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# Phase field modeling of hydraulic fracture in heterogeneous media with interfacial damage

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**Résumé** — In this work, we propose a numerical framework for modeling hydraulic fracturing or cracking in saturated porous media, taking into account : (a) the presence of heterogeneities ; (b) interfacial damage between the inclusions and the matrix ; (c) the possibility to define the microstructures in the form of regular grids of voxels e.g. as obtained from experimental imaging techniques. The developed numerical framework employs the phase field method with a regularized description of both bulk and interface cracks, extended in a fully hydro-mechanical context. We present applications in both 2D and 3D complex hydro-mechanical microcracking initiation and propagation in voxel-based models of heterogeneous media with interfacial damage between the inclusions and the matrix.

**Mots clés** — fracture, phase field, heterogeneous materials, hydro-mechanical coupling, interfaces.

## 1 Introduction

The computational modeling of fracturing in fluid-saturated porous media is of essential importance for numerous practical applications in geotechnical, environmental, petroleum engineering and biomechanics. Typical applications include the caprock integrity during the geological gas storage [10], nuclear waste storage and hydraulic fracturing for oil and gas extraction. The computational modeling of hydraulic fracturing, the so-called “fracking”, has attracted a special research attention due to the growing interest of the petroleum industry (see e.g. [1]). To optimize the hydraulic fracturing processing so as to maximize the extraction while preventing potential environmental contamination, there is a great necessity to develop efficient and robust numerical methods for the modeling of the hydraulic fracturing processing.

Several methods have been developed to simulate the hydraulic fracturing or crack propagation in fluid-saturated porous media such as the cohesive zone model, adaptive meshing strategies [9], approaches based on lattice, particle models, or discrete elements [2, 3] or extended finite element method (XFEM) [8, 6].

In this work, we propose an extension of the phase field model for the modeling hydraulic fracturing or cracking in saturated porous media, taking into account : (a) the presence of heterogeneities ; (b) interfacial damage between the inclusions and the matrix ; (c) fluid flow within both matrix cracks and interfacial cracks ; (d) the possibility to define the geometry of the heterogeneous media in the form of regular grids of voxels e.g. as obtained from experimental imaging techniques. The developed numerical framework employs the phase field method with a regularized description of both bulk and interface cracks, extended to a fully coupled hydro-mechanical framework.

## 2 Diffuse approximation of interfacial discontinuities

Let  $\Omega \in \mathbb{R}^D$  be an open domain  $D \in [2, 3]$  describing an heterogeneous medium composed of an homogeneous porous matrix including elastic inclusions. The external boundary of  $\Omega$  is denoted by  $\partial\Omega \in \mathbb{R}^{D-1}$ . The internal interfaces between the porous medium and the inclusions are collectively denoted by  $\Gamma^I$ . Crack which may propagate in the porous medium and pass through the interfaces during the loading as depicted in Fig. 1 are collectively denoted by  $\Gamma$ . In this work, we adopt the regularized framework proposed e.g. in [4] for regularized representation of both crack and interface. In this regularized

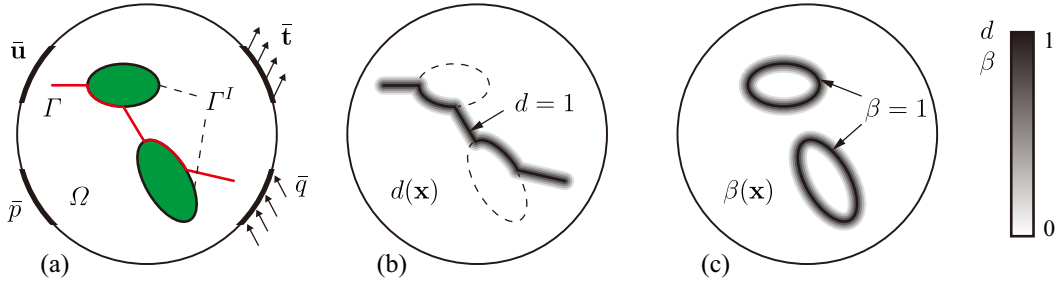


FIGURE 1 – Diffuse approximation of cracks and interfaces : (a) a porous medium containing two inclusions and a crack passing through the interfaces together with prescribed boundary conditions ; (b) diffuse representation of the crack ; (c) diffuse representation of the interfaces.

framework, the cracks are approximately represented by a scalar phase field  $d(\mathbf{x}, t)$  (see e.g. [4]) and the interfaces are represented by a fixed scalar function  $\beta(\mathbf{x})$ .

