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NON-LINEAR MODAL ANALYSIS APPLIED TO AN INDUSTRIAL STRUCTURE

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ABSTRACT

A nonlinear modal analysis based on the concept of nonlinear modes is applied in the context of an aircraft landing gear study. Swept sine tests at constant force level have shown typical phenomena: frequency shifts at low level and jumps for higher ones. Clearances and friction give a physical reason for this behaviour as the system is composed of many components connected by complex joints. A nonlinear modal model is proposed to analyse experimental data. It consists in the expression of a dynamic nonlinear compliance composed of several nonlinear modes contributions. Identification is made on the basis of multi-level, multi-channel, up and down frequency responses, minimising a global error function between theory and test data using numerical methods. Mode shapes, modal mass, natural frequencies and damping ratios are functions of modal amplitudes. Results are in good agreement with all experimental data and are described by a reduced set of parameters. This nonlinear modal data can be used in some application such as updating friction and clearance parameters of the landing gear finite element model.

NOMENCLATURE

$[M]$	mass matrix
$[C]$	viscous damping matrix
$[H]$	hysteretic damping matrix
$\{\phi\}_r$	r^{th} mass-normalised normal mode shape
m	total number of linear normal modes
p_r	r^{th} non-linear modal coordinate
$\{p\}$	vector of non-linear modal coordinates
$\{\bar{\phi}(p_r)\}_r$	r^{th} non-linear normal mode shape vector
\bar{m}	total number of non-linear modes
$\bar{\omega}_r(p_r)$	r^{th} non-linear eigen-frequency
$[\bar{K}(\cdot)]$	non-linear stiffness matrix
$\bar{m}_r(p_r)$	r^{th} non-linear modal mass
$\bar{c}_r(p_r)$	r^{th} non-linear effective viscous damping
$\bar{h}_r(p_r)$	r^{th} non-linear hysteretic damping
$\bar{f}_r(p_r)$	r^{th} non-linear modal force
$\{F\}$	excitation force vector
N_f	total number of force levels
$\Omega, \Omega_1, \Omega_i, \Omega_{N_\Omega}$	forcing frequencies
N_Ω	number of discretised forcing frequencies
$p_r^i, \bar{\omega}_r^i, \{\bar{\phi}(p_r)\}_r^i$	discretised non-linear modal parameters
$[\bar{\alpha}(\Omega, \{p\})]$	non-linear dynamic compliance matrix
$\{U\}$	displacement vector
U_j	j^{th} displacement component
\dot{U}_j	j^{th} velocity component
N	number of displacement components
t, t_o, t_1, T	instants time and time period
j, k, l, r, u	integers

1 INTRODUCTION

Modal analysis for mechanical systems is widely used in numerical and experimental domains when a structure has to be dynamically characterised. The usefulness and efficiency of resulting products, namely natural frequencies, mode shapes and damping ratios, doesn't need to be demonstrated. Some applications are: prediction of responses, model reduction and component mode synthesis, model updating, Modal analysis is based on the assumption that the systems under study behaves like a linear one so that concepts of normal or complex modes could be successfully used ^[1]. Unfortunately, actual structures often exhibit non-linear behaviours which may arise for example from non-linear material, large motions, friction Thus, the nonlinear theory can no longer be applied, so, we lost the opportunity of having an experimental model because of non-linearities. Moreover, the definition of a non-linear parametric model can be very difficult especially for complex structures including several non-linearities. In particular, damping properties often have to be experimentally determined. To overcome these problems, one could think about modal extended concepts for non-linear systems. In this paper, we present an applied non-linear analysis of an industrial structure (a landing gear) based on the concept of non-linear modes ^[2]. After showing the necessity of non-linear analysis for this example by test results, we introduce a non-linear frequency response model based on non-linear modes. Then, non-linear modal parameters are identified by a curve-fitting technic using measured data.

2 EXPERIMENTAL TESTS

The landing gear is rigidly fixed using its three attachment points which are actually used to connect it to the plane's wing (see figure 1). An electrodynamic shaker applies an harmonic driving force on the structure, using different points of entry in order to have the first "modes" exited separately. The force level is measured and is kept at a constant value by means of a control loop including the power amplifier shaker. The frequency is slowly increased from low to higher values (up swept sine) and then slowly decreased back to low frequencies (down swept sine). Several force levels are tested. About twenty accelerometers are spread on the structure to have motion description in the three directions of space.

