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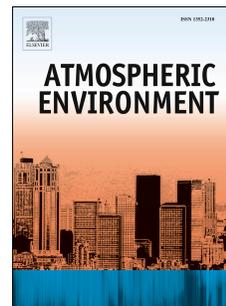
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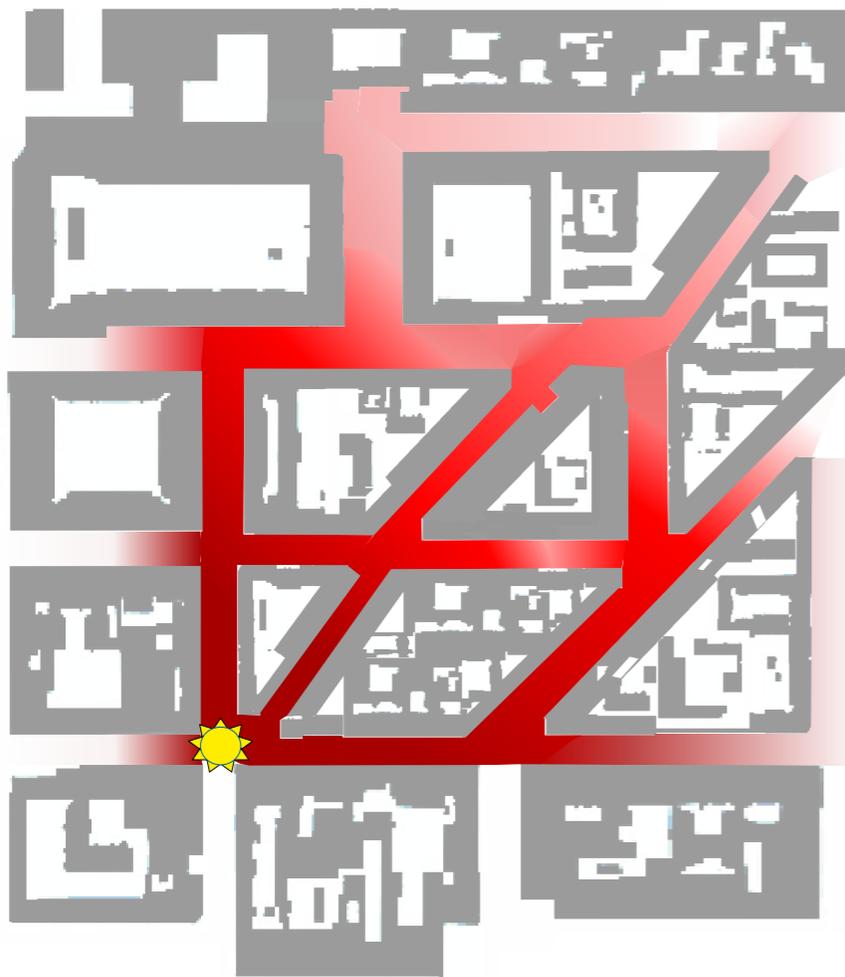
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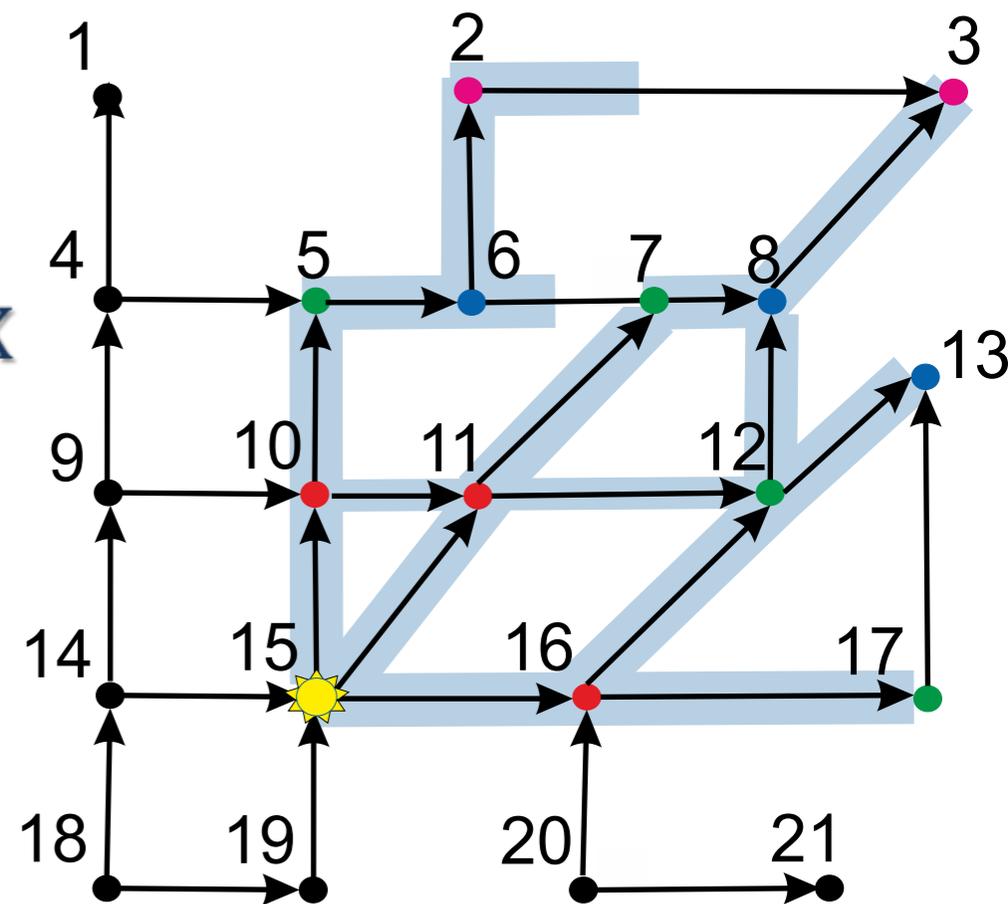
Dispersion in the urban atmosphere



COMPLEX NETWORK
PERSPECTIVE



Spreading process on a complex network



1 Propagation of toxic substances in the urban
 2 atmosphere: a complex network perspective

3 Sofia Fellini^{a,b,*}, Pietro Salizzoni^b, Lionel Souhac^b, Luca Ridolfi^a

4 ^a*Department of Environmental, Land, and Infrastructure Engineering, Politecnico di
 5 Torino, Corso Duca degli Abruzzi 24, 10129 Turin, Italy*

6 ^b*Laboratoire de Mécanique des Fluides et d'Acoustique, UMR CNRS 5509, Université de
 7 Lyon, Ecole Centrale de Lyon, INSA Lyon, Université Claude Bernard Lyon I, 36, avenue
 8 Guy de Collongue, 69134 Ecully, France*

9 **Abstract**

10 The accidental or malicious release of toxic substances in the urban atmo-
 11 sphere is a major environmental and safety problem, especially in large cities.
 12 Computational fluid dynamics codes and simplified modelling tools have been
 13 used in the last decades to model pollutant dispersion in urban areas. These
 14 studies have shown that propagation is strongly influenced by the layout of
 15 buildings and, therefore, by the street topology of the city. This work presents
 16 a novel approach to the study of toxic propagation within the urban canopy
 17 based on the theory of complex networks. The urban canopy is modelled as
 18 a network, where the streets and the street intersections represent respectively
 19 the links and the nodes of the network. The direction and the weights of the
 20 links contain the geometrical characteristics of the street canyons and their wind
 21 conditions, so that all the key variables involved in pollutant dispersion are rep-
 22 resented in a single mathematical structure that is a weighted and directed
 23 complex network. Thanks to this mathematical interpretation, propagation is
 24 modelled as a spreading process on a network and a depth-first search algorithm
 25 is used to rapidly delimit the zone of influence of a source node. This zone is
 26 the set of streets that are contaminated from the source. As a case study, the
 27 proposed model is applied to the urban tissue of the city of Lyon. The algorithm
 28 simulates a toxic release in all the nodes of the network and computes the num-
 29 ber of people affected by each propagation process. In this way, the nodes with

*Corresponding author

30 the most dangerous spreading potential are identified and vulnerability maps of
31 the city are constructed. Moreover, various wind and concentration scenarios
32 are easily implemented. The results highlight how the proposed method is ef-
33 fective in assessing the most vulnerable points in a city with a computational
34 time that is up to three orders of magnitude lower than that of existing models.
35 Moreover, the proposed approach paves the way to future applications of tools
36 and metrics from the complex network theory for a deeper comprehension of
37 the mechanisms that drive pollutant dispersion in urban areas.
38 *Keywords:* Urban air pollution, Street network, Complex networks,
39 Vulnerability, Accidental releases, Spreading on networks

40 1. Introduction

41 Large cities are particularly vulnerable to air pollution as they exhibit both a
42 large number of potential sources and a high density of people exposed (Brunekreef
43 and Holgate, 2002; Heinrich and Wichmann, 2004). Urban air pollution is
44 mainly linked to human activities and in particular to vehicular traffic, heating of
45 buildings and industrial emissions (Mayer, 1999). Moreover, accidental releases
46 such those related to gas leaks, industrial plants or the transport of dangerous
47 goods are particularly critical in densely populated environments. Besides un-
48 intentional releases, the current political situation also raises the possibility of
49 terrorist attacks aimed at the dispersal of toxic or pathogenic substances in the
50 air (Tucker, 2000; McLeish, 2017).