The diffuse approximation of damageable interfaces was introduced in [7]. The scalar interface function  $\beta(\mathbf{x})$  is determined in the same manner as  $d(\mathbf{x}, t)$  by solving the boundary value problem subjected to Dirichlet conditions  $\beta = 1$  on the interfaces :

$$\begin{cases} \beta(\mathbf{x}) - \ell_\beta^2 \nabla^2 \beta(\mathbf{x}) = 0, & \text{in } \Omega \\ \beta(\mathbf{x}) = 1, & \text{on } \Gamma^I \\ \nabla \beta(\mathbf{x}) \cdot \mathbf{n} = 0, & \text{on } \partial\Omega, \end{cases} \quad (1)$$

where  $\ell_\beta$  is a length scale parameter that governs the width of the regularization zone of the interface. Similarly, Eqs (1) corresponds to the Euler-Lagrange equation of the variational problem :

$$\beta = \text{Arg} \left\{ \inf_{\beta \in S_\beta} \Gamma_\beta(\beta) \right\} \quad \text{with} \quad \Gamma_\beta(\beta) = \int_{\Omega} \gamma_\beta(\beta) dV, \quad (2)$$

where  $S_\beta = \{\beta | \beta(\mathbf{x}) = 1, \forall \mathbf{x} \in \Gamma^I\}$  and  $\gamma_\beta$  is defined as

$$\gamma_\beta(\beta) = \frac{1}{2\ell_\beta} \beta^2 + \frac{\ell_\beta}{2} \nabla \beta \cdot \nabla \beta. \quad (3)$$

The regularized interface representation  $\Gamma_\beta$  from the above variation principle converges for  $\ell_\beta \rightarrow 0$  to the exact sharp interface  $\Gamma^I$ . In the following, both length scale parameters  $\ell_d$  and  $\ell_\beta$  are assumed to be identical, i.e.  $\ell_d = \ell_\beta = \ell$ . In addition, the interfaces are assumed to be fixed, i.e.  $\beta(\mathbf{x})$  is kept unchanged during the simulation. We introduce the approximate displacement jump field as :

$$[[\mathbf{u}(\mathbf{x})]] \approx \mathbf{w}_\beta(\mathbf{x}) = h \nabla \mathbf{u}(\mathbf{x}) \cdot \mathbf{n}^\phi. \quad (4)$$

where,  $h$  is a characteristic length parameter and  $\mathbf{w}(\mathbf{x})$  denotes the regularized approximation of the displacement jump and  $\mathbf{n}^\phi$  is the normal to the diffuse discontinuity field.

### 3 Phase field modeling of hydraulic fracturing with interfacial damage

In this section, we extend the phase field hydraulic fracturing framework developed recently by Miehe et al. [5] to heterogeneous media by accounting for interfacial damage [7].

The two main strong form equations for hydro-poro-elasticity are summarized as follows :

$$\begin{cases} \nabla \cdot \boldsymbol{\sigma} = \mathbf{0}, & \text{with } \boldsymbol{\sigma} = \boldsymbol{\sigma}_{\text{eff}} - bp\mathbf{1}, \\ \frac{1}{M} \dot{p} + b \nabla \cdot \dot{\mathbf{u}} - \mathbf{k} \nabla^2 p = 0, \end{cases} \quad (5)$$

together with the prescribed boundary conditions on displacement and pressure fields. We present in the following the modifications of the effective stress and the permeability tensor due to the presence of the smeared cracks and interfaces together with the cohesive interface modeling.

