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Errors or didactic resources:

A teacher education task in the context of probability

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In this paper we discuss a teacher education task centred on the request to interpret the reasoning of students addressing a ‘throw of the dice’ task. The task for teachers was designed after an analysis of class excerpts, carefully selected by our research group because of the interesting educational reflections to which they gave rise. The goal of the task is to support prospective teachers to develop interpretive ability in order to use errors as a didactic resource and to help students build their mathematical knowledge starting from their reasoning.

Keywords: Probability, teacher education, task design.

Introduction

In the framework of PISA 2015 the domain “Uncertainty and Data” is put at the core of the mathematical cultural experience. Items about this topic are, by now, present in all the national assessment tests and the bad students’ results to these items, both in national and international tests, put in evidence the need to improve the teaching of Probability and Statistics in school. But this request, as is often the case, is not adequately satisfied, at least in Italy, by a specific plan of teacher education.

Many studies (see, e.g., Batanero, Godino & Roa, 2004) have highlighted that teachers do show interest for this topic but, at the same time, declare to feel a lack of knowledge and experience in it. Indeed, on one side probability was not included in school curricula until a few years ago, and, on the other side, many teachers, even with a Master Degree in Mathematics, did not take the exams of Probability or Statistics in their plan of studies, as in the past it was an optional exam. Nevertheless, it is well known that the success of any educational design aimed at developing students’ knowledge and skills in Probability, as well as for any other topic, depends in great part on teachers’ knowledge and attitude toward the topic itself.

In this study, we refer to Mathematical Knowledge for Teaching (MKT) framework that serves as a resource for specifically addressing the mathematical demands of teaching (Ball, Thames & Phelps, 2008). More and more researches all around the world are demonstrating the effectiveness of this frame for teacher education.

Several studies (Tversky & Kahneman, 1974; Jones, Langrall & Mooney, 2007), starting from the fact that situations of uncertainty are widely present in daily life, show that very often the common sense or the experience linked to gamble can conflict with the mathematical management of the situation. For this reason we think that it is crucial to design practices of teacher education in this topic which take into account students’ answers and look at them as didactic resources (Borasi,

1994), even when these answers seem to be in conflict with the formalization goal of the educational design.

In this paper we discuss a particular teacher education task in teaching Probability. Its design and implementation is rooted in the “modus operandi” of our research team, in which university researchers, in-service school teachers and Master students are involved as a co-learning community throughout an inquire research experience (e.g., Jaworski & Goodchild, 2006). We will explain the rationale that guided us in the choices of some transcripts of classroom discussions, used to design a task for in-service teachers that, in our opinion, should be considered the final product of a long training path and an effective tool to develop their interpretive skills. Finally, we try to answer to the question whether this particular type of task could be used also with perspective teachers, in order to scale up the reflections and the outcomes developed in our co-learning community.

Theoretical framework

Teacher knowledge and, in a broader sense, teachers’ attitudes and goals, play a key role in any effort of educational innovation. Among the different conceptualizations of teacher knowledge, the MKT effectively describes the particular features of mathematical knowledge needed for teaching (Ball, et al., 2008). In particular, many studies underline that, in order to improve the teaching of Probability, it is crucial to support teachers’ training not only in the Common Content Knowledge (CCK), subdomain of MKT, i.e. the general knowledge of a math topic, but also in terms of the Specialized Content Knowledge (SCK), that is the knowledge about the topic specifically connected with its teaching. However, to develop SCK in the context of Probability, different aspects need to be taken into account (see, e.g., Ponte, 2008): i) epistemological reflections on the meaning of concepts (e.g., different meanings of probability); ii) awareness of the closed connections between probabilistic and statistical issues; iii) students’ learning difficulties, errors, obstacles and counterintuitive ideas in this field; iv) necessity to plan assessment tests and instruments for interpreting students’ responses.

In this study, we focus on the third point of the previous list, looking at errors and non-standard reasoning not as something to avoid but rather as a source to be capitalized, that really shapes the dynamics in mathematics educational process (Borasi, 1996). Indeed, in our perspective, one of the main task for a mathematics teacher is to grasp the “meaning” of students’ answers, in order to develop their mathematical knowledge starting from students’ reasoning.