51 For these reasons, local authorities are urged to adopt not only instruments
52 for air quality control but also predictive tools for the management of dangerous
53 situations due to accidental and malicious releases. These actions are in line
54 with the current challenge of building resilient metropolises able to cope with
55 emergencies (Berke et al., 2009; Ahern, 2011).

56 Dispersion models are commonly used to predict urban pollution and to es-
57 timate concentration of toxic substances. Computational fluid dynamics (CFD)
58 simulations are the most accurate tool for modelling dispersion in a complex

59 geometry like the urban fabric, since they solve the velocity and the concen-
60 tration fields in the whole domain (Blocken, 2015). However, these models
61 require a huge computational cost and therefore long simulation times and high
62 performance computers. To reduce the computational cost, several modelling
63 approaches have been developed in the last decades (Di Sabatino et al., 2013).
64 These include street network models (e.g., Carruthers et al., 2000; Soulhac et al.,
65 2011) based on a simplified description of the building geometry and modelling
66 the mass exchange within and above the urban canopy by parametrising a few
67 key transfer processes. Their formulation rely on the basic idea that the urban
68 structure of the city, the orientation of the streets and their connectivity play a
69 major role in determining the intensity of these transfer processes. The street
70 network approach was originally proposed by Soulhac (2000), who developed
71 the operational model SIRANE (Soulhac et al., 2011). Recently, Kim et al.
72 (2018) presented a multi-scale model of urban air pollution that combines SIR-
73 ANE with a comprehensive treatment of atmospheric chemistry. On the other
74 hand, several efforts have been devoted to further simplify the model. Hamlyn
75 et al. (2007) constructed a simple street network model for the dispersion of
76 pollutants within a regular array of cubic obstacles. Belcher et al. (2015) and
77 Goulart et al. (2018) derived an analytical formulation for atmospheric disper-
78 sion in a regular geometry and showed that the process can be described by basic
79 geometrical and flow properties. Tzella and Vanneste (2016) developed a large
80 deviation theory for concentration prediction in a rectangular network. These
81 studies have highlighted the need for viable models able to capture the funda-
82 mental aspects of pollutant dispersion within urban-like geometries, in order to
83 give quantitative estimates of mean concentration with a reduced computational
84 cost and starting from few input data.

85 In this framework, we introduce a novel approach for the assessment of the
86 vulnerability of a dense city to the release of harmful airborne pollutants. Going
87 beyond the representation of the urban spatial domain as a street network, in
88 this work all the key variables involved in the dispersion process are viewed as
89 properties of a complex network. Thanks to this mathematical interpretation,

90 the modern techniques of the theory of complex networks can be adopted and
91 propagation phenomena in the streets are modelled as a transport process on a
92 network.

93 In complex network theory (e.g., Boccaletti et al., 2006; Newman, 2010),
94 complex systems are traced back to a set of entities (nodes) that interact with
95 each other. Interactions are represented as links between the nodes and may
96 have weights that describe the strength of these interactions. Complexity does
97 not lie in the elements that form the system, but rather in their topology and
98 within the pattern of their interconnections. In the last few years, the theory of
99 complex networks has gained a great deal of attention in countless fields, from
100 the social sciences (e.g., Borgatti et al., 2009) to engineering (e.g., Carvalho
101 et al., 2009; Yazdani and Jeffrey, 2011; Giustolisi and Ridolfi, 2014). Recently,
102 a network approach has been adopted for the description of geophysical fluid mo-
103 tion and associated transport phenomena. Gelbrecht et al. (2017) have proposed
104 a complex network representation of wind flows to study regional meteorology
105 systems, while Ser-Giacomi et al. (2015) have represented mixing and dispersion
106 processes in the Mediterranean as a transportation network.

107 In this work we focus on the pollutant dispersion at the local urban scale.
108 The topology of the city is modelled as a network, within which the propagation
109 of the airborne toxic pollutant occurs. The streets and the street intersections
110 are respectively the links and the nodes of the network. The direction and the
111 weight of the links contain the fluid-dynamical properties of the flow within
112 the streets and the geometrical characteristics of the bordering buildings. In
113 this way, the topology of the city and the wind conditions along the streets
114 are all represented in a single mathematical structure, that is a weighted and
115 directed complex network. Differently from street network models as SIRANE,
116 a numerical solution of a system of mass balance equations is here not required.
117 Given an initial point, i.e. a pollutant source, propagation along the streets is
118 modelled as an epidemic on a network. In particular, as in epidemic applications
119 (Newman, 2002), a search algorithm on networks is adopted to delimit the zone
120 of influence of a source node, that we define as the part of the network affected

121 by toxic propagation. By counting the number of people that reside in the
122 zone of influence, a vulnerability index of the source node can be estimated.
123 The procedure is applied to each node of the network and vulnerability maps
124 are easily constructed, in order to reveal at a glance the urban areas with the
125 highest spreading potential.

126 The method is much faster than previous models, and can be applied to
127 entire cities with a low computational cost and with few input parameters.
128 Given the simplified assumptions underlying the method, the results are not
129 expected to be as accurate as the ones obtainable with more complete models.
130 However, a very detailed estimate of pollutant concentration in every point of
131 the city is generally not required for the assessment of urban vulnerability. The
132 aim of this work is to provide a reliable and rapid technique to identify the
133 most vulnerable points in the city, i.e. the source points from which the toxic
134 spreading can affect the greatest number of people, and to understand the effect
135 of meteorological conditions (namely wind direction) on urban vulnerability.
136 Moreover, this work lays the foundation for further applications of tools from
137 complex network theory. In particular, the new complex network perspective
138 facilitates the investigation of the fundamental aspects that drive dispersion in
139 the urban environment. In fact, the model outcome is not the overall result
140 of a set of parametrizations and mass balance equations but is the result of
141 a spreading process on a network characterized by well defined adjacency and
142 weight matrices. Simple metrics (e.g., centrality metrics) and analyses from
143 complex network theory can thus be applied to understand the role of the key
144 variables (e.g., the topology of the city, the fluid-mechanical and geometrical
145 characteristics of the streets) in the dispersion process.

146 The work is organized as follows. In Section 2, we describe the physical as-
147 sumptions adopted to model pollutant propagation in the urban canopy. Then,
148 the network perspective is introduced in Section 3. The basic steps to con-
149 struct the street network and its weight matrices are thus described and the
150 algorithm for spreading on networks is presented. Subsequently, the proposed
151 general method is applied to the city of Lyon (France). Vulnerability maps

152 for a district of Lyon are shown and analysed in Section 4. Finally, the main
153 conclusions obtained from the present work are summarized.

154 **2. Physical assumptions about the propagation of a toxic substance** 155 **in the urban environment**

156 Transport and mixing processes in the urban environment are characterized
157 by complex fluid structures due to the interaction between the atmospheric
158 flow and the city. The presence of buildings and vegetation highly affects the
159 structure of the urban boundary layer, characterized by the generation of a
160 shear layer at the top of the canopy, wake diffusion behind buildings, and form
161 drag due to the pressure differences across the roughness elements (Roth, 2000).
162 Moreover, the flow field in the streets is altered by the convective fluxes due
163 to the differential solar irradiance on building walls and to the heat sources
164 related to human activities (Oke, 1982; Arnfield, 2003). The simulation of all
165 these dynamical effects on the flow field within the urban canopy is nowadays a
166 challenge for modellers that adopt sophisticated computational tools, typically
167 CFD codes (Tominaga and Stathopoulos, 2012). These require a huge amount of
168 input data and high computational costs, which limit their use when analysing
169 a large number of emission scenarios. In this latter case, alternative simulation
170 strategies should be adopted, based on a simplified description of the flow and
171 of the dispersion phenomena occurring within the urban area.

172 The physical assumptions for the propagation model presented in this work
173 are inspired by the street network model approach (e.g., Namdeo and Colls,
174 1996; Soulhac et al., 2011). In street network models, the geometry of the ur-
175 ban canopy is simplified to a network of streets (Fig. 1.a), and the streets are
176 represented as urban canyons, i.e. cavities of rectangular section with length
177 l , height h and width w (Fig. 1.b). Following the approach of Soulhac et al.
178 (2011), the main transport phenomena for an airborne pollutant in the urban
179 canopy are (i) the advective mass transfer along the street due to the mean wind
180 along the longitudinal axis, (ii) the vertical transfer from/toward the external

181 atmosphere and (iii) the transport at street intersections. In street network
182 models, pollutant dispersion is then described by a series of mass balance equa-
183 tions within streets, at street intersections and above the urban canopy. This
184 system of equations is numerically solved to evaluate pollutant concentrations
185 within the streets and above them.