Including crack and interface phase fields  $d(\mathbf{x})$  and  $\beta(\mathbf{x})$ , the effective stress is defined according to :

$$\boldsymbol{\sigma}_{\text{eff}} = ((1-d)^2 + \kappa) \boldsymbol{\sigma}_{\text{eff}}^+ + \boldsymbol{\sigma}_{\text{eff}}^- \quad (6)$$

where  $\kappa \ll 1$  is a small positive parameter introduced to prevent the singularity of the stiffness matrix due to fully broken parts,  $\boldsymbol{\sigma}_{\text{eff}}^+$  and  $\boldsymbol{\sigma}_{\text{eff}}^-$  are the tensile and compressive stresses defined as :

$$\boldsymbol{\sigma}_{\text{eff}}^\pm = \lambda \langle \text{tr}[\tilde{\boldsymbol{\varepsilon}}] \rangle_\pm \mathbf{1} + 2\mu \tilde{\boldsymbol{\varepsilon}}^\pm, \quad (7)$$

with  $\lambda$  and  $\mu$  the Lamé coefficients of the solid phase. Only tensile damage degradation is taken into account in the elastic energy (6) through a decomposition of the elastic strain  $\tilde{\boldsymbol{\varepsilon}}$  into tensile and compressive parts [4] :

$$\tilde{\boldsymbol{\varepsilon}} = \tilde{\boldsymbol{\varepsilon}}^+ + \tilde{\boldsymbol{\varepsilon}}^- \quad \text{with} \quad \tilde{\boldsymbol{\varepsilon}}^\pm = \sum_{i=1}^3 \langle \tilde{\boldsymbol{\varepsilon}}^i \rangle_\pm \mathbf{n}^i \otimes \mathbf{n}^i. \quad (8)$$

In the above,  $\langle x \rangle_\pm = (x \pm |x|)/2$ , and  $\tilde{\boldsymbol{\varepsilon}}^i$  and  $\mathbf{n}^i$  are the eigenvalues and eigenvectors of  $\tilde{\boldsymbol{\varepsilon}}$ . The degradation of the stress in (6) is assumed to occur only in the the effective stress within the solid skeleton while the pore fluid pressure  $p$  is not affected by the fracturing process following the same treatment suggested in [5].

Note that in Eq. (6) an alternative notation  $\tilde{\boldsymbol{\varepsilon}}$  is adopted for the strain rather than  $\boldsymbol{\varepsilon}$ . This is because within the continuously defined phase field framework, there exists no distinct separation between the bulk and the interface kinematics and thus the strain induced by the approximate displacement jump  $\mathbf{w}_\beta$  needs to be subtracted from the strain tensor [11, 7] :

$$\tilde{\boldsymbol{\varepsilon}} = \nabla^s \mathbf{u} - \mathbf{n}^\phi \otimes^s \mathbf{w}_\beta \quad (9)$$

where the first term is the classical strain tensor defined as :

$$(\nabla^s \mathbf{u})_{ij} = (u_{i,j} + u_{j,i})/2 \quad (10)$$

and the latter represents the strain induced by the diffused displacement jump

$$(\mathbf{n}^\phi \otimes^s \mathbf{w}_\beta)_{ij} = (n_i^\phi w_j + w_i n_j^\phi)/2. \quad (11)$$

Detailed derivations of (9) can be found in both [11] and our recent work [7].

A Poiseuille-type fluid flow is assumed [5] within the cracks and the crack phase field  $d(\mathbf{x}, t)$  is adopted to serve as an indicator for the evolving anisotropic permeability. The permeability tensor  $\mathbf{k}$  in the second equation of (5) is splitted into two parts using the crack phase field as an indicator

$$\mathbf{k} = \mathbf{k}_{\text{homo}} + d^\varepsilon \mathbf{k}_{\text{crack}} \quad (12)$$

where  $\varepsilon \geq 1$  is an additional material parameter applied to localize the increased permeability along the fracture. Above,  $\mathbf{k}_{\text{homo}} = (k_{\text{homo}}/\eta) \mathbf{1}$  is the isotropic permeability tensor of the homogeneous porous phase where  $k_{\text{homo}}$  is the scalar intrinsic permeability and  $\eta$  the dynamic viscosity of the fluid. The anisotropic permeability  $\mathbf{k}_{\text{crack}}$  dependent on the crack opening is defined given by :

$$\mathbf{k}_{\text{crack}} = \left( \frac{(w_d^n)^2}{12\eta} - k_{\text{homo}} \right) (\mathbf{1} - \mathbf{n}^\phi \otimes \mathbf{n}^\phi). \quad (13)$$