This aspect is even more crucial in Probability than in other fields, considering that all the typical technical terms (probable, possible, random, event) are widespread common words. The epistemological consequent conflict can be faced only if teachers get aware that, according to Borasi (1996), ambiguities, in particular linguistic ones, can play a useful role in the development of mathematics learning. This peculiarity makes students, and, sometimes also teachers, more susceptible to counterintuitive ideas, which, in addition, arise in Probability already at an early level, more than in other branches of mathematics (Borovcnik & Peard, 1996).

We are firmly convinced that improving students’ mathematical knowledge on the basis of their arguments, even when they seem naïve, requires that teachers activate a real process of interpretation, shifting from an *evaluative listening* to a more flexible *hermeneutic* listening (Davis,

1997). In this peculiar field, the wrong answers of students often arise from well-known misconceptions (Ang & Shahrill, 2014), the same ones that afflict teachers, even in service, as discussed in (Batanero, Godino & Cañizares, 2005). According with this vision, we have proposed a design of tasks to develop teachers' SCK, specifically addressed to support them in making didactic choices fit to develop students' mathematical knowledge, starting from their reasoning (Ribeiro, Mellone Jakobsen, 2016). The tasks we have designed address the difficulties encountered by students in coordinating the classic and frequentist approaches to probability, above all with respect to the representativeness of the sample, and in recognizing the notion of equiprobability as a basic aspect of any probabilistic judgement. These tasks come as a final result of a training path in our research group, with the aim to enhance teachers' professional development. But, after experiencing the task within our community, we also decided to propose a similar task for teachers' learning, because we agree with (Robutti et al., 2016) that it is necessary to scale up the significant experienced practices for teacher professional development.

Methodology

The particular task we are going to present was developed during the meetings of our research group. In the last two years, our group have been working as a co-learning community, where educators :

(...) create opportunity to work with teachers, to ask questions and to see common purposes in using inquiry approaches that bring both groups closer in thinking about and improving mathematics teaching and learning. (Jaworski & Goodchild, 2006, p. 354)

This group is composed by five university researchers (two in mathematics education and two in physics education), fifteen in-service teachers with a long experience (one kindergarten teacher, five teachers from primary school, two from first order secondary school and 7 from second order secondary school), and some Master students in Mathematics or Education who have been working in their master thesis with us. Most of the teachers have been collaborating with the researchers since 2004, some were also involved in a three-year project (PDTR, Professional Development of Teacher-Researchers, www.pdtr.eu), financed by the European Community, aiming at “engaging classroom teachers of mathematics in the process of systematic, research-based transformation of their classroom practice” (Malara & Tortora, 2009 for the Italian contribution). It is worthwhile to say that we have found several similarities with the method of work of the mathematics learning communities in Norway described in (Robutti et al., 2016).

In the last two years, we decided to start a reflection about Probability and we used to meet once a month for three hours at the Department of Mathematics of the University of Naples. The training path was divided into two parts. Firstly, we organized several theoretic seminars on the epistemological bases of Probability, the connections between Probability and Statistics and on the necessity to enhance the study of Probability at school. Secondly, our meetings were devoted to plan didactic activities in Probability, from primary to secondary school. Each teacher had the possibility to adapt the didactic paths to his/her school level according with the national curricula. Successively, the teachers were asked to illustrate the outcomes of the didactic activities in their classrooms, sharing with the community audio and video recordings of class discussions, and samples of students' productions.

In this paper, we focus on some excerpts of the discussions occurred in Piera's (one of the author of this paper, and expert secondary teacher of the group) classroom of 10th grade students.

The class discussions were registered and transcribed thanks to the help of Laura (another author of the paper) who have been working to her Master Degree Thesis in Mathematics on the experimental educational activities on Probability, carried out in Piera's classes (Afeltra, 2015). Piera had already involved her two 10th level classes in inquiry based activities about descriptive statistics and her students were quite used to discuss each other, in order to deepen their understanding of what they were asked to study. The didactic path had been planned in the learning community with the aim to check if her students would have evidenced the predicted difficulties, also considering that most of them had studied the early elements of Probability in the previous years, often with a formula based approach. Among the others, we selected those class excerpts that gave rise to deep discussions during our meetings, about the possible interpretation of students' answers, as we will illustrate in the following. In order to mobilize teachers' SCK, we focused our attention on the following two frequent students' errors: to view different outcomes of an event as always equally likely (within a classic approach to Probability); and to consider a too small sample as representative to estimate the probability (within a frequentist approach).