186 To assess the vulnerability of urban areas to toxic releases, a detailed quan-
187 tification of the concentration in the streets and in the street intersections is gen-
188 erally not required. Rather, an operational model based on few input data and
189 with very low computational cost is sought, so that it can be easily and quickly
190 implemented for the assessment of large city vulnerability, even in emergency
191 situations. For these reasons, numerical solution of mass balance equations and
192 the estimation of a large number of physical parameters are here avoided. Con-
193 versely, we introduce an analytical solution for dispersion along a street canyon,
194 we adopt a simplified model for propagation in street intersections and we use
195 concepts from complex network theory to simulate toxic spreading from one
196 street to the adjacent ones. To this aim, further simplifications are required
197 compared to those adopted in well-known street network models (e.g., Soulhac
198 et al., 2011). These simplifications concern the physico-chemical processes of
199 pollutants in the street canyons, vertical transfers between the canyon and the
200 atmosphere, and pollutant fluxes in the intersections.

201 In the following, the transport mechanisms along a street canyon and at
202 street intersections are described in detail. The adopted physical assumptions
203 are introduced and discussed. The emission scenario consists of a release from a
204 point source (s in Fig. 1) at ground level within the urban canopy. The external
205 wind blowing on the city with direction Φ is the driving force for propagation
206 processes.

207 *2.1. Transport along a street canyon*

208 Consider a source s releasing a gaseous substance that results in a concen-
209 tration c_0 at the beginning of a street canyon, as illustrated in Fig. 1.b-c.
210 The main transport processes are the advective flux along the longitudinal axis,

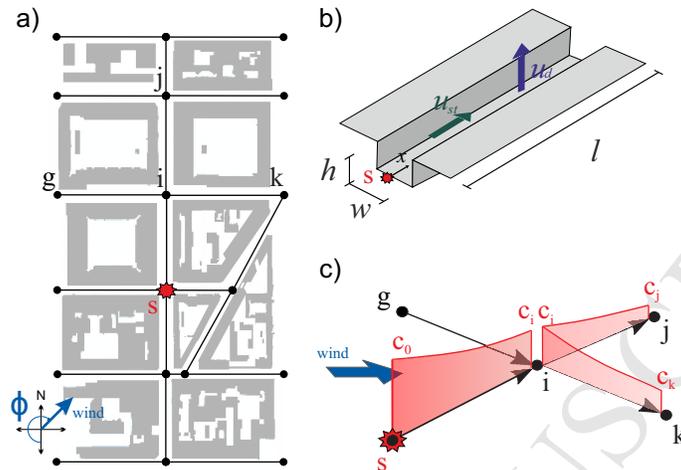


Figure 1: a) Toxic source s within a network of streets. Propagation is driven by the wind blowing on the city with direction Φ . b) Representation of a street canyon with the main variables of the model. c) Transport of a contaminated flow in street intersections.

211 and the vertical turbulent exchanges between the street and the overlying at-
 212 mosphere (Soulhac et al., 2013). In fact, turbulent longitudinal diffusion is
 213 marginal compared to the longitudinal advection, while physico-chemical pro-
 214 cesses are generally negligible as their typical time scales are longer than the
 215 time needed for propagation. Under these assumptions, the transport along a
 216 street canyon can be modelled with the one-dimensional transport equation

$$\frac{\partial c}{\partial t} + u_{st} \frac{\partial c}{\partial x} + \frac{u_d}{h} (c - c_{ext}) = 0, \quad (1)$$

217 whose solution is the function $c(x, t)$, i.e. the concentration along the longitu-
 218 dinal coordinate x over time t .

219 The first two terms in (1) describe the advective transport driven by the
 220 spatially averaged wind velocity along x (u_{st}). This velocity is assumed to
 221 be given by a balance between the stress imposed at the canopy top by the
 222 external atmospheric flow and the drag due to the roughness of the canyon walls
 223 (neglecting the role of pressure gradients). Under this assumption, Soulhac
 224 et al. (2008) derived an analytical formulation for u_{st} , as a function of the
 225 external wind intensity and direction, the geometry of the street canyon and the

226 aerodynamic roughness of building walls (see Appendix A). The third term in (1)
 227 models the mass transfer from/to the canyon to/from the overlying atmosphere
 228 by means of a bulk exchange velocity u_d (Salizzoni et al., 2009; Soulhac et al.,
 229 2011) estimated as

$$u_d = \frac{u_*}{\sqrt{2\pi}}, \quad (2)$$

230 where u_* is the friction velocity of the overlying boundary layer flow. We underline that the specific physical models adopted here for u_{str} and u_d could be easily replaced if better ones become available. Moreover, a simple model for physico-chemical transformation could be easily implemented in (1) by adding a physico-chemical transformation rate to the exchange velocity u_d .

235 Since our focus is on a localized ground-level release only - therefore inducing maximal concentration at the street level -, we will assume that the vertical flux of pollutants from the canopy to the external atmosphere will induce significantly low concentrations, c_{ext} , above roof level, due to the high dilution occurring in the lower part of the boundary layer compared to that within the streets. As a consequence, we will consider that $c - c_{ext} \simeq c$, therefore ignoring the role of any re-entrainment of mass from the external flow to the streets. The effects of re-entrainment have been discussed by different authors (Hamlyn et al., 2007; Belcher et al., 2015; Goulart et al., 2018). Their studies highlighted that re-entrainment effects are physically significant in the so called far-field region, i.e. the region extending from the third intersection downwind the source. However, the vertical flux is in general less than a quarter of the horizontal advective flux (Goulart et al., 2018). For these reasons, the possible inclusion of re-entrainment effects in the model is not expected to bring significant improvements to the results.

250 Equation (1) was analytically solved for both an instantaneous and a continuous release in the source (see Appendix B). In the first case, the initial condition is set as a rectangular pulse with height c_0 . The solution describes this initial step travelling along the street with velocity u_{st} and undergoing an

254 exponential decay of concentration. In the second case, the initial condition is a
 255 continuous release with constant concentration c_0 . The solution is a front that
 256 spreads along the street with velocity u_{st} . Although the analytical solutions are
 257 different, the concentration at the end of the street ($x = l$) in both cases is the
 258 same:

$$c_l \left(x = l, t = \frac{l}{u_{st}} \right) = c_0 e^{-\frac{l}{u_{st}} \frac{u_d}{h}}. \quad (3)$$

259 Thus, the concentration at the beginning of the street (c_0) undergoes an expo-
 260 nential decay driven by the ratio between the advection time (l/u_{st}) that the
 261 toxic front spends to reach the end of the street, and the vertical transfer time
 262 (h/u_d). The terms in the exponent summarize all the information about the
 263 geometry of the canyon and the flow dynamics in it.

264 2.2. Transport in the street intersections

265 The flow field in the street intersections is driven by complex physical pro-
 266 cesses that depend on multiple geometrical and meteorological parameters. Sev-
 267 eral studies (e.g. Hunter et al., 1990; Robins et al., 2002; Soulhac et al., 2009)
 268 have demonstrated that even slight variations in the building geometry and
 269 wind direction can affect significantly the redistribution of the incoming fluxes
 270 over the outgoing fluxes. On the basis of these observations, Soulhac et al.
 271 (2009) have developed a model, quantifying the balance of the time-averaged
 272 incoming and outgoing fluxes at the street intersection. This model requires
 273 the estimation of exchange coefficients between the streets, and the formula-
 274 tion of a mass balance equation for each intersection. In street network models
 275 (e.g., SIRANE), these balance equations are solved numerically together with
 276 the equations for transport along the canyons.

277 As mentioned in Section 2, our aim is to propose a simplified methodology
 278 for pollutant dispersion modelling, that simulates propagation as a spreading
 279 process on a network. In order to adopt this approach, an analytical solution
 280 for the physical transport mechanisms is required. For these reasons, we define

281 a simple exchange model in the intersections that avoid the use of mass balance
 282 equations. Given the uncertainty of flow redistribution in street intersections,
 283 a conservative approach is adopted in developing this minimalistic model.