Recall that the normal  $\mathbf{n}^\phi$  and the approximate displacement jump field  $\mathbf{w}_\beta$  defined above, the crack opening can be approximated by

$$w_\beta^n \approx \mathbf{w}_\beta \cdot \mathbf{n}^\phi. \quad (14)$$

Considering the problem defined in the previous section, the total energy in of a medium embedding cracks and cohesive interfaces in a standard framework reads :

$$J(\boldsymbol{\varepsilon}, \boldsymbol{\theta}) = \int_{\Omega} W_{\text{bulk}}(\boldsymbol{\varepsilon}, \boldsymbol{\theta}) \, dV + \int_{\Gamma'} \boldsymbol{\psi}'([\![\mathbf{u}]\!]]) \, dA + \int_{\Gamma} g_c \, dA \quad (15)$$

in which  $\Psi^I$  is the strain energy density function depending on the displacement jump  $[[\mathbf{u}]]$  across the interface  $\Gamma^I$  (the history parameter  $\alpha$  has been omitted, see [7]) and  $g_c$  is the critical fracture energy density, also named as Griffith's critical energy release rate.

In the present regularized framework, we introduce the fields  $d(\mathbf{x})$  and  $\beta(\mathbf{x})$  for the representation of crack and interface and replace also the corresponding strong displacement jumps  $[[\mathbf{u}]]$  by the approximation  $\mathbf{w}(\mathbf{x})$  given in (4). Then (15) is replaced by a *total* pseudo-energy defined in the form :

$$J(\varepsilon, \theta, \beta, \mathbf{w}; d) = \int_{\Omega} W_{\text{bulk}}(\tilde{\varepsilon}, \theta; d) dV + \int_{\Omega} \Psi^I(\mathbf{w}_{\beta}) \gamma_{\beta} dV + \int_{\Omega} (1 - \beta)^2 g_c \gamma_d dV \quad (16)$$

where  $\gamma_{\beta}$  and  $\gamma_d$  are the surface densities. The factor  $(1 - \beta)^2$  is introduced here following our original proposition in [7] to ensure that the constitutive behavior along interfaces is dominated by the applied cohesive model.

With Eq. (16) at hand, the *total* pseudo-energy potential or free energy  $W$  can be identified as :

$$W(\varepsilon, \theta, \beta, \mathbf{w}; d) = W_{\text{bulk}}(\tilde{\varepsilon}, \theta; d) + W_{\text{inter}}(\beta, \mathbf{w}_{\beta}) + W_{\text{frac}}(\beta, d) \quad (17)$$

with

$$W_{\text{inter}} = \Psi^I(\mathbf{w}_{\beta}) \gamma_{\beta} \quad \text{and} \quad W_{\text{frac}} = (1 - \beta)^2 g_c \gamma_d \quad (18)$$

and the bulk contribution as

$$W_{\text{bulk}} = (1 - d)^2 \Psi_{\text{eff}}^+(\tilde{\varepsilon}) + \Psi_{\text{eff}}^-(\tilde{\varepsilon}) + \Psi_{\text{fluid}}(\tilde{\varepsilon}, \theta). \quad (19)$$

The bulk contribution  $W_{\text{bulk}}$  is composed of three parts including the tensile and compressive effective strain energy of the solid skeleton of the porous medium,

$$\Psi_{\text{eff}}^{\pm} = \lambda \langle \text{tr}[\tilde{\varepsilon}] \rangle_{\pm}^2 / 2 + \mu \text{tr}[\tilde{\varepsilon}^{\pm}]^2, \quad (20)$$

and the contribution related to the fluid

$$\Psi_{\text{fluid}} = \frac{M}{2} (\text{tr}[\tilde{\varepsilon}]^2 - 2\theta[\tilde{\varepsilon}] + \theta^2), \quad (21)$$

where the degradation applies only to the tensile effective strain energy in line with the assumption in (6). The small parameter  $\kappa$  appearing in (6) is omitted here in (20) to simply the notation.