The system has shown two types of typical non-linear behaviours. First, at low level, some of the resonant pics are "sliding" toward low frequencies with distorsion (see figure 2) and then at higher level the frequency shift goes toward high frequencies and responses exhibit distorsion and jump phenomena (see figure 3). These types of behaviour, have been noticed in simulations results ^[3] by numerical dynamic models including friction and/or clearance which are the most common non-linearities met in actual structures such as the landing gear we studied. This type of frequency responses have been analysed for modal parameters but more often in a SDOF approach ^[4]. Our aim here is to have information on modal parameters such as damping ratios, natural frequency and also about mode shapes. In or-

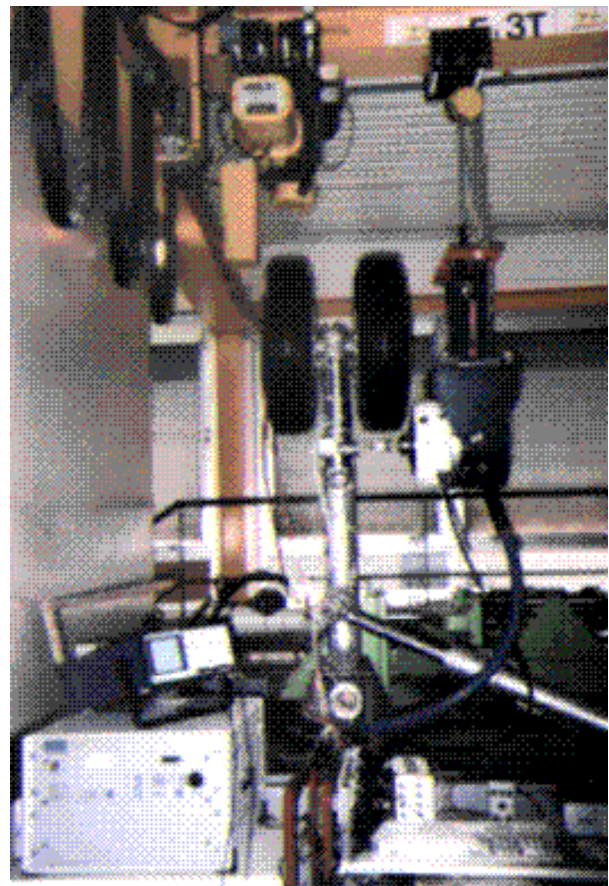


Figure 1: Experimental setup for landing gear testing.

der to represent the non-linear behaviour of our structure, these modal parameters have to change with amplitude of motion.

3 THEORETICAL TOOLS

3.1 Non-linear normal modes

Our non-linear modal model will be based on non-linear normal modes theory. This concept have been introduced by Rosenberg ^{[5] [6] [7] [8] [9]} and overviewed in ^[2] for so-called admissible discret conservative systems. Normal non-linear modes are an extension of their linear counterparts. In a general case, the normal oscillations are defined as periodic motions where all coordinates reach the equilibrium point and their maximum values simultaneously in two distinct times and can be represented by curved modal relations expressing all coordinates as function of an arbitrarily chosen one in configuration space :

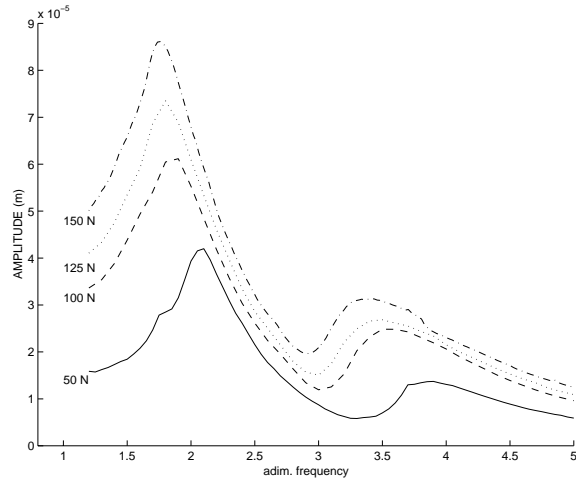


Figure 2: "Softening type" responses, amplitude versus normalised frequency

$$\begin{aligned}
 \forall t, \forall j \in \{1, \dots, N\}, \quad U_j(t) &= U_j(t + T) \\
 \forall k, j \in \{1, \dots, N\}, \quad U_j &\equiv U_j(U_k) \\
 \forall t, \exists! t_o \in [t, t + T/4] \quad / \quad \{U(t_o)\} &= 0 \\
 \forall t, \exists! t_1 \neq t_o \in [t, t + T/4] \quad / \quad \{\dot{U}(t_1)\} &= 0
 \end{aligned}$$