284 Consider a simple intersection with a single incoming and one outgoing street
 285 canyon. The volumetric air flow rate entering the intersection with a concentra-
 286 tion c_{in} is Q_{in} , while the volumetric air flow rate outgoing the intersection with
 287 a concentration c_{out} is Q_{out} . Two cases are possible for the mass balance in
 288 the intersection. In the first case, the mass flow rate from the incoming canyon
 289 ($\dot{m}_{in} = c_{in}Q_{in}$) is lower than the mass flow rate toward the outgoing canyon
 290 ($\dot{m}_{out} = c_{out}Q_{out}$). In order for the mass balance to be satisfied, an external
 291 mass flow rate (\dot{m}_{ext}) from the atmosphere above the canopy enters the inter-
 292 section vertically with a volumetric air flow rate Q_{ext} and a concentration c_{ext} ,
 293 so that

$$\dot{m}_{in} + \dot{m}_{ext} = \dot{m}_{out} \rightarrow c_{in}Q_{in} + c_{ext}Q_{ext} = c_{out}Q_{out}. \quad (4)$$

294 In Section 2.1, we stated that re-entrainment from the atmosphere is negligible
 295 in our model. Thus, the mass flow rate from the atmosphere makes a zero con-
 296 tribution in (4). The concentration c_{out} at the beginning of the street outgoing
 297 the intersection is therefore given by $c_{out} = c_{in}Q_{in}/Q_{out}$. As the volumetric
 298 flow rate balance in the intersection is $Q_{in} + Q_{ext} = Q_{out}$, the ratio Q_{in}/Q_{out} is
 299 lower than 1 and thus $c_{out} < c_{in}$. In the second case, $\dot{m}_{in} > \dot{m}_{out}$ and the mass
 300 flow rate \dot{m}_{ext} leaves the intersection vertically, in order for the mass balance
 301 to be satisfied:

$$\dot{m}_{in} = \dot{m}_{ext} + \dot{m}_{out} \rightarrow c_{in}Q_{in} = c_{ext}Q_{ext} + c_{out}Q_{out}. \quad (5)$$

302 We consider that the concentration leaving the intersection is the same for
 303 both the upwards flow and the flow towards the outgoing street canyon, i .e.
 304 $c_{ext} = c_{out}$. Applying (5) and the volumetric flow rate balance equation ($Q_{in} =$
 305 $Q_{out} + Q_{ext}$), we find that $c_{out} = c_{in}$.

306 These arguments show that in the case of an intersection with a single in-

307 coming and one outgoing street canyon, the concentration c_{out} is equal or lower
308 than the concentration c_{in} . To maintain a conservative approach, in our model
309 we assume that the concentration of pollutants at the beginning of the street
310 outgoing the intersection is the one at the end of the incoming contaminated
311 canyon.

312 In the case of several streets crossing, we assume that when a contaminated
313 puff with concentration c_{in} reaches the intersection, then the concentration at
314 the beginning of the streets exiting the intersection is c_{in} . According to this
315 assumption, we consider that when a contaminated flow reaches the intersection,
316 it does not divide between the different outgoing streets but tends to spread
317 towards one of them. Given the uncertainty associated with the trajectory taken
318 by the pollutant puff (Scaperdas, 2000), we consider all the possible statistical
319 realizations, and thus we assume that all the outgoing streets are contaminated
320 by the incoming concentration c_{in} .

321 The assumptions adopted for the transport in street intersections are clearly
322 illustrated in Fig.1c. The contaminated flow from street (s, i) propagates to-
323 wards streets (i, j) and (i, k) . According to this scheme, the concentration at
324 the beginning of streets (i, j) and (i, k) is the one at the end of street (s, i) .

325 **3. A network perspective**

326 In the model presented above, toxic substances move in the urban environ-
327 ment driven by the wind blowing along the street canyons. The street canyons
328 behave like upward leaking transport channels and their geometry, position and
329 connectivity strongly influence the propagation. In big cities, streets cross each
330 other to compose intricate patterns (Fig. 2). Given the spatial extent and the
331 high number of elements, these urban fabrics can be seen as complex networks
332 (Porta et al., 2006; Barthélemy, 2011). Links stand for the street canyons,
333 while nodes represent the street intersections. The direction and the weight of
334 the links describe the geometrical and fluid-dynamical properties of the street
335 canyons. Within this approach, the tools of network theory provide interesting

336 information about the propagation phenomena.

337 3.1. Construction of the network

338 The urban canopy is modelled as a network of N nodes (intersections) and
 339 M links (streets). Fictitious nodes can be created to divide a street into two
 340 links in case there is a significant change in the street properties. Each link is
 341 directed according to the orientation of the mean wind along the street (u_{st}).
 342 Thus, the network structure represents both the topological properties of the
 343 urban fabric and the directions in which the propagation processes take place.
 344 The connectivity of the street canyons is described by the adjacency matrix \mathbf{A} ,
 345 a $N \times N$ square matrix whose element A_{ij} is equal to 1 if a directed link from
 346 node i to node j exists, is equal to 0 otherwise (see Appendix C for an example
 347 of adjacency matrix). Since the links have a specific direction, the adjacency
 348 matrix is asymmetric.

349 According to this network representation, the geometry and fluid-dynamical
 350 properties of the street canyons are stored efficiently in matrices. \mathbf{L} and \mathbf{H} are
 351 the symmetrical matrices of the length of the streets (l) and the average height
 352 of the buildings overlooking the streets (h). The wind velocity along the streets
 353 (u_{st}) and the velocity of the vertical transfer towards the external atmosphere
 354 (u_d) are enclosed in the matrices \mathbf{U} and \mathbf{U}_d .

355 As mentioned in the Introduction, the main purpose of this work is to es-
 356 tablish a methodology for the rapid assessment of urban vulnerability to the
 357 ground-level release of toxic gases. The vulnerability index (V_s) for a generic
 358 node s can be defined as the number of people affected by the toxic propaga-
 359 tion if the release takes place in s . The adopted network approach and matrix
 360 notation make it easy to calculate V_s as follows,

$$V_s = \sum_i^N \sum_j^N D_{ij} P_{ij}, \quad (6)$$

361 where \mathbf{P} is the matrix that associates at each link (i, j) the number of inhabitants
 362 per unit street length and \mathbf{D} is the matrix that associates at each link (i, j)



Figure 2: Snapshot of the street network of Paris (a), New York (b), Rome (c), and Lyon (d).

363 the contaminated length of the street. This last matrix represents the zone of
 364 influence of the source node. The meaning of this matrix and its construction
 365 process will be widely described in Subsections 3.2 and 3.3.

366 An interesting advantage of this compact notation is that changes to the
 367 network properties can be easily implemented. By modifying matrices \mathbf{U} and
 368 \mathbf{U}_d , we can simulate different meteorological scenarios, while variations in pop-
 369 ulation distribution (e.g., differences between weekdays and holidays) can be
 370 considered by adjusting \mathbf{P} . Furthermore, new buildings and structural changes
 371 are included in the model by revising the single elements of \mathbf{L} and \mathbf{H} .

372 3.2. Propagation of a toxic substance in the street network

373 Consider the release of a toxic substance in a street intersection (see Fig.
 374 3.a). According to the hypotheses of our model (Section 2), (i) the substance
 375 propagates along the adjacent streets depending on the direction of the wind, (ii)
 376 the concentration decays exponentially along the streets, (iii) the concentration
 377 at the end of each street can be estimated using (3), (iv) the concentration
 378 remains unchanged in the street intersections, and (v) from a contaminated
 379 street intersection the gas spreads further to the adjacent streets.

380 Given this description, the pollutant dispersion within the canopy can be
 381 then easily seen as a spreading process on a network (e.g., Newman, 2002;
 382 Comin and da Fontoura Costa, 2011). In Fig. 3.b the urban canopy is rep-
 383 resented as a network. The links are directed according to the direction of the
 384 wind in the streets and the release is modelled as a source node s . From a
 385 network perspective, transport from s towards a generic node u is possible if
 386 there is a link directed from s to u , i.e. if (i) the two nodes are physically con-
 387 nected by a street canyon, and (ii) the wind is blowing from the source towards
 388 the target node. Once infected, the target node u is modelled as a new source
 389 and the spreading process carries on towards the farthest nodes. According to
 390 this scheme, propagation is a recursive process that spontaneously expands to
 391 the topological boundaries of the network. Physically, the extent of the con-
 392 taminated zone can be delimited based on a threshold concentration value c_{th} :
 393 when the concentration falls below c_{th} , the contamination process is irrelevant.
 394 Thus, a stopping rule for the spreading process on the network is introduced:
 395 at each propagation hop the concentration at the target node is estimated. If
 396 this concentration is higher than c_{th} , then the propagation carries on.

397 Considering the example in Fig. 3.b, a toxic substance is released in the
 398 source node $s = 15$ and propagates towards the first neighbours of s : nodes 10,
 399 11, and 16. The concentration in the first neighbours is evaluated using Equa-
 400 tion (3), as a function of the geometrical and wind characteristics of the street
 401 canyons associated to links (15, 10), (15, 11) and (15, 16). Since the concentra-
 402 tion in the first neighbours is greater than the predefined threshold c_{th} , nodes
 403 10, 11 and 16 act as source nodes and the spreading process carries on towards
 404 the second neighbours of s . In the same way, the third and fourth neighbours
 405 are affected by the toxic propagation until the concentration in the nodes falls
 406 below c_{th} .

