimates.

[SIAM Journal of Mathematical Analysis 2016](#)



The stability constant is bounded

$$\frac{1}{4\omega^2} e^{K_1 N^{1/5}} \leq \mathcal{C} \leq \frac{1}{\omega^2} e^{(K(1+\omega^2 B_2) N^{4/7})} \quad (2)$$

- ▶ depends on the partitioning  $N$  and the frequency  $\omega$



E. Beretta, M. V. de Hoop, F. and O. Scherzer

Inverse boundary value problem for the Helmholtz equation: quantitative conditional Lipschitz stability estimates 2016

## Conditional Lipschitz stability for Cauchy data

In the case of partial Cauchy data ( $p$  and  $\partial_\nu p$ ), we have that, we can obtain a Lipschitz type stability:

$$\|c_1^{-2} - c_2^{-2}\| \leq \mathcal{C}(\mathcal{J}_G(c_1^{-2}, c_2^{-2}))^{1/2}$$

Where  $c_1^{-2}$  and  $c_2^{-2}$  are piecewise linear.