These properties are used in order to build normal modes trajectory equations to be solved in addition with boundary conditions; this is called the geometrical method. An interesting feature is, as it is shown by Rosenberg, that resonance of forced vibration occurs in the neighbourhood of normal modes. This provides a useful tool for studying non-linear systems even in non-linearisable cases. But, even if there exists some "integrable" cases, the general geometrical computing of normal modes seems to be difficult to apply in practical cases. To overcome this drawback, Szemplińska-Stupnika^[10] developed non-linear normal modes by means of Ritz-Galerkin approximation method for discrete and continuous systems and showed the need of taking the mode shape variations into account, in order to have a good prediction of response for the case of lightly damped systems. Based on Ritz-Galerkin method^[10], approximated non-linear mode is defined by its modal parameters $\bar{\omega}_r(p_r)$ and $\{\bar{\phi}(p_r)\}_r$ which are functions of the magnitude of these modal non-linear coordinate^[10] p_r .

When one has a mathematical model, these are found by solving a non-linear eigen-problem of the following form^[10]:

$$[\bar{K}(p_r \{\bar{\phi}(p_r)\}_r)] \{\bar{\phi}(p_r)\}_r = \bar{\omega}_r^2(p_r) [M] \{\bar{\phi}(p_r)\}_r \quad (1)$$

This can be achieved using a continuation method^[11] taking p_r as the parameter, with a zero order predictor and a Newton-Raphson corrector. We will use in addition, the non-linear modal mass defined by:

$$\bar{m}_r(p_r) = \{\bar{\phi}(p_r)\}_r^T [M] \{\bar{\phi}(p_r)\}_r. \quad (2)$$

3.2 Forced response

In order to introduce forced response we will need to define non-linear modal viscous or hysteretic effective damping by:

$$\bar{c}_r(p_r) = \{\bar{\phi}(p_r)\}_r^T [C] \{\bar{\phi}(p_r)\}_r \quad (3)$$

$$\bar{h}_r(p_r) = \{\bar{\phi}(p_r)\}_r^T [H] \{\bar{\phi}(p_r)\}_r, \quad (4)$$

which makes appear that effective damping may be non-linear even if damping is linear in nature. The response of the structure under harmonic load is expressed using a non-linear complex modal coordinate which is implicitly defined (for hysteretic damping) by:

$$p_r = \frac{\{\bar{\phi}(p_r)\}_r^T \{F\}}{\bar{m}_r(p_r) (\bar{\omega}_r^2(p_r) - \Omega^2) + i \bar{h}(p_r)}. \quad (5)$$

In few words, the response of the mechanical non-linear lightly damped system in principal resonance condition $\Omega \approx \bar{\omega}_r(p_r)$, is near the following form :

$$[U(t)] = \text{Re}(p_r \{\bar{\phi}(p_r)\}_r e^{i\Omega t}) \quad (6)$$

This assumption is the basis of the "single non-linear normal mode method"^[10] used to an approximated solution of forced resonance.

4 MODAL PARAMETER EXTRACTION METHODS

4.1 Step by step frequency domain method

Few methods exist which can allow modal parameters extraction for non-linear systems for MDOF systems. Sétio and Jézéquel^[12] developed a "step by step frequency domain" extraction method which needs the knowledge of a linear eigenvector set in order to represent non-linear modal shapes. This can be obtained experimentally by means of very low force level tests and standard modal parameter extraction. We will recall the fundamental of this method as a basis to introduce a different technique which could be named "global frequency domain fitting".