These excerpts had a significant impact on the professional development of all in-service teachers, since most teachers, with a master degree in mathematics, seemed not able to act as students, because of a clear prevalence of their CCK on the SCK, necessary to predict students' learning difficulties. In our learning community, Piera's classroom excerpts were considered particularly interesting to be used for a pre service teachers' training activities, as they show, even on a small sample, that these misconceptions are a common way of thinking, so a teacher has to be ready to appreciate the opportunity to transform a wrong answer into a challenging one.

A teacher education task

In the following, we present the teacher education task based on excerpts of a class discussion on the game of tossing two dice, indeed, a very rich and complex context, in spite of its apparent simplicity. In fact, it is one of the typical contexts used to introduce Statistics and Probability, nevertheless several research studies show that both students and teachers can run into counterintuitive ideas about the representativeness of the sample, when rolling one die, and about the item of equiprobability, when playing with two dice (see, e.g. Batanero et al., 2005). Moreover, this context offers many interesting experiences involving the necessity to handle at the same time different approaches to Probability. It is necessary that students become aware of the intrinsic limits of the two approaches: in the classical case, it may be all but easy to identify all the possible equally probable cases, whilst in the frequentist approach it may be not possible to repeat a large number of trials at the same conditions. On the other hand, students who have already studied some elements of Probability in their previous scholar levels can get quite confused about the probability of an event, when they may calculate it in two different ways, for example when playing with dice, but with a small sample. Indeed, the game with dice, as all kinds of gamble, allows to introduce the Law of the Large Numbers in a significant way.

The task is organized as a questionnaire composed of three items, two of them centred on class discussion's excerpts. In the following, we comment on each item, and, in particular, on the class

excerpt, to illustrate the reasons why we consider them meaningful to be interpreted and interesting from the teacher education's point of view.

Item 1

Read and analyse the following class excerpt.

Teacher: Playing with one die, how can we measure the probability of each side?

Daniele: There is no favourite side, so for each of them the probability is $1/6$.

Federica: But rolling the die, we have got five 8 times over a total of 63. So the probability is 8 over 63, that is approximately 0,127.

Daniele: One sixth is 0,167, they are not the same!

Federica: Yes, we can use both definitions... but we have studied that in this case it is certainly one sixth!

- a) For each students' answer discuss if, in your opinion, it is mathematically correct or not;
- b) Plan a suitable didactic action to develop students' learning, starting from their answers.

We consider this excerpt as a good starting point to discuss the link between the classic approach and the frequentist one to Probability. The transcript shows that Federica is firmly convinced that the classical definition of Probability is prevalent over the frequentist approach. On the other hand, the play with one die is a very well known situation, and the "a priori" probability is so simple and evident that Federica's answer is quite "reasonable". In our opinion, the thought-provoking thing is her absolute indifference to the value of the relative frequency obtained as the result of rolling the die several times. Many speculations are possible: Federica's certainty can come from a previous formula-based approach to probability, experienced in previous years, or, instead, from the awareness that the frequentist definition needs a large number of data. In both cases, the item allows to reflect on the necessity that teachers clarify the question with their students. Moreover, it shows the importance to involve students in activities of data collection, which gives the chance to significantly introduce the Law of Large Numbers, also, maybe, with the aid of a digital simulator, to work with large samples.

Item 2

During a lesson on Probability, a teacher submits to his/her students the following question: "When two dice are simultaneously thrown, what are the chances of obtaining 7 as the sum of the two sides up?"

- a) Answer the question, as you are able to do; b) Which possible answers, correct or not, in your opinion, will be given by students? c) Discuss about the possible difficulties that, in your opinion, a student will encounter when addressing this question.