The evolution of the damage variable  $d(\mathbf{x}, t)$  can then be determined by the variational derivative of  $W$ . In a rate-independent setting with the consideration of the reduced Clausius-Duhem inequality, the evolution criterion is provided by the Kuhn-Tucker conditions :

$$\dot{d} \geq 0; \quad -\delta_d W \leq 0; \quad \dot{d}[-\delta_d W] = 0 \quad (22)$$

yielding

$$-\delta_d W = 2(1 - d) \Psi_{\text{eff}}^+(\tilde{\varepsilon}) - (1 - \beta)^2 g_c \delta_d \gamma_d = 0 \quad (23)$$

with [4]

$$\delta_d \gamma_d = d / \ell_d - \ell_d \delta d. \quad (24)$$

The damage evolution criterion can be expressed in the following form :

$$(1 - \beta)^2 \frac{g_c}{\ell_d} [d - \ell_d^2 \nabla^2 d] = 2(1 - d) \max_{t \in [0, T]} \{ \Psi_{\text{eff}}^+(\mathbf{x}, t) \} \quad (25)$$

with a record of the maximum effective tensile strain energy during the loading process to account the loading and unloading history. The criterion (25) is a monotonously increasing function of the strain  $\tilde{\varepsilon}(\mathbf{x}, t)$  that induces unnecessary stress degradation even at low strain values. To avoid this issue, an energetic damage evolution criterion with threshold has been introduced in [5]

$$(1 - \beta)^2 2 \Psi_c [d - \ell_d^2 \nabla^2 d] = 2(1 - d) \max_{t \in [0, T]} \{ \langle \Psi_{\text{eff}}^+(\mathbf{x}, t) - \Psi_c \rangle_+ \} \quad (26)$$

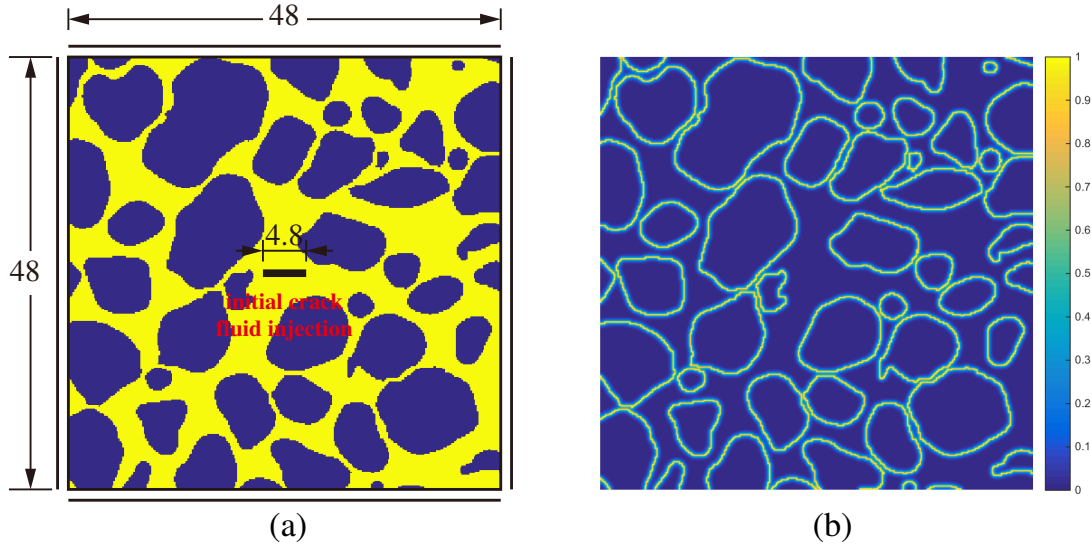


FIGURE 2 – Geometry of the highly heterogeneous medium : (a) segmented data obtained from a microtomography image, (b) the corresponding interface phase field  $\beta(\mathbf{x})$ .