The expression of the analytical non-linear dynamic compliance for a viscous damped system will be expressed as a linear combination of single mode contributions^[12]:

$$\bar{\alpha}_{jk}^A(\Omega, \{p\}) = \sum_{r=1}^{\bar{m}} \frac{\bar{\phi}_{jr}(p_r) \bar{\phi}_{kr}(p_r)}{\bar{m}_r (\bar{\omega}_r^2(p_r) - \Omega^2) + i \Omega \bar{c}_r}. \quad (7)$$

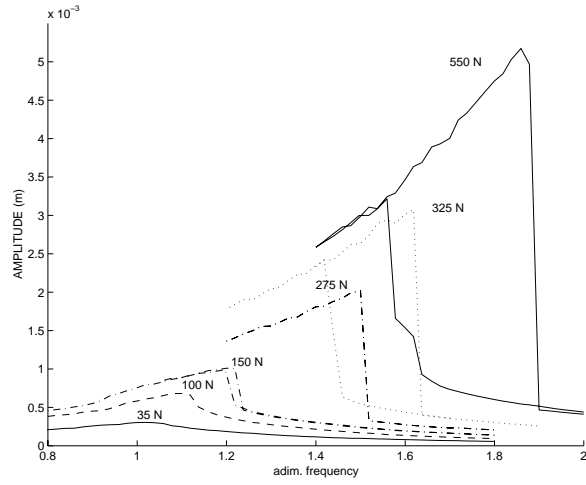


Figure 3: "Hardening type" responses, amplitude versus normalised frequency

The non-linear mode shape $\{\bar{\phi}(p_r)\}_r$ will be expressed by means of some participation coefficients $b_{r,l}(p_r)$:

$$\{\bar{\phi}(p_r)\}_r = \sum_{l=1}^m b_{r,l}(p_r) \phi_l \quad (8)$$

with the normalisation condition :

$$b_{r,r} = 1. \quad (9)$$

so that equation 7 can be re-written :

$$\bar{\alpha}_{jk}^A(\Omega, \{p\}) = \sum_{r=1}^{\bar{m}} \dots \frac{\sum_{l=1}^m \sum_{u=1}^m b_{r,l}(p_r) b_{r,u}(p_r) \phi_{jl} \phi_{ku}}{\sum_{l=1}^m b_{r,l}(p_r)^2 (\bar{\omega}_r^2(p_r) - \Omega^2) + i\Omega \sum_{l=1}^m b_{r,l}(p_r)^2 c_l} \quad (10)$$

Then, the parameters to be identified are the $\bar{\omega}_r(p_r)$'s and the $b_{r,l}$'s. This is done by minimising an error function between 10 and corresponding measured data. To do so, we use Newton-Raphson algorithm which is initiated with linear modal parameters for a Ω_i far from resonance. For each frequency step $\Omega_{i+1} = \Omega_i + \Delta\Omega$ we take the previous estimation $\bar{\omega}_r^i$ and $b_{r,l}^i$ as a starting point to find $\bar{\omega}_r^{i+1}$ and $b_{r,l}^{i+1}$ with the minimisation procedure. After the whole frequency band has been "swept", the values $\bar{\omega}_r^i$ and $b_{r,l}^i$ are associated with their corresponding modal amplitudes p_r^i for $i = 1, \dots, N_\Omega$ determined approximatively from 6 as follows:

$$p_r^i = \frac{\{U^X\}^i}{\{\bar{\Phi}\}_r^i} \quad (11)$$

or using pseudo-inverse :

$$\{p\}^i = [\bar{\Phi}^i]^+ \{U^X\}^i \quad (12)$$

This method has the advantage of simplicity and easyness of programming but involves only one test at a time, given for a specified single force level.

4.2 Global frequency domain fitting method

In order to take more data into account simultaneously: different frequency points, force levels, sweeping directions (up, down), and different sensors, we will try a more global approach based on frequency curve fitting. Then, for given modal parameter functions, we need to generate theoretical responses in order to fit them with measured ones. As this is a rather easy task in linear case, it can be seen from 5 that computing non-linear modal coordinate for a given frequency Ω is an implicit non-linear problem which must be solved numerically. Here, in order to have a fast synthesis routine, we do not solve 5 for each Ω but we rather seek for the frequency Ω the corresponding modal coordinate p_r . This is done by "inverting" 5 and one obtains in the case of hysteretic damping (a little more complicated expression is obtained from viscous damping which can be expressed in the same manner) the following:

$$\Omega = \sqrt{\bar{\omega}_r^2(p_r) \pm \frac{1}{\bar{m}_r(p_r)} \sqrt{\frac{|\bar{f}_r(p_r)|^2}{|p_r|^2} - \bar{h}_r^2(p_r)}} \quad (13)$$

where :

$$\bar{f}_r(p_r) = \sum_{l=1}^m b_{r,l}(p_r) \{\phi\}_l^T \{F\} \quad (14)$$

and where the symbol \pm is used to make distinction between different Ω solutions when they are found before or after maximum magnitude which has to be found first by solving numerically :

$$|p_r^*| = \sqrt{\frac{\bar{f}_r}{\bar{h}_r}(p_r^*)} \quad (15)$$

Thus by solving only 3 non-linear problems : 15 and 5 for $\Omega = \Omega_1$ and $\Omega = \Omega_{N_\Omega}$, which leads respectively to p_r^*, p_r^1 and $p_r^{N_\Omega}$, and taking discretised p_j 's in interval $[\min(p_r^1, p_r^{N_\Omega}), p_r^*]$, one is able to draw the non-linear response in $\Omega - p_r$ by means of equation 13.

Once the synthesized compliances have been computed, one has to define a cost function to be minimized in order to find

modal parameters which will be obtained directly as functions of their modal coordinate. The general cost function could be :

$$\epsilon = \sum_{i=1}^{N_{\Omega}} \sum_{l=1}^{N_f} \sum_{u=up}^{down} \sum_{j=1}^N |\bar{\alpha}_{jk}^A(\Omega_i) - \bar{\alpha}_{jk}^X(\Omega_i)|_{l,u} \quad (16)$$

Given that theoretical responses can be multi-valued because of jumps, "unstable branches" have to be removed in order to build analog curves as these obtained experimentally by up and down frequency sweeping.

4.3 Application test

In order to apply this theory to measured data on the landing gear, we have change for a slightly different version of equation 10. First, in the vicinity of a particular resonance mode, only one non-linear contribution is taken, effect of other modes being approximated by mass and stiffness terms [1]. Second, as the damping can be suspected to be non-linear in nature, an independant effective (hysteretic) damping term is introduced:

$$\alpha_{jk}^A(\Omega) = -\frac{M_{jk}}{\Omega^2} + \frac{\sum_{l=1}^m \sum_{u=1}^m b_{rl}(p_r) b_{ru}(p_r) \phi_{jl} \phi_{ku}}{\sum_{l=1}^m b_{rl}^2(\bar{\omega}_r^2(p_r) - \Omega^2) + i\bar{h}(p_r)} + K_{jk} \quad (17)$$

Unknown modal parameter functions can be represented as polynomial or piecewise polynomial function (spline functions). Now, parameters optimisation needs an initial guess: mode shape is initialised by linear one, the frequency $\bar{\omega}_r(p_r)$ is approximated by just picking values at resonances and expressed as function of p_r using 11. Then effective damping is estimated to have correct magnitude at resonance and is also expressed versus p_r .

4.4 Modal parameter results

We show now results of identification for our test plant. Figure 4 represents the evolution of natural frequencies for the first three modes of the structure with respect to modal amplitude.

Figures 5 and 6 show the comparison between measured responses and synthesized ones using identified modal parameters for two different sensors locations.

Figure 7 gives us three of the components of the mode shape of the first identified non-linear mode versus its modal amplitude.

Each of these modal parameters need no more than 3 or 5 parameters to be defined using polynomial or rational fractions.

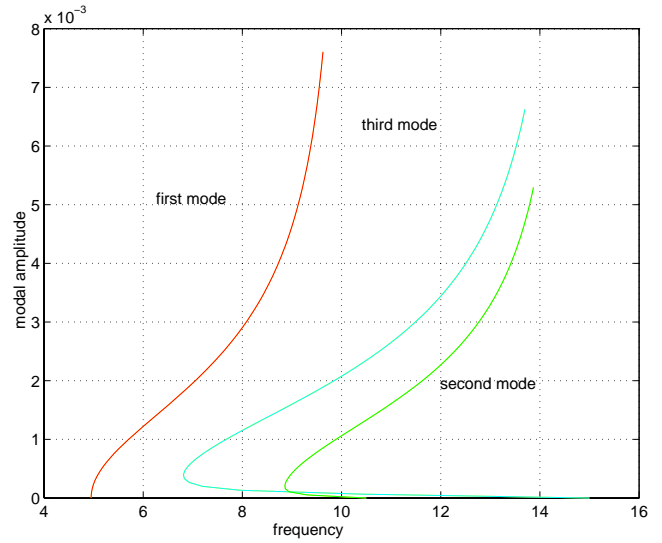


Figure 4: Non-linear identified natural frequencies for first three modes.