407 According to this network interpretation, the spreading of a toxic gas in
 408 the urban environment is governed by two properties of the network: (i) the
 409 topological connectivity of the network, given by its adjacency matrix \mathbf{A} , and
 410 (ii) the concentration decay along the links, given by a combination of the

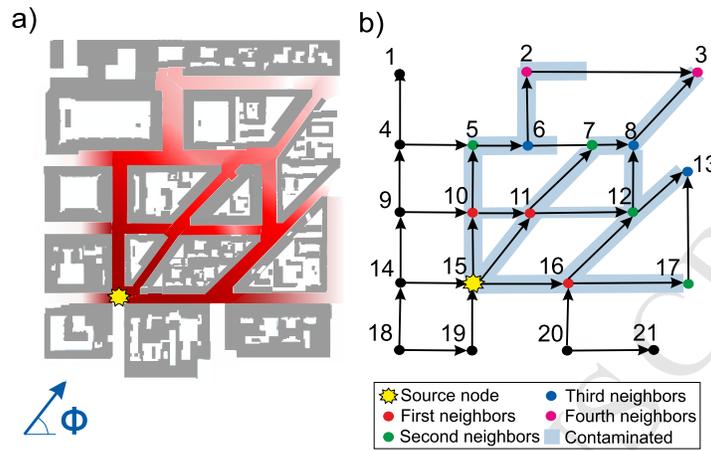


Figure 3: Analogy between the physical propagation of a toxic substance in the urban environment (a) and the spreading process on a network (b). Φ is the direction of the external wind blowing on the city.

411 geometrical characteristics and the flow dynamics in the street canyons.

412 This spreading results in the zone of influence of the source node s , i.e.
 413 the set of links contaminated from the propagation process originated in s (the
 414 elements highlighted in blue in Fig. 3). The number of people living in the zone
 415 of influence represents the vulnerability index of s (V_s). Notice that this index
 416 contains both information on population density and meteorological conditions
 417 in the city. In fact, as will be shown in the following sections, its value changes
 418 drastically with the wind direction.

419 3.3. Algorithm

420 Given a source node s with a concentration c_s , the set of its first neigh-
 421 bours $N_1(s)$ can be derived from the non-zero elements of the s -th row of
 422 the adjacency matrix \mathbf{A} . In Fig. 3, $s = 15$ and thus $N_1(s) = \{10, 11, 16\}$. For
 423 each node u belonging to the set $N_1(s)$, the algorithm calculates the distance
 424 D_{su}^{pot} . This length is the potential distance (hence the superscript *pot*) that the
 425 contaminated front can reach along the link (s, u) with a concentration higher

426 than the predefined threshold c_{th} . According to (3), D_{su}^{pot} is

$$D_{su}^{pot} = -\frac{H_{su}}{U_{d,su}} U_{su} \log\left(\frac{c_{th}}{c_s}\right), \quad \forall u \in N_1(s). \quad (7)$$

427 In general, D_{su}^{pot} is different from the physical length of the street (L_{su}) asso-
 428 ciated to the link (s, u) . D_{su}^{pot} is lower than L_{su} if the concentration undergoes
 429 the threshold c_{th} before the propagation front has travelled the entire street.
 430 Conversely, it is higher if the front reaches node u with a concentration above
 431 the threshold. As a consequence, the effective contaminated distance (D_{su}) is
 432 the minimum between the reachable distance D_{su}^{pot} and the effective length of
 433 the street, i.e.

$$D_{su} = \min[D_{su}^{pot}, L_{su}] \quad (8)$$

434 If $D_{su} = L_{su}$, the substance has reached the target node u with a concen-
 435 tration equal to or higher than c_{th} . As a result, node u is contaminated. Con-
 436 versely, if the front reaches u with a negligible concentration (i.e. $D_{su} < L_{su}$),
 437 then node u remains uncontaminated. In both cases, the algorithm stores the
 438 effective contaminated length D_{su} as the (s, u) element in the matrix \mathbf{D} . As
 439 introduced in Section 3.1, this matrix defines the zone of influence of the source
 440 node s .

441 We define $\hat{N}_1(s)$ the set of the first neighbours of s that have been contam-
 442 inated,

$$\hat{N}_1(s) = \{u \in N_1(s) \mid D_{su} = L_{su}\}. \quad (9)$$

443 Referring to Fig. 3, $\hat{N}_1(15) = \{10, 11, 16\}$ since all the first neighbours of
 444 s are contaminated. The algorithm estimates the concentration in the nodes
 445 belonging to $\hat{N}_1(s)$ using (3), as

$$c_u = c_s e^{-\frac{U_{d,su}}{H_{su}} \frac{L_{su}}{U_{su}}}, \quad \forall u \in \hat{N}_1(s). \quad (10)$$

446 These nodes behave as new source nodes. Thus, the algorithm repeats the
 447 above presented steps, replacing in Equations (7)-(10) node s with the nodes
 448 belonging to $\hat{N}_1(s)$. For example, once node 16 (Fig. 3) has been contaminated
 449 from the initial source node 15, the algorithm finds the set of its first neighbours,
 450 i.e. $N_1(16) = \{12, 17\}$. Equations (7)-(8) estimate the effective contaminated
 451 length along links (16, 12) and (16, 17), while (9) identifies the set of the first
 452 neighbours of node 16 that have been infected, i.e. $\hat{N}_1(16) = \{12, 17\}$. Finally,
 453 the concentration reached in nodes 12 and 17 is determined by (10). This
 454 procedure is repeated recursively until the concentration in each node of the
 455 network falls below the threshold c_{th} .

456 Notice that for nodes 15 and 16 the sets N_1 and \hat{N}_1 are identical. However,
 457 this is not true in general. Consider node 6 in Fig. 3b. The set of its first neigh-
 458 bours is $N_1(6) = \{2, 7\}$, while $\hat{N}_1(6) = \{2\}$ because node 7 cannot be reached
 459 by the propagation along link (6, 7).

460
 461 The algorithm explores the nodes of the network starting from a root node
 462 and progressively visiting the adjacent nodes. In computer science, this process
 463 is called tree traversal (Valiente, 2013) since the result of the exploration is a
 464 tree structure that is a subgraph of the initial graph. In our study, this tree
 465 corresponds to the zone of influence of the source node. There are multiple
 466 ways to perform a tree traversal, according to the order in which the nodes
 467 are visited. The algorithm we use explores the nodes of the network using a
 468 depth-first search analysis. The algorithm starts at the source node and goes
 469 as far as it can down a given branch before backtracking. Referring to Fig. 3,
 470 the algorithm starts at node 15, selects the first node 10 in the set $\hat{N}_1(15)$ and
 471 deepens the analysis in the first element of the set $\hat{N}_1(10)$, i.e. node 5. Following
 472 this method, the algorithm visits the nodes in the order (15, 10, 5, 6, 2). The
 473 in-depth analysis along this path ends when the concentration falls below the
 474 threshold c_{th} . Formally, the path (15, 10, 5, 6, 2) ends because the set $\hat{N}_1(2)$
 475 is empty. Once the first branch has been explored, the algorithm backtracks to
 476 node 6's next available neighbour, i.e. node 7.

477 From the numerical point of view, depth-first search analysis requires less
 478 memory and it is more efficient in finding trees on networks compared to other
 479 algorithms, such as breath-first search (Kozen, 1992).

480 Notice that, referring to the example in Fig. 3b, link (11, 7) is affected
 481 by toxic propagation twice, both along propagation path $\gamma_1=(15, 11, 7)$ and
 482 $\gamma_2=(15, 10, 11, 7)$. As a consequence the algorithm calculates two different
 483 values of $D_{11,7}$, since the concentration reached at node 11 (c_{11}) along γ_1 is
 484 generally different from the one obtained along γ_2 . As the aim of the method is
 485 to determine the extent of the zone of influence, the algorithm considers all the
 486 possible passages through a generic link and stores the longest distance reached
 487 by the toxic substance. More precisely, for each path γ_α the algorithm compares
 488 $D_{su}^{(\gamma_\alpha)}$ with the distance $D_{su}^{(\gamma_{\alpha-1})}$ obtained along the previously explored path
 489 $\gamma_{\alpha-1}$ passing through the link (s, u) . Equation (8) is, thus, refined as:

$$D_{su}^{(\gamma_\alpha)} = \max[\min[D_{su}^{pot}, L_{su}], D_{su}^{(\gamma_{\alpha-1})}], \quad (11)$$

490 where γ_α is an index for the order in which the path is explored. We underline
 491 that, given the assumptions in Subsection 2.2, the two contaminated flows along
 492 paths γ_1 and γ_2 do not physically meet in node 11. Rather, γ_1 and γ_2 are two
 493 of the possible contaminated paths originated by the pollutant emission.