in which  $\psi_c$  is a specific fracture energy density of the solid skeleton, which can be further related to a critical fracture stress  $\sigma_c$

$$\psi_c = \frac{1}{2E} \sigma_c^2 \quad (27)$$

in terms of a Young's modulus related parameter  $E$  (see more details in [5]). The above evolution criterion in (26) can be further stated as

$$(1 - \beta)^2 2\psi_c [d - \ell_d^2 \nabla^2 d] = 2(1 - d) \mathcal{H}(\mathbf{x}, t) \quad (28)$$

with the introduction of a strain energy history function [5]

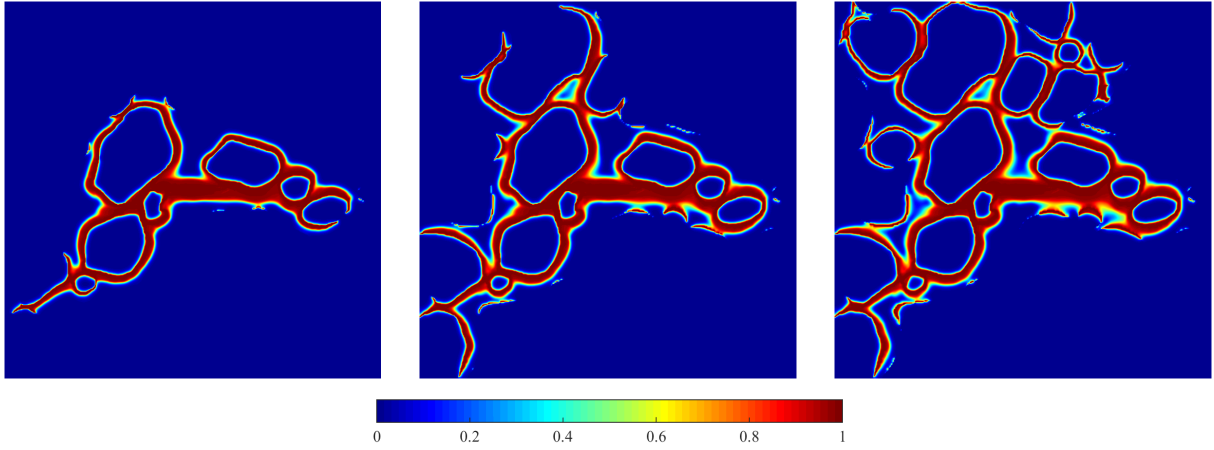
$$\mathcal{H}(\mathbf{x}, t) = \max_{t \in [0, T]} \{ \langle \psi_{\text{eff}}^+(\mathbf{x}, t) - \psi_c \rangle_+ \}. \quad (29)$$

## 4 Numerical example

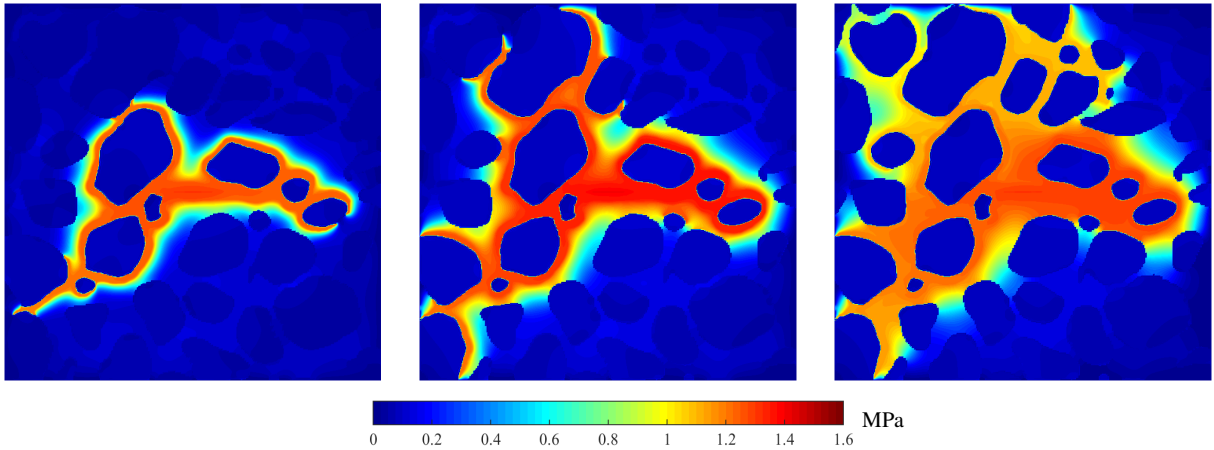
### 4.1 Hydraulic fracturing of realistic heterogeneous medium from microtomography