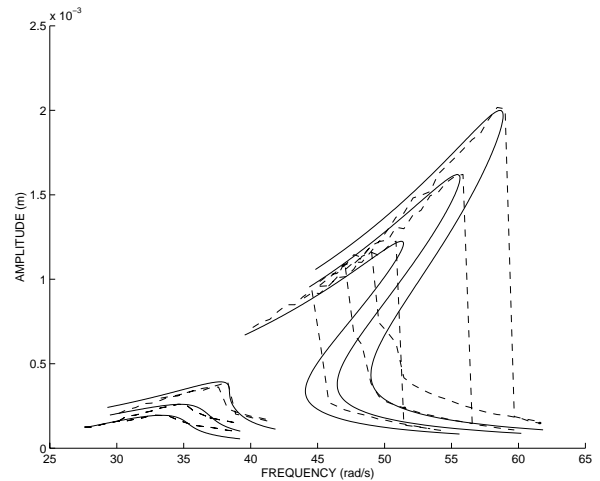


Figure 5: Non-linear resonance curves fitting (sensor 15).

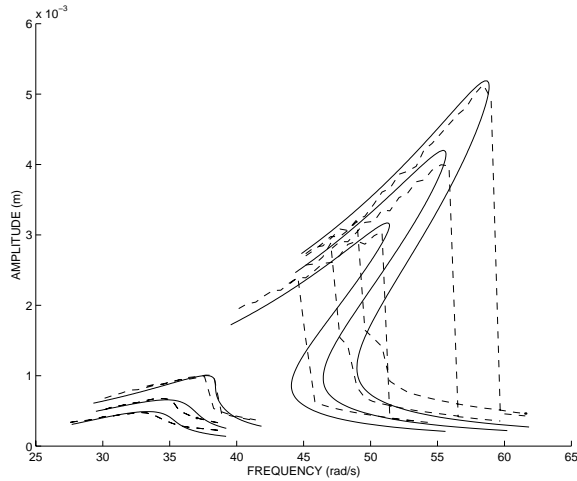


Figure 6: Non-linear resonance curves fitting (sensor 6).

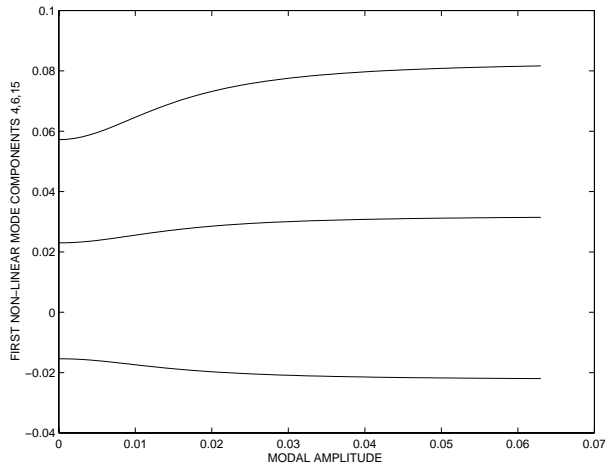


Figure 7: First non-linear mode components $\tilde{\phi}_{15}(p_1)$, $\tilde{\phi}_6(p_1)$, $\tilde{\phi}_4(p_1)$

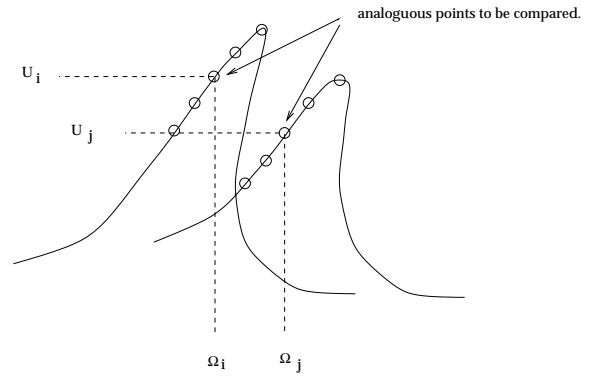


Figure 8: Comparison of two non-linear frequency response curves.