494 4. Results

495 The potential of the proposed approach is discussed through a case study.
 496 The model is applied to assess urban vulnerability of the city of Lyon (France)
 497 to the release of a toxic gas. Lyon (Fig. 4) is located in east-central France and
 498 it is the third-largest urban agglomeration in France after Paris and Marseille,
 499 with a population of approximately 1.5 million inhabitants. In this work, the
 500 analysis is limited to a part of the city (Fig. 4c) that presents an intricate urban
 501 fabric and tall buildings on the edge of the streets. These characteristics are
 502 consistent with the model representation of the streets as a network of street
 503 canyons. The study area has an extent of about 6.5 km^2 , it hosts a population of

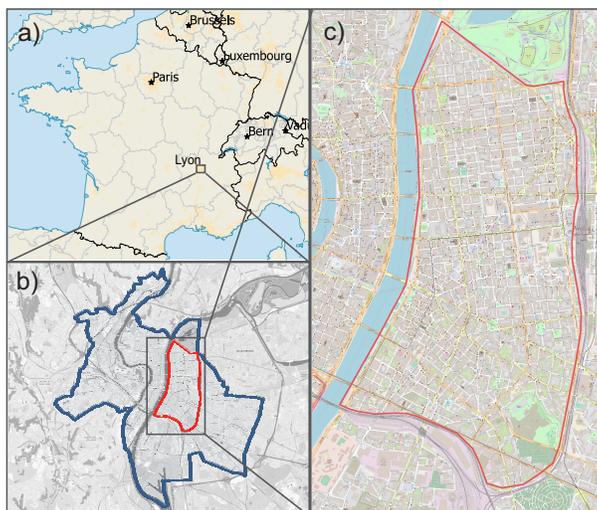


Figure 4: The case study area. a) Location of the metropolitan area of Lyon within France. b) Location of the study area within the municipality of Lyon. c) The study area.

504 about 140,000 inhabitants and is delimited by natural and artificial boundaries
 505 (rivers, parks and railways). These boundaries determine a discontinuity in the
 506 dispersion of pollutants along the street canyons.

507 The street canyons and the street intersections result in a network of 750
 508 nodes and 1110 links. The geometrical characteristics of the street canyons, the
 509 population density, the longitudinal mean wind and the vertical transfer velocity
 510 in the streets are stored in the matrices \mathbf{H} , \mathbf{L} , \mathbf{P} , \mathbf{U} and \mathbf{U}_d , respectively.

511 4.1. The zone of influence of a source node

512 The first outcome of the model is the identification of the zone of influence
 513 of a source node, i.e. the set of links contaminated by the toxic substance with
 514 a concentration above a defined threshold, stored in matrix \mathbf{D} (see Section 3).

515 As an example, Fig. 5 shows the zone of influence of a source node for differ-
 516 ent wind directions ($\Phi = 45^\circ, 135^\circ, 225^\circ$ and 315°) and for two different initial
 517 concentration values. For the sake of generality, the concentration scenarios are
 518 defined by the ratio between the concentration in the source node (c_0) and the
 519 limit concentration (c_{th}).

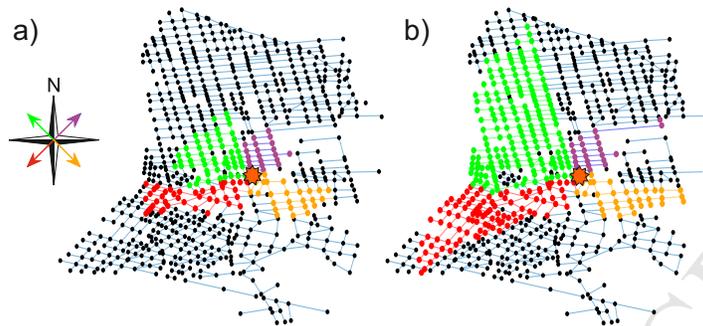


Figure 5: The zone of influence of a generic source node (orange star) in the network for different wind directions (red, green, violet and yellow refer to the wind directions $\Phi = 45^\circ$, 135° , 225° and 315° , respectively). Panel (a) and (b) refer to a concentration ratio $c_0/c_{th} = 10$ and $c_0/c_{th} = 100$, respectively.

520 The urban topology, the wind direction and the initial concentration shape
 521 the zone of influence of the source node. Variations in the mean velocity of
 522 the external wind (\bar{u}) as well as the stability conditions (as determined by the
 523 Monin-Obukhov length L_{MO}) are instead irrelevant. As stated in (2) and (A.1)-
 524 (A.2), both u_d and u_{st} depend linearly on the friction velocity u_* . Since our
 525 propagation model evaluates the pollutant spreading as a function of their ratio
 526 (3), variations of u_* , and thus of L_{MO} and \bar{u} , will not be effective in determining
 527 the zone of influence.

528 4.2. Spatial and frequency distribution of urban vulnerability

529 The spatial pattern of urban vulnerability can be analysed at a glance using
 530 vulnerability maps. Given the geometry of the urban fabric, the conditions
 531 of the external wind, the spatial distribution of the citizens and the emission
 532 scenario, the model provides a map that associates at each node of the street
 533 network its vulnerability index. The computational time is 2 (in the case of
 534 $c_0/c_{th} = 100$) or 3 (in the case of $c_0/c_{th} = 10$) orders of magnitude lower than
 535 the time taken by well-known street network models as SIRANE. As mentioned
 536 in Section 3, this is achieved by first applying the algorithm for the spread of
 537 toxic substances on networks (Section 3) and by computing the matrix of the
 538 contaminated street lengths \mathbf{D} (i.e. the zone of influence) for all nodes of the

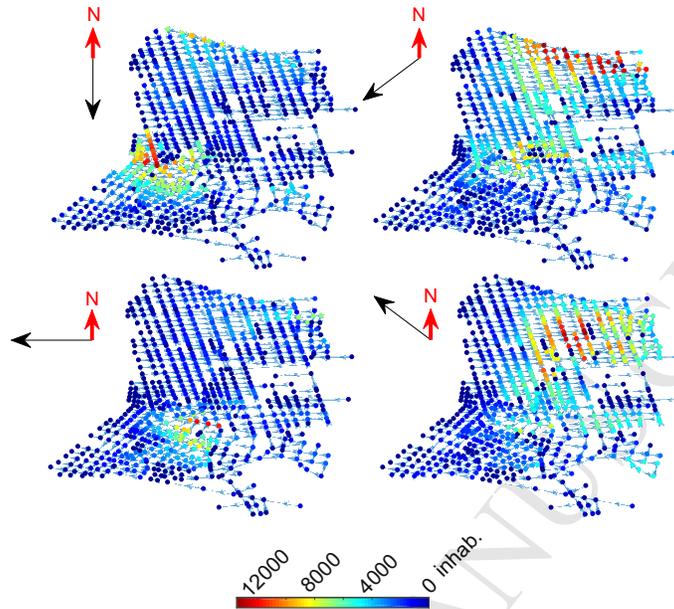


Figure 6: Vulnerability maps for different wind directions and with a concentration ratio c_0/c_{th} equal to 10.

539 network. Next, the vulnerability index defined in Equation (6) is calculated
 540 for each node, taking into account both the extent of the zone affected by the
 541 propagation originated in the node (\mathbf{D}) and the number of people leaving in that
 542 zone (\mathbf{P}). Finally, a vulnerability map is constructed by associating a colour to
 543 each node based on its vulnerability index.

544 Sixteen vulnerability maps for the case study area are obtained (see the
 545 supplementary material) by varying the direction of the external wind (Φ) and
 546 the concentration ratio c_0/c_{th} . These parameters affect the node vulnerability
 547 by shaping its zone of influence (matrix \mathbf{D}), as mentioned in Section 4.1. In
 548 this study, the number of inhabitants in the streets (\mathbf{P}) was kept constant in
 549 the different scenarios and was derived from the map of the resident citizens in
 550 the city of Lyon. Future works should consider how the spatial distribution of
 551 the population varies on different days of the week or at different times of the
 552 day. These variations could be easily implemented in the model by modifying
 553 the \mathbf{P} matrix and would increase the number of cases (i.e. maps) considered.

