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## Teaching geometry to students (from five to eight years old)

### “All that is curved and smooth is not a circle”

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*The ERMEL team (IFé, ENS-Lyon) in France, builds complete engineering to teach math in elementary school. For several years, we have been experimenting with teaching situations on space and geometric learning for 5-8 years old students. In this paper, we focus on the results about the relationship between actions on objects, graphs and first geometric properties about curved line. Our methodology involves an analysis of the student's way of solving problems and thus their abilities, but it also allows us to make a hypothesis on what it is that teachers need in order to carry these learning situations out.*

*Keywords: Geometry, teaching, learning, primary education, spatial ability.*

#### **Issue of this paper**

Our research takes place in the French context of geometry teaching in primary school. Our goal is to build a proven, complete and reliable teaching engineering and thus to improve geometry teaching.

In CERME 9, we reported our results about the knowledge of straight line for the same kind of students, not as an element of geometrical figure but as a usual or new component of the pupil's practice. Several point of view in the WG4 showed the similar approach and questions were shared:

- The relation between everyday and geometrical concept, perception, language and manipulation
- How to start with low level, and long planning time
- What kind of tools for the research and tools for teaching geometry are there? (How to help teacher to know what student are able to do)

In this paper, we aim to make our contribution to the three first topics of the WG4 group: what is doing, learning and teaching geometry at school?

#### **Why working on geometry teaching?**

A starting point of our research is a finding of deficiencies in the geometry teaching practices in the early grades of elementary school. A short analysis of easily available French resources for teaching geometry to young pupils shows that two mains goals are pursued: learning geometric words and drawing abilities. Spatial activities also exist but without much problem solving, and also unrelated to the pupil's initial knowledge or mathematical activity. This research is also based on the idea that students' abilities are insufficiently taken into account in geometry teaching in primary school. Thus we have to identify the knowledge at stake in this learning and take the students' already acquired knowledge into account.

## About our team

ERMEL is the research team on mathematics education in primary school (in French “Équipe de Recherche en Mathématiques à l’École Élémentaire”) belonging to the French institute of education (IFé). ERMEL team is made up of primary school teachers, teachers’ trainers and researchers working in different regions of France. Results of these researches lead to comprehensive books publications including complete teaching engineering. Studies on teaching and learning conducted since 1999 are about geometry teaching more precisely on the analysis of spatial and geometric skills that students from primary school to GS (5-6 years old) CE1 (7-8 years old) can build. Key issues of these researches are knowledge creation and resources production for teachers and teachers’ trainers as well as the study of the appropriation of these resources. In CERME 9 we have clarified our theoretical framework and the steps of our methodology: a didactic engineering, based on an experiment conducted in many classrooms for several years (Douaire & Emprin, 2015). In this paper, we try to explain the transition between space and graphic knowledge which is often underestimate.

## Purpose of the study

Our previous studies (Douaire & Emprin, 2015) showed that the knowledge developed by pupils in meso-space by solving problems does not necessarily build a geometric knowledge usable on paper. This lead us to ask many questions: in particular, what are the opportunities for pupils to understand the underlying patterns of drawings on a sheet of paper?

We have to clarify the relationship between two types of knowledge, spatial and geometric, and especially the discovery of the meanings of lines on a sheet of paper, and how these plots can provide information about objects in space. If several works address the distinction between drawing or figure at the beginning of the “college” (Parzysz, 1988) or the apprehension of the components of a figure at the end of primary school (Duval, 2005) (Perrin & al, 2013), our research concerns the emergence of graphic representations a few years earlier.

In this working group we will identify the contributions of space experiments in the construction of geometrical knowledge. Our goal is that pupils overcome the overall perception of a figure and develop the analysis of its components.

We analyze learning involved by a problem solving of closed curves figures construction, the characteristics of different implementations, as well as the needs of teachers. We present insights of learning situations for space experiments and we question the relevance of a resource based on the needs of teachers for its implementation identified in the context of ongoing Ermel research.