5 CONCLUSION

A non-linear modal model have been identified using frequency responses in an application context of an actual complex structure. The result shows correct agreement between measured resonance responses and synthetized ones which are defined on the basis of modal functions with few parameters. The identified modal model allows reproducing most of the dynamic features of the actual system, especially jumps, frequency shifts on several space-points of the structure and amplitude dependancy of mode shape of vibration. This work his being followed by further improvements and developpements: First, experimental setup should be improved to have more accurate datas, especially for keeping constant force level with better precision.

Second, further technics in the identification procedure could be tried , in regard of modelling functions of modal parameters and definition of the cost function. About the later point, an alternative for expression 16, based on the comparison procedure illustrated by figure 8 seemed to be more efficient than function 16 wich can exhibit irregularities because of jumps when parameters are continuously varied. Here, (see figure 8), we can define error quantities of the form

$$\epsilon_{ij} = \alpha | u_j - u_i | + \beta | \Omega_j - \Omega_i |$$

using amplitudes and also corresponding frequencies between "analogous" points on two responses to be compared. The analogous points are for exemple the maxima and some regularly spaced other points from the maxima.

Finally, further aspects to be examined are validation of the modal model in prediction of un-measured responses with respect to force entry point for example; improvement of the method in order to fit responses with closed resonances, and parametric updating of non-linear finite element models by means of the identified modal data.

REFERENCES

- [1] **Ewins, D. J.**, Modal testing: Theory and application, John Wiley, 1984.
- [2] **Rosenberg, R. M.**, *On Nonlinear Vibration of Systems with Many Degrees of Freedom*, Advance Applied Mechanics, pp. 155–242, 1966.
- [3] **Tomlinson, G. R.** and **Lam, J.**, *Frequency response characteristics of structures with single and multiple clearance-type non-linearity*, Journal of Sound and Vibration, Vol. 96, No. 1, pp. 111–125, 1984.
- [4] **Vakakis, A. F.** and **Ewins, D. J.**, *Effects of weak nonlinearities on modal analysis*, Mechanical System and Signal Processing, Vol. 8, No. 2, pp. 175–198, 1994.
- [5] **Rosenberg, R. M.** and **Atkinson, F. o. n.-l. r. C. P.**, *On the natural modes and their stability in nonlinear two-degree-of-freedom systems*, Transaction of the ASME, Journal of Applied Mechanics, Vol. 26, pp. 377–385, 1959.
- [6] **Rosenberg, R. M.**, *Normal modes of nonlinear dual-mode systems*, ASME Journal of Applied Mechanics, Vol. 27, pp. 233–268, 1960.
- [7] **Rosenberg, R. M.**, *On normal vibrations of a general class of nonlinear dual-mode systems*, ASME Journal of Applied Mechanics, pp. 275–283, 1961.
- [8] **Rosenberg, R. M.**, *The normal modes of nonlinear N-degree of freedom systems*, ASME Journal of Applied Mechanics, pp. 7–14, 1962.
- [9] **Rosenberg, R. M.** and **Kuo, J.**, *Nonsimilar normal mode vibration of nonlinear systems having two degrees of freedom*, ASME Journal of applied Mechanics., pp. 283–290, 1964.
- [10] **Szemplińska-Stupnika, W.**, The Behaviour of Non-Linear Vibrating Systems, Volume II: Advanced Concepts and Methods, Kluwer Academic Publishers, 1990.
- [11] **Nayfeh, A. H.** and **Balachandran, B.**, Applied nonlinear dynamics : Analytical, computational, and experimental methods, John Wiley and sons, Inc., 1995.
- [12] **Setio, S.**, **Setio, H.** and **Jezequel, L.**, *A method of nonlinear modal identification from frequency response tests*, Journal of Sound and Vibration, Vol. 158, No. 3, pp. 497–515, 1992.