554 Fig. 6 shows four vulnerability maps for a wind direction varying between
 555 $\Phi = 0^\circ$ (north wind) and $\Phi = 135^\circ$. The maps are obtained simulating for
 556 each node a release with a concentration equal to ten times the threshold value,
 557 i.e. $c_0/c_{th} = 10$. Node vulnerability, defined in terms of number of affected
 558 people, varies between 0 (blue) and 12000 (red) inhabitants. The maps reveal
 559 at a glance the most susceptible zones and the global vulnerability of the urban
 560 area in the different wind direction scenarios. From Fig. 6, it can be seen that
 561 the vulnerability index is not distributed homogeneously. On the contrary, the
 562 vulnerability tends to be maximal (red nodes) in a defined area of the map and
 563 its value gradually decreases in the nodes around it. Moreover, the position
 564 of the most vulnerable nodes varies strongly with small variations in the wind
 565 direction. As an example, notice the difference between the two scenarios related
 566 to $\Phi = 0^\circ$ and $\Phi = 45^\circ$ in Fig. 6.

567 To better understand this behaviour, Fig. 7.a summaries the results of the
 568 sixteen simulated scenarios. Each scenario is represented by an arrow oriented
 569 with the wind direction, coloured according to the concentration ratio and po-
 570 sitioned in the area of greatest vulnerability for that scenario. It is evident that
 571 for Φ equal to 0° , 90° , 180° and 270° (the cardinal directions) the most vulner-
 572 able nodes are located in the southern part of the network, while for Φ equal to
 573 45° , 135° , 225° and 315° (the transversal directions) the most vulnerable areas
 574 are in the northern part of the network.

575 As the geometrical characteristics of the street canyons and the population
 576 density are rather homogeneously distributed in the street network, this differ-
 577 ent vulnerability pattern is related to the orientation of the streets with respect
 578 to the wind direction. In fact, the northern part of the street network is mainly
 579 oriented according to the cardinal directions (North-South and West-East ori-
 580 ented streets) and experiences the highest vulnerability when the wind blows
 581 according to one of the transversal directions. Conversely, the southern part
 582 of the street network is oriented mainly according to the transversal directions,
 583 and a greater vulnerability occurs when the wind is blowing in the cardinal di-
 584 rections. Thus, if the wind blows obliquely with respect to street orientations

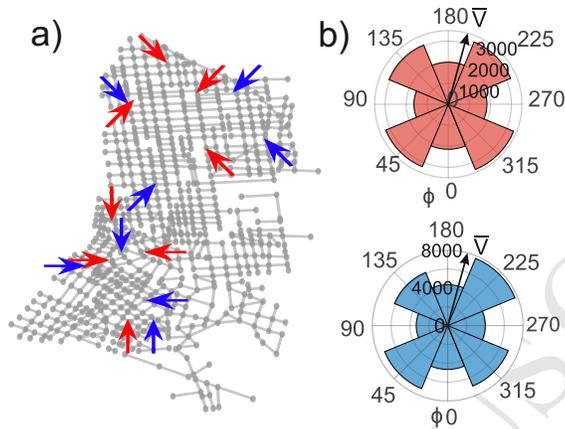


Figure 7: a) Most vulnerable areas in the street network for different wind directions and concentration ratios. Each scenario is represented by an arrow oriented according to the wind direction, coloured according to the concentration ratio (red arrows for $c_0/c_{th} = 10$ and blue arrows for $c_0/c_{th} = 100$) and positioned in the area of greatest vulnerability on the map. b) Polar histogram (red for $c_0/c_{th} = 10$ and blue for $c_0/c_{th} = 100$) of the average node vulnerability for the different wind directions

585 then vulnerability increases.

586 For each of the sixteen scenarios analysed, the average vulnerability over the
 587 entire network (\bar{V}) is calculated as

$$\bar{V} = \frac{1}{N} \sum_i^N V_i, \quad (12)$$

588 where V_i is the vulnerability index for the i -th node and N is the number of
 589 nodes in the network. The two polar histograms of Fig. 7.b report these average
 590 vulnerability values for the two concentration ratios considered. Each sector in
 591 the polar histograms refers to a different wind direction. As expected, higher
 592 concentration ratios correspond to higher vulnerability values. Moreover, since
 593 most of the streets ($\sim 60\%$) are oriented in the cardinal directions, the average
 594 vulnerability is greater when the wind blows in the transversal directions.

595 Fig. 8 gives an overview of the long-term vulnerability of the urban fabric
 596 to toxic releases. The maps depict nodes vulnerability weighted by the annual

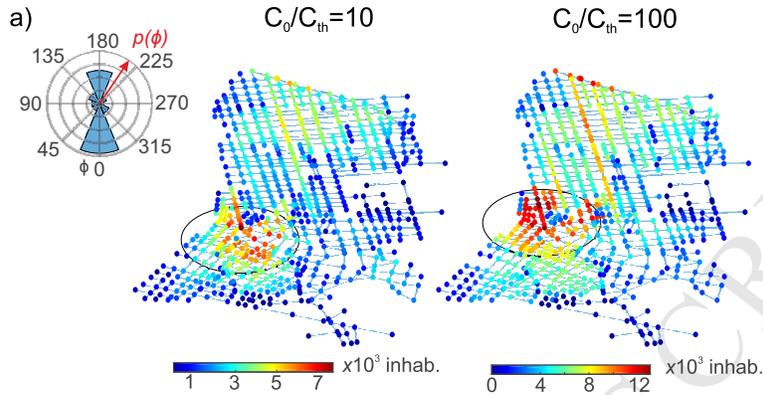


Figure 8: Maps of vulnerability weighted by the annual frequency of the wind directions. The polar histogram (inset *a*) shows the occurrence of wind directions in the city of Lyon in terms of annual relative frequency $p(\Phi)$.

597 frequency of the wind directions over the city of Lyon. As detailed in the inset of
 598 Fig. 8, the dominant wind direction in Lyon is North-South. As a consequence,
 599 the highest vulnerabilities are located in the circled area, corresponding to the
 600 most critical area when North-South oriented winds blow (see $\Phi = 0^\circ$ and
 601 $\Phi = 180^\circ$ scenarios in the supplementary material).

602 Results in Fig. 6 suggest that, for each scenario, a restricted area of the urban
 603 fabric is characterized by high levels of vulnerability, while most of the nodes
 604 have a low spreading potential. Thus, both spatial and frequency distribution of
 605 node vulnerabilities are not trivial. A frequency analysis of vulnerability values
 606 was performed to identify the statistical distribution of the data. For each
 607 scenario, node vulnerability values were classified in ten equal size intervals.
 608 Then, the relative frequency (p) of each class was calculated. Fig. 9 presents
 609 the relative frequency of node vulnerability for the different scenarios in a log-
 610 log plot. For the sake of graphic clarity, vulnerability values were normalised to
 611 the maximum one (V_{max}). Generally, the data show a linear behaviour in the
 612 log-log plot, thus exhibiting a power law trend. The power law confirms that
 613 vulnerability distribution is heterogeneous, with few nodes being much more
 614 critical than the others. This configuration suggests that, for each wind scenario,
 615 the entire neighbourhood could be protected with security interventions targeted

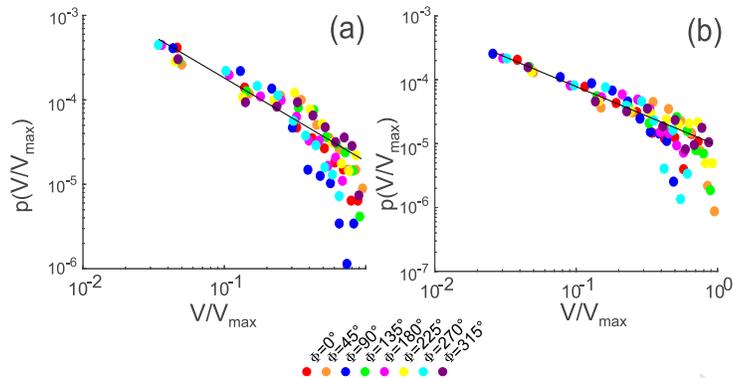


Figure 9: Relative frequency of vulnerability for different wind directions and for a concentration ratio $c_0/c_{th} = 10$ (a) and $c_0/c_{th} = 100$ (b). The display is in logarithmic scale.

616 on small urban areas. From the opposite point of view, it demonstrates that a
 617 malicious release of gaseous substances can have a much more dramatic effect
 618 if it occurs in points characterized by maximum vulnerability.

619 Notice that many real-world networks (e.g., the Internet, World Wide Web,
 620 scientific citations) present a power law distribution of nodes degree (Boccaletti
 621 et al., 2006). The degree defines the importance of a node in the network
 622 in terms of its connectivity. In this case, a power law results from the non-
 623 trivial interaction of multiple factors that define the vulnerability index, i.e.
 624 the topological connectivity of the network, the geometry characteristics of the
 625 street canyons, the toxic spreading process and the population distribution in
 626 the city.