## Presentation of the experimentation

Experimentation concern procedures (graphics, practices, discourse ...) that can be developed by 7-year-olds pupils to distinguish circles from ellipses and other rounded shapes. We are not trying to develop early knowledge of the circle, but to promote the passage from a global perception of drawings and shapes to a geometric analysis of geometric shape underlying.

Some questions concern the comparison procedures: what are the abilities of students of CP (6 years old) or CE1 (7 years old) to distinguish a circle from another closed curve (an "almost circle")? What use of superposability as a validation procedure for that?

We present an experiment in progress: students have to produce closed rounded shapes, and must prove if they are different from others.

### Proposed situation

The problem is to build closed shapes using four circular or elliptic arcs (quarter of a big circle, little circle or an ellipse figure 1). Identifying that shapes built are different lead students to develop and formulate analytical geometric criteria.

Two major phases of this situation are analyzed successively, the first concerns the problem solving phase to produce shapes: we briefly analyze the productions. The second concern comparison of production. We also present some exchanges during the validation of the solutions.

Finally, a brief summary will address the explicit needs of teachers.

### Presentation of the activity

Each pair of students has a deck of 16 cards: 4 quarts of a small circle, 4 quarters of a large circle and 8 quarters of an ellipse (shapes figure 1 are cut following dotted lines like in figure 2). The major axis of the ellipse is the diameter of the large circle. The minor axis of the ellipse is the diameter of the small circle. Thus, shapes can be linked.

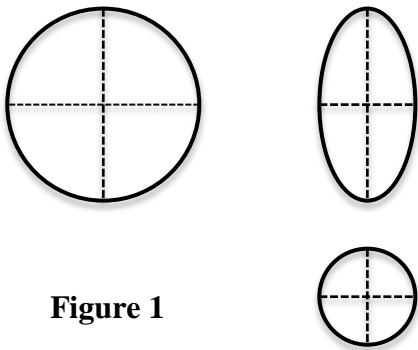


Figure 1

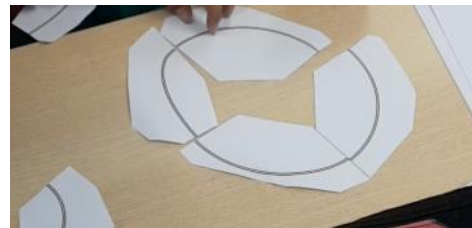


Figure 2

### Several successive phases in this situation:

- 1- To ensure the pupils' appropriation of the constraints of the problems they are experimented one by one. Each pupil must first assemble pieces to form a closed shape. Then they verify that these solutions are really closed. These discussions lead to clear assembly instructions: "Are only accepted shapes that are joined edge to edge (assembly like in figure 3 are rejected)". In this first phase, the students do not need to draw, but to assemble pieces of heavy paper. Then pupils have to make closed shapes using exactly four arcs. The findings, confirmed by teachers and observers in classes are that all students are aware of the goal, namely produce closed curves ("tracks") consisting of four arcs at the end of devolution phase. Students have understood the problem's rules.