In this example, we investigate the capabilities of the method to simulate hydro-mechanical cracks in realistic microstructures such as obtained by experimental imaging techniques, like X-ray microtomography for small dimensions of heterogeneities. The geometry is constructed by projecting a voxel-based image of a real cementitious material obtained in [7] on a regular finite element mesh. The 2D segmented image consists in a single slice extracted from the 3D model. The image resolution is  $480 \times 480$  and is depicted in Fig. 2 (a). Note that to fit with realistic conditions of hydraulic fracture in geomaterials and to maintain the Poiseuille-type flow law assumption, the dimensions have been upscaled to the order of tens of meters. The dimensions of the domains have been chosen as  $48 \times 48 \text{ m}^2$ . The properties of the matrix and inclusions are the same as in the previous example. For interfaces, the cohesive model II of the previous example is used. The mesh matches the pixels of the original image and thus consists in  $480 \times 480$  quadrilateral bilinear elements. As in the previous examples, the displacements are fixed over the external boundary and the fluid pressure along the boundaries is set to zero. A constant fluid flow of  $0.003 \text{ m}^2/\text{s}$  is injected on a initial crack of length 4.8 m during 48 s with a constant time step  $\Delta t = 0.1 \text{ s}$ . The characteristic length scale parameters for both the crack phase field and the interface phase field are set to  $\ell_d = \ell_\beta = 0.2 \text{ m}$ . The function  $\phi(\mathbf{x})$  for the geometry of the microstructure depicted in Fig. 2 (a) is provided in Fig. 2 (b).

The evolution of both the crack phase field and the fluid pressure during the hydraulic fracturing test are depicted in Fig. 3. As expected, the crack propagates preferably along the interfaces. A very complex



(a) Crack phase fields from left to right at  $t = 12s, t = 24s, t = 48s$ .



(b) Fluid pressure fields from left to right at  $t = 12s, t = 24s, t = 48s$ .

FIGURE 3 – (a) Evolution of crack phase and (b) pressure during the hydraulic fracturing of the realistic heterogeneous medium considering the interfacial cohesive model II.

crack pattern results from the hydraulic fracturing simulation due to highly heterogeneous nature of the medium, and both crack branching and joining can be observed from Fig. 3(a). We further note from Fig. 3(b) that higher fluid pressure takes place around the injection position whereas the fluid pressure decreases gradually towards the domain edges with the crack propagation.

## 5 Conclusion

In this work, we have proposed a phase field method to hydraulic fracturing to take into account the following features : (a) the presence of heterogeneities ; (b) interfacial damage and (c) the possibility to model the initial geometry and the cracks in regular grids of voxels as arising from experimental imaging techniques. For this purpose, we have extended the framework proposed in [7] to hydro-mechanical coupling. In [7], the formulation allowed interaction between bulk cracks and interfacial damage within the phase field and regular meshes for arbitrary morphologies of heterogeneities through an appropriate regularized framework of both interface and bulk crack discontinuities. In the present paper, this framework has been extended to modeling of anisotropic fluid flow within bulk and interfacial cracks and the coupling between the fluid flow within the porous matrix and the crack initiation and propagation. The obtained framework avoids the burden of remeshing during crack initiation and propagation, and is well adapted to simulations within voxel-based models of heterogeneous media as arising from experimental imaging. We have validated the method by a series of benchmark tests and have applied it to hydraulic fracturing of highly heterogeneous media composed of a porous matrix and rigid inclusions with complex geometrical shapes. To our best knowledge, the presented simulations involving hydraulic fracturing, with interfacial damage and realistic voxel-based models of heterogeneous media have been

presented here for the first time, and seems to constitute a very promising framework for predicting initiation, propagation of complex microcracking in a hydro-mechanical context in highly heterogeneous media, such as concrete or geological media.

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