627 5. Conclusions

628 We have presented a complex network approach for the study of diffusion
 629 processes in the urban atmosphere. Within the urban canopy, the wind flows
 630 along the street canyons and the transport of pollutants is strongly influenced
 631 by the structure of the city, i.e. by the orientation of the streets and by their
 632 interconnections. Based on these considerations, we have modelled the inter-
 633 action between the city and the external wind as a weighted complex network
 634 whose links and nodes represent the streets and the street intersections, respec-

635 tively. The direction of the links and their weights describe the direction and
636 the intensity of the wind along the streets and the geometrical properties of the
637 buildings that surround the street canyons. Using a depth-first search analysis,
638 we have implemented a spreading model on networks that simulates the propa-
639 gation process from a point source. As an example, the developed method has
640 been adopted to create vulnerability maps of the city of Lyon (France). These
641 maps highlight the most vulnerable areas of the city, i.e. the areas from which
642 the spread of a toxic substance, released at ground level, can harm more people.
643 We found that the spatial and frequency distribution of urban vulnerability is
644 heterogeneous and is strongly influenced by the alignment between the direction
645 of the external wind and the orientation of the streets.

646 The model proved to be fast-to-run and functional, and therefore useful for
647 the analysis of multiple scenarios that take into account various meteorological
648 conditions, or different distributions of the residing population, and also tem-
649 porary or permanent changes in the urban structure (e.g., the construction of a
650 new building or the raising of a temporary wall to close a street during a public
651 event). In particular, the computational time was estimated to be up to 3 orders
652 of magnitude lower than that of the well-known street network model SIRANE.
653 Thanks to these characteristics, this method is appropriate for the prediction
654 and management of emergency scenarios, due to accidental or harmful releases
655 of toxic substances in the urban atmosphere.

656 This work is in line with the efforts made in recent years (Hamlyn et al.,
657 2007; Soulhac et al., 2011; Belcher et al., 2015; Goulart et al., 2018) to develop
658 operational modelling tools for the prediction of pollutant transport in large
659 urban areas. Moreover, the study demonstrate that the theory of complex
660 networks can be adopted to this aim. The rate at which new techniques and
661 applications are developed in the field of complex networks (in particular, in the
662 field of epidemic spreading on networks) makes this new perspective particularly
663 promising.

664 Future work is aimed at comparing different cities to understand how urban
665 topology influences the dispersion and therefore the vulnerability of the city. In

666 this way, it will be possible to understand which cities are the most structurally
 667 fragile. Further analyses should be conducted to consider variations in urban
 668 vulnerability due to changes in the presence of people in the different times of
 669 the day or during the different periods of the week. Finally, the results of this
 670 proof of concept study encourage the use of metrics and tools from complex
 671 network theory (e.g., centrality metrics) to investigate the role of the different
 672 key variables involved in urban dispersion, in order to better understand the
 673 fundamental mechanisms that drive this phenomenon.

674 **Appendix A. Longitudinal wind velocity along a street canyon**

675 According to Soulhac et al. (2008), the average velocity u_{st} along the longi-
 676 tudinal axis of a street canyon reads

$$u_{st} = u_H \cos \Phi \frac{\delta_i^2}{hw} \left[\frac{2\sqrt{2}}{C} (1 - \beta) \left(1 - \frac{C^2}{3} + \frac{C^4}{45} \right) + \beta \frac{2\alpha - 3}{\alpha} + \left(\frac{w}{\delta_i} - 2 \right) \frac{\alpha - 1}{\alpha} \right] \quad (\text{A.1})$$

$$\text{with} = \begin{cases} \alpha = \ln \left(\frac{\delta_i}{z_{0,build}} \right) \\ \beta = \exp \left[\frac{C}{\sqrt{2}} \left(1 - \frac{h}{\delta_i} \right) \right] \\ u_H = u_* \sqrt{\frac{\pi}{\sqrt{2}k^2 C} \left[Y_0(C) - \frac{J_0(C)Y_1(C)}{J_1(C)} \right]} \\ C \text{ solution of } \frac{z_{0,build}}{\delta_i} = \frac{2}{C} \exp \left[\frac{\pi}{2} \frac{Y_1(C)}{J_1(C)} - \gamma \right] \\ \delta_i = \min \left(h, \frac{w}{2} \right) \end{cases} \quad (\text{A.2})$$

677 where Φ is the external wind direction with respect to the street longitudinal
 678 axis, h and w are the height and the width of the street canyon, $z_{0,build}$ is
 679 the aerodynamic roughness of the canyon walls, u_* is the friction velocity of the
 680 external atmospheric boundary layer flow, J_0 , J_1 , Y_0 and Y_1 are Bessel functions,
 681 k is the von Kármán constant, and γ is the Euler constant. The friction velocity
 682 u_* is determined using the Monin-Obukhov similarity theory to model the flow

683 in the external boundary layer.

684 **Appendix B. Solution for the one-dimensional transport equation**

685 Under the assumption that c_{ext} is negligible with respect to the concentration
686 in the street canyon, the one-dimensional transport equation (1) becomes

$$\frac{\partial c}{\partial t} + u_{st} \frac{\partial c}{\partial x} + \frac{u_d}{h} c = 0. \quad (\text{B.1})$$

687 By introducing the substitution

$$g(x, t) = c(x, t) \exp\left(\frac{u_d}{h} t\right), \quad (\text{B.2})$$

688 (B.1) yields

$$\frac{\partial g}{\partial t} + u_{st} \frac{\partial g}{\partial x} = 0. \quad (\text{B.3})$$

689 The general solution for (B.3) is found by introducing the new coordinates
690 $\tau = t$, $\xi = x - u_{st}t$ and using the chain rule:

$$\frac{\partial g(x, t)}{\partial t} = \frac{\partial g(\xi, \tau)}{\partial \tau} - u_{st} \frac{\partial g(\xi, \tau)}{\partial \xi} \quad (\text{B.4})$$

$$\frac{\partial g(x, t)}{\partial x} = \frac{\partial g(\xi, \tau)}{\partial \xi}. \quad (\text{B.5})$$

691 Equation (B.3) becomes

$$\frac{\partial g(\xi, \tau)}{\partial \tau} = 0, \quad (\text{B.6})$$

692 and, therefore,

$$g(\xi, \tau) = F(\xi) \quad \rightarrow \quad g(x, t) = F(x - u_{st}t), \quad (\text{B.7})$$

693 where F is a derivable function. The solution of (B.1) is obtained returning to
 694 the original function,

$$c(x, t) = g(x, t) \exp\left(-\frac{u_d}{h}t\right) = F(x - u_{st}t) \exp\left(-\frac{u_d}{h}t\right). \quad (\text{B.8})$$

695 Function F is found using the initial or boundary conditions of the problem.

696 For a continuous release c_0 in the source node starting from $t \geq 0$, the
 697 boundary condition is defined as

$$c(0, t) = c_0 \Theta(t), \quad (\text{B.9})$$

698 where Θ is the Heaviside function. For $x = 0$, (B.8) and (B.9) yield

$$c(0, t) = F(-u_{st}t) \exp\left(-\frac{u_d}{h}t\right) = c_0 \Theta(t). \quad (\text{B.10})$$

699 Function F is obtained from the previous one and the solution for the continuous
 700 release is thus

$$c(x, t) = F(x - u_{st}t) \exp\left(-\frac{u_d}{h}t\right) = c_0 \exp\left(-\frac{u_d}{u_{st}h}x\right) \Theta\left(t - \frac{x}{u_{st}}\right). \quad (\text{B.11})$$

701 For a quick release in the source node, the initial condition is set as a rect-
 702 angular pulse with width a and height c_0

$$c(x, 0) = c_0 [\Theta(x) - \Theta(x - a)]. \quad (\text{B.12})$$

703 When a tends to zero, the release is almost instantaneous. Following the same
 704 reasoning as in (B.10) and (B.11), but considering this time $t = 0$, we obtain
 705 the solution for a quick release:

$$c(x, t) = c_0 \exp\left(-\frac{u_d}{h}t\right) [\Theta(x - u_{st}t) - \Theta(x - u_{st}t - a)]. \quad (\text{B.13})$$

706 **Appendix C. Adjacency matrix**

707 Consider the network in Fig.1c, extracted as a subgraph from the network
 708 of streets in Fig.1a. The adjacency matrix of this simple four-links graph reads

$$\mathbf{A} = \begin{matrix} & \begin{matrix} s & g & i & j & k \end{matrix} \\ \begin{matrix} s \\ g \\ i \\ j \\ k \end{matrix} & \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}. \quad (\text{C.1})$$

709 Note that the matrix is asymmetric since the network is directed (i.e. the links
 710 have a specific direction).

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- A complex network approach to model dispersion in the urban atmosphere is presented
- Dispersion from a source is modelled as a spreading process on a complex network
- The polluted area and the number of people affected by the release are assessed
- Maps of urban vulnerability to the release of toxic gases are rapidly constructed
- The street topology of the city is a key driver in the dispersion process

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