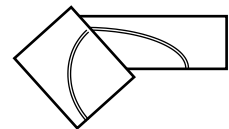


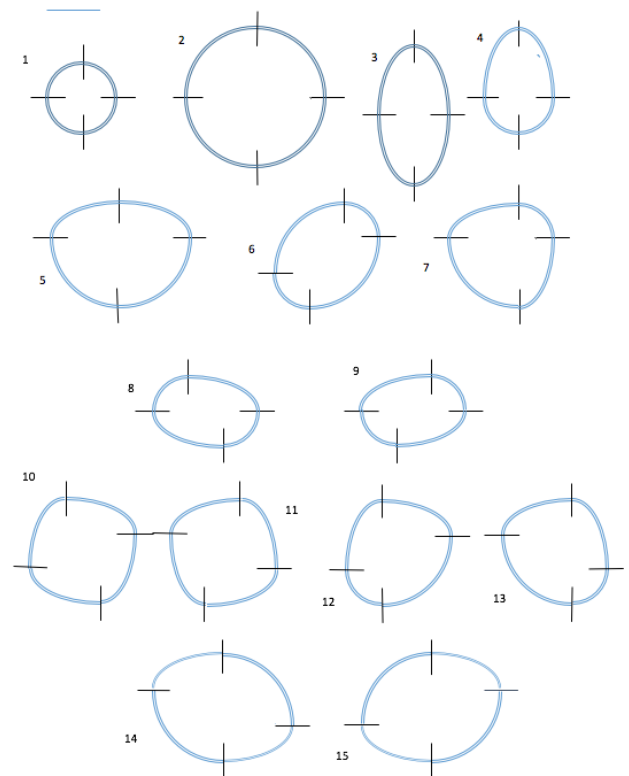
Figure 3

- 2- Then each student looks for new solutions. In order to save their shapes and be able to make new assembly pupils are asked to draw on tracing paper (or lite paper) the outline of each new shape found. When students believe they cannot find new shapes the search stops, and solutions are pooled: are they different? If a student thinks he has found another solution, it is displayed and compared with previous. Students explain why they think it is different or it's the same as another already displayed. The goals of the pooling are to identify products that meet or not the constraints and identify the identical solutions.

## Possible shapes

The solutions are:

- reconstruction of three basic shapes (large circle, small circle, ellipse) solutions 1-3
- combining two half ellipses contiguous or half a small circle (ovoid: solution 4) a large circle (such as "roly poly" or "roller" or solution ... 5). Radius or half axes being concurrent.
- combining alternative quarters of small circle and large circle (solution 6)
- combining two quarters of alternating ellipses with a quarter of a small circle and a quarter wide circle (solution 7)
- combining two quarter ellipses and a small circle (8, 9) or large circle (14,15)
- juxtaposing four quarter ellipses (10, 11)
- juxtaposing two quarters ellipse with on one side a quarter of large circle, and on the other side a quarter of small (12,13).



**Figure 4: 15 different shapes were found**

Of course, the goal of this situation is not to find all solutions but finding shapes from 1 to 5 with also one or two shapes they cannot name globally is enough for pupils to learn.

### Description of pupils' strategy to produce shapes

- choose pieces of random way;
- if unsuccessful start from scratch;
- put two pieces, then try the other two;
- replace one or two pieces in an assembly already achieved;
- place the fourth piece by estimating its size.

### Description of the comparison procedure

Solutions are shown on the blackboard (some of them may be identical but differently oriented).

We describe comparison strategy used by pupils:

1. Use of the overall look of the drawing (perceptual validation):
  - a. recognition of known shape (circle, round, egg ...);
  - b. rely on variables: overall size, width;
  - c. rely on differences of regularity in curves.
2. Identification of the elements that compose the shape (analytical aspect).
3. Recognition of identical shapes by rotation or reversal.
4. Use of construction processes, with the possibly of remaking to the class.
5. Use of symmetry properties of the shape, mention of the folding ...
6. Use of a practical validation by overlapping.

Strategies based on perception are meanly used.

Those results confirm that spatial abilities are often neglected and that it is a great challenge of our current research.

This pooling highlight some questions linked with the drawing:

- the impact of the thickness of the lines on strategy using perception,
- the acceptable tolerance to judge the compliance with constraint, for example the fact that the curve is closed.

The fact is that those questions emerge are important because it lead pupils to progressively give up perception in favor of the analysis of the shape. This dialogue during the validation of solutions illustrated this aspect.

### **Exchanges during the validation**

The solutions are exposed to the blackboard and students check if the set is suitable.

Student (S2) I believe we cannot do it ...

Teacher (T) The others, are you sure?

S2 appears to confirm

T This drawing there you think that doing it is impossible?

S2 show the drawing of the shape and try to express something.

T So, what could you do to know?

S2 Take the pieces

M So, the team who have made this track, you come back to the dashboard.

Resuming the question of drawings validity

S It looks like ...

T With four parts

S2 But then there are bumps

S ...but in all there are bumps ... because we do not succeed in drawing....

T Yes, there is the problem of drawing, we will not be going back to that. Is this circuit possible to do? ”

(Many S.) Yes

We can see in this transcript that pupils manage to move away from the drawing to analyze the shape. plutôt un”.” que “;”

After a pupil has the drawing of his shape on one hand and follows the lines of the set of pieces to check if the shapes are the same (we will show a video of this moment).

### **Analysis of learning at stake**

Firstly, the effects of the situation on pupils learning we can observe are:

- 1- A change in perception of the role of vocabulary:
  - a. The familiar vocabulary for describing the known forms, is not effective for others. Many different shapes can look like an egg ...
  - b. Since drawing on paper is not always successful students have to describe the shapes by analyzing the way they have been built: the arcs used and their sequence of use.
- 2- The progressive understanding of the role of sheet layouts to work on the lines (here curves, but straight in other situations)
- 3- A better knowledge of the circle, based on the development of procedures compared with other forms (Artigue & Robinet, 1982)
- 4- The practice of displacement to produce new solutions and to recognize identical figures arranged differently
- 5- Transition from a practical validation based on the superposition to a validation based on the analysis of the properties
- 6- The perseverance in research: the students have to rely on perseverance and go to the end of the task, and of course explain, justify, criticize, debate.

We think that, at the end of this situation, students are able to explain, with their words, several of their learnings.

Secondly, we have analyzed what are the difficulties, and what is possible to propose to pupils. We think, after several experiences, that the main obstacles are not epistemological; they do not come from the inability of students to analyze forms or working on their uncluttered representation. But it is rather didactic obstacles created by neglecting their knowledge and solving capacity. There is a necessary transition from a perceptive approach of the shapes to the analysis of their geometrical characteristics (size, composition, curvature...). In our progression of learning we also offer other situations which contribute to this passage from "global" (for instance, students perceive the regularity of form) to "analytic".

Thirdly we try to find condition of a real use in classrooms. For those of teachers who are not satisfied with their way of teaching geometry we think it is important to propose activities that are real problems (where pupils have to produce new strategies) and to clarify learnings at stake. But we think that it would be unrealistic for a teacher whose main objective would be to set a vocabulary to embrace the process. Potential learning is related to the possibility for the teacher to understand the issues: allow students to implement the different comparison procedures.

This workshop proposes to enlighten those issues.

In this presentation we have not detailed the many changes in the description of the situation, related to successive and necessary experiments for students to produce the specified procedures. On this aspect of the construction of a teaching situation, we simply discuss the question of the context: is it necessary for such young students to evoke a familiar context? The way quarter-circles or ellipses are drawn have also been chosen differently during the experiment: either simple lines (parts of the cited figures), double lines to evoke real-world objects: railway circuits. But does the latter choice provide a better understanding of the constraints (in particular the continuity of lines)? We currently believe that, on the contrary, this approach makes the problem harder to understand.

## Conclusions and prospects

Let us return briefly to the issues addressed:

On learning targeted: what learnings can be developed based on the perception (regularity of a shape) to contribute to the analysis of geometrical properties? We mentioned a shift from a parts assembly problem to a drawing problem (a graphical problem); to that extent, the sensitive space has changed. First it is the space of action on the shapes to assemble, which has been among the first personal procedures of the students in the resolution phase. During the pooling, pupils focus on curves continuity and sensitive space become the place where graphical plots are questioned: they are new geometrical objects.

On taking initial knowledge of students into account: how knowledge, language, gesture, participate in the apprehension of the common elements to the diverse types of spaces?

How are first procedures and knowledge of the "graphical-space" combined with previous experience on objects? In particular, this prior knowledge is not primarily "declarative" but relies on gestures (eg the difference between rotate and reverse or the use of drawing with instruments without necessarily aiming to represent a geometric object ...)

They are also expressed in the language forms, as part of a language used by the student to control his actions or to communicate about a production (formulation of a procedure, checking of constraints validation of a solution ...). The importance of this learning is often underestimated in education in favor of the use of stereotyped and offline vocabulary.

Our research aims to analyze and develop, not only in this specific example, general student abilities via experience and actions on objects with graphical plots. Thus they evolve from a spatial perception to geometric characteristics of the shapes. Our concerns are also those of Swoboda (2015) about problem solving mainly with older students: "Therefore, the problem of bringing students to the ability of making mental transformations I treat as an educational task. In the literature, there is no explicit opinion on what educational level there is possible to create such skills. »



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