The first item of the task aims to put teachers at the place of students. The whole task involves teachers in a brainstorming activity, mobilizing their CCK about the topic. In this way, teachers can recognize a didactic obstacle, and, consequently, realize the need to design an educational path to face it. Moreover, in the following item, the teachers can confront their own answers with those given by the students.

Item 3

In the following excerpt it is reported a discussion between a teacher and his/her students about the previous question.

Teacher: When two dice are simultaneously thrown, what are the chances of obtaining 7 as sum of the two numbers?

Gianluigi: There are 11 possible sums, from 2 to 12, so the probability is $1/11$.

Ludovica: No, it would be true if we used a die with 11 sides. We have to consider all the possible couples, such as one and one, one and two and so on.

Teacher: Ok. And then, how many?

Students work in groups for a while until one of them affirms:

Massimo: There are 21 possible couples.

- a) For each students' answer, discuss if, in your opinion, it is mathematically correct or not;
- b) Identify a set of possible questions that you would pose to students to support them their learning process.

According to the classic definition of probability, Gianluigi's answer has to be considered wrong, since the 11 values of the sum are not equally likely. But Gianluigi's answer opens up the reflection about the difference between the sample space and the possible cases. The transcripts were enlightening for most teachers in our community. Even Piera, one of the author, witnessed that, in the previous years, on the basis of her CCK only, she would have rejected this answer, immediately underlining that the eleven possible outcomes are not equally possible. In the light of the reflections previously made in our community, this time she rather decided to give space to the classroom discussion. Moreover, Ludovica smartly refers to another context, where 11 would have been the correct answer, and this seems to us a significant process to support the construction of meaning, above all because it arose from students.

Even Massimo's answer, which is caused by the common belief that, throwing two dices, the order is not relevant (see, e.g., Batanero et al., 2005), is wrong but interesting. Indeed, we have selected this excerpt because, in our meetings, it triggered many discussions about the didactic choice, for example, to use or not dice of different colours, in order to avoid this error. Finally, we agreed on the opportunity to use dice of the same colours in order to open up the possibility that reasoning as Massimo's one emerge.. In this way, a teacher has the possibility to ask students to look for contexts different from the dice game, where the order turns to be relevant. In our meetings, many ideas were proposed: the most popular examples were contexts of sport competitions, while a more sophisticated context could be the Bose–Einstein statistics. Only on the basis of her learning's development, due to the participation in our co-learning community, the teacher was able to guide her students in finding a possible context in which Massimo's answer could be correct, activating in this way an hermeneutic listening (Davis, 1997) Her actions helped students not only to better understand the crucial role of equiprobability, but also to describe mathematical models and formulate hypotheses about them.

Conclusions

For several years researchers in probability education have been investigating the counterintuitive ideas arising when people formulate judgements in real situations involving uncertainty (Tversky & Kahneman, 1974). These ideas are often in conflict with the mathematical formalization of Probability, so they can cause, if not recognized, paradoxical results. It is widely known that a correct probabilistic reasoning needs a specific instruction (Fischbein, 1975), therefore, it is fundamental that a teacher is able to use students' intuitions, their counterintuitive answers or incorrect reasoning as resources for learning (Borasi, 1994). We think that this work is particularly useful for Probability, even more than for other mathematics topics, since, in this field, widespread counterintuitive ideas arise from many daily situations.

Our research group has been working as a co-learning community (Jaworski & Goodchild, 2006), where the professional development of all the participants was enhanced throughout inquiry activities, in order to improve learning from experience and reflection. In this paper we have focused on the process which led us to design a teacher education task, starting from the analysis of excerpts of classroom activities. We selected the excerpts most challenging to be converted in interpretative tasks for teacher training, with the aim to improve teachers' SCK about some crucial notions of Probability. The deep impact observed on the Math teachers of our community encourages us to redesign a similar path to be used for teacher training on a larger scale, as an attempt to face the issue of dissemination of the significant practices experienced in a co-learning community, as described in (Robutti et al., 2016). In this direction many questions are still open, and need further investigation. However, we are just now exploring the real effectiveness of this types of task with in-service and perspective teachers, their difficulties and